Economic analysis inversion mechanism taking into account argument inerrelation

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Abstract — The article deals with solving inverse economic analysis problems using inverse calculations where there is dependence between the arguments of the function. It offers a solving algorithm for an inverse problem with stochastic dependence between arguments, which includes the optimal solution. It provides a description of the program for solving a problem of fast food restaurant’s profit generation.

Keywords — economic analysis, inverse calculations, regression, contribution margin

I. INTRODUCTION

The concept of the inverse problem was first used by A.N. Tikhonov, who defined the content of such problems as the recovery of unknown values by a given consequence [1]. Inverse problems in various statements became widely used in economics [2–4]. B.E. Odintsov offered an inverse calculation instrument for solving the following inverse economic problems: determination of the increments of the function arguments based on initial values of the arguments, given value of the function and expert information, by way of indicator trends and relative importance coefficients.

In case of a two-argument function \( y = f(x_1, x_2) \), solving an inverse problem by inverse calculations involves solving the following simultaneous equations [5]:

\[
\begin{align*}
y \pm \Delta y &= f(x_1 \pm \Delta x_1, x_2 \pm \Delta x_2) \\
\Delta x_1/\Delta x_2 &= \alpha/\beta; \\
\alpha + \beta &= 1;
\end{align*}
\]

where \( y \) is the initial value of the function;

\( x_1, x_2 \) are the initial values of the arguments;

\( \Delta x_1, \Delta x_2 \) are argument increments;

\( \alpha, \beta \) are argument priority coefficients, \( x_1, x_2 \) respectively.

The sign before the increment shows the indicator trend: increase ("+"), decrease ("-").

Solution to a problem allows to determine how to achieve the desired performance of an economic entity. The obtained information can be used in management decision making. By changing expert information, it is possible to consider various options for achieving the goal.

If any restrictions are imposed on the argument values, which may be caused, for example, by company’s limited resources, then finding the solution is reduced to multiple-solving simultaneous equations (1), while changing the resulting indicator by a small value until the restrictions are violated. Further, the work of the algorithm either ends, or the function changes due to other arguments (in case of their interchangeability).

B.E. Odintsov also considered solving the inverse problem taking into account the “golden proportions” of enterprise performance (part-whole ratio is equal to 1.618). For this purpose, the solutions are sequentially adjusted with the specified targets and taking into account the “golden proportions”, depending on the priority of the ways to achieve the goal.

II. INVERSE SOLUTION IN CASE OF A DETERMINISTIC DEPENDENCE BETWEEN THE FUNCTION ARGUMENTS

Expert information (relative importance coefficients and indicator trends) determines the dependence between the arguments of the function. In the modified inverse calculations method, a linear connection is built between the arguments \( (a, b) \) are the parameters):

\[
x_1 = a + bx_2.
\]

Equation parameters are determined by the type of dependence between the arguments (direct or inverse), which corresponds to the indicator trend in the classical inverse calculations method.

If there is a given deterministic dependence between the arguments, simultaneous equations will be written as:
\( x_2 + \Delta x_2 = f(x_1 + \Delta x_1); \) \hspace{1cm} (3)

\[(x_1 + \Delta x_1) (x_2 + \Delta x_2) = y \pm \Delta y. \] \hspace{1cm} (4)

In other words, instead of the importance coefficients, it is necessary to indicate the equation of dependence between the function arguments.

Let’s consider the revenue generation problem:

\[ r = p \cdot c \] \hspace{1cm} (5)

where \( r \) is revenue;
\( p \) is quantity;
\( c \) is price.

Suppose there is the following hyperbolic dependence between the arguments (Fig. 1):

\[ p = 10 + 10/c. \] \hspace{1cm} (6)

Initial data: \( r = 50 \) standard monetary unit, \( p = 12.5 \) standard units, \( c = 4 \) standard units. It is necessary to determine the values of price and quantity, which will provide the revenue value of 100.

Figure 1 shows 50 and 100 level lines. This means that any point of the given graph, formed by the quantity and price values, will provide the corresponding revenue value (50 or 100). Therefore, the solution of the inverse problem with the deterministic dependence between the function arguments is reduced to finding the point of intersection of the function argument dependence graph with the line of a given level.

Simultaneous equations for solving the problem are written as:

\[ p + \Delta p = 10 + 10/(c + \Delta c); \] \hspace{1cm} (7)

\[ (p + \Delta p) (c + \Delta c) = 100. \]

The following values will be the solution of simultaneous equations (point B, Fig. 1): \( \Delta c = 5, \Delta p = -1.3889, c = 9, p = 1.1111. \)

In the absence of a solution (there is no point of intersection of the argument dependence graph with the line of a given level), point \( x_1^* \) can be found where the distance between the argument dependence function \((x_2'(x_1^*)) \) and the level line \( x_2(x_1^*) \) is minimal:

\[ (x_2(x_1^*) - x_2'(x_1^*))^2 \rightarrow \text{min.} \] \hspace{1cm} (8)

The obtained value \( x_1^* \) suggests that:

\[ x_2^*(x_1^*) \] is the value of the argument, which, under the existing dependence, provides the solution closest to the target.

\[ x_2(x_1^*) \] is the value of the argument, which, when the target is satisfied, is closest to the existing dependence.

![Hyperbolic dependence between the function arguments](image)

**Fig. 1.** Hyperbolic dependence between the function arguments

III. **Inverse Solution in Case of a Stochastic Dependence Between the Function Arguments**

Let’s consider a modification of the inverse calculation instrument to take the stochastic dependence between the function arguments into account. To do that, the argument equation is build: statistics data for the previous periods is collected and \( \theta_0 \) and \( \theta_1 \) parameters are determined by the least square method. In case of a linear dependence, the dependence formula will be as follows:

\[ x_2 = \theta_0 + \theta_1 x_1, \] \hspace{1cm} (9)

where \( x_2 \) is the dependent variable;
\( x_1 \) is the independent variable.

Upon the given value of the explanatory variable, the value of the explained variable can belong to an interval. The lower and upper boundaries of such a predictive interval are determined by:

\[ l_i = x_2 + s \cdot t_{\alpha}, \] \hspace{1cm} (10)

\[ l_b = x_2 - s \cdot t_{\alpha}, \]

where \( l_i \) is the upper interval boundary;
\( l_b \) is the lower interval boundary;
s is the standard deviation of the individual value of the dependent variable forecast error;

\(x_2\) is the point estimation of the dependent variable forecast;

\(t_{\alpha}\) is the table value of the t-test at \(\alpha\) significance level.

Solution, taking into account the possible values of the explained variable by way of one of the function arguments \(y = f(x_1, x_2)\), is determined by the following steps:

Step 1. Using the initial \(x_1^0, x_2^0\) data as the starting point, find \(x_1^1, x_2^1\) point to reach the specified \(y^1\) function value. The point can be found, for example, by specifying the relative importance coefficients (1), or by minimizing the argument increments.

Step 2. Build the predictive interval for \(x_2^1\) with \(x_1 = x_1^1\). If \(x_2^1\) belongs to the obtained interval, the solution is considered found and the algorithm work is completed, otherwise we move to step 3 (\(x_1^1 = x_1^1\)).

Step 3. Change \(x_1^1\) value by the specified step \(h\), determine the corresponding \(x_2^*\) value (expressed in terms of \(f(x_1, x_2)\) function), the upper \(l_2\) and the lower \(l_2\) boundaries of the predictive interval for \(x_2^*\).

Step 4. Checking the break condition: if \(x_1^1 \geq x_{\max}\), the value enumeration stops and we move to step 5, otherwise we move to step 3 (with a negative step, the minimum \(x_{\min}\) value is set).

Step 5. \(x_1^1\) point selection by solving the optimization problem:

\[
\begin{align*}
\min f(x_1^1) &= k_1 \gamma_{\text{norm}}(x_1^1) + k_2 \lambda_{\text{norm}}(x_1^1), \\
\gamma(x_1^1) &= (x_1^* - x_1^1)^2 + (x_2^* - x_2^1)^2, \\
\lambda(x_1^*) &= \frac{\gamma(x_1^1) - \gamma(x_1^*)}{\gamma(x_1^1)}, \\
\gamma_{\text{norm}}(x_1^1) &= \frac{\min(\gamma(x_1^1))}{\max(\gamma(x_1^1))}, \\
\lambda_{\text{norm}}(x_1^1) &= \frac{\min(\lambda(x_1^1))}{\max(\lambda(x_1^1))},
\end{align*}
\]

where \(k_i\) is \(i\) indicator importance coefficient (is equal to zero if the indicator is disregarded);

\(\gamma(x_1^1)\) is the location indicator relative to the initial solution;

\(\lambda(x_1^1)\) is the location indicator relative to the midpoint of the predictive interval value;

\(\gamma_{\text{norm}}(x_1^1), \lambda_{\text{norm}}(x_1^1)\) are standardized value of \(\gamma(x_1^1)\) and \(\lambda(x_1^1)\) indicators, respectively.

The objective function has two parts. \((x_1^1 - x_1^1)^2 + (x_2^* - x_2^1)^2\) value characterizes the remoteness from the solution found at step 1 corresponding to the given targets. At constant boundaries \((l_i - x_2(x_1^*)) (x_2(x_1^*) - l_2)\) / \((l_i - l_2)\), expression will take the maximum value when \(x_2(x_1^*)\) value is in the middle of the predictive interval (corresponds to the point estimation). The value is standardized according to the size of the predictive interval \((l_i - l_2)\) difference in the denominator is the size of predictive interval. Also, a penalty for increasing the predictive interval value may be introduced. In this case, the expression must be multiplied by the interval value and divided by the value of the maximum interval.

Table 1 shows a problem solution example. The starting point \(x_1^1 = 5, x_2^1 = 10\) (point A), solution found at the first step: \(x_1^1 = 8.215, x_2^1 = 12.172 (x_2 = 100 / x_1)\), the step is 0.5, the maximum value \(x_{\max} = 0.937 (\alpha = 0.05, k_1 = 0.3, k_2 = 0.7)\).

<table>
<thead>
<tr>
<th>(x_1^*)</th>
<th>(x_2^*)</th>
<th>(l_2)</th>
<th>(l_1)</th>
<th>(\gamma(x_1^*))</th>
<th>(\lambda(x_1^*))</th>
<th>(\lambda_{\text{norm}}(x_1^*))</th>
<th>(\gamma_{\text{norm}}(x_1^*))</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.215</td>
<td>12.172</td>
<td>10.072</td>
<td>12.072</td>
<td>0</td>
<td>0.105</td>
<td>0.489</td>
<td>0.269</td>
</tr>
<tr>
<td>8.715</td>
<td>11.474</td>
<td>10.238</td>
<td>12.338</td>
<td>0.737</td>
<td>-0.472</td>
<td>0.962</td>
<td>0.318</td>
</tr>
<tr>
<td>9.215</td>
<td>10.852</td>
<td>10.405</td>
<td>12.405</td>
<td>2.742</td>
<td>-0.347</td>
<td>0.747</td>
<td>0.000</td>
</tr>
<tr>
<td>9.715</td>
<td>10.293</td>
<td>10.572</td>
<td>12.572</td>
<td>5.781</td>
<td>0.318</td>
<td>0.000</td>
<td></td>
</tr>
</tbody>
</table>

The increments in the first step are determined using the following formula:

\[
\Delta x_1 = x_1^* - x_1^1 = 8.215 - 8.215 = 0, \quad \Delta x_2 = x_2^* - x_2^1 = 12.172 - 12.172 = 0.
\]

Finally, the objective function is:

\[
\gamma(8.215) = 0^2 + 0^2 = 0; \\
\gamma_{\text{norm}}(8.215) = (0.737 - 0) / (0.737 - 0) = 1, \\
\lambda(8.215) = - (12.072 - 12.172) (12.172 - 10.072) / (12.072 - 10.072) = 0.105, \\
\lambda_{\text{norm}}(8.215) = (0.318 - 0.105) / (0.318 - (-0.472)) = 0.269, \\
f(8.215) = 0.31 + 0.7 	imes 0.269 = 0.489.
\]

According to the obtained results, the solution to the problem with the highest value of objective function will be: \(x_1^1 = 8.715, x_2^1 = 11.474\) (Fig. 2, point B).
The main problem in forecasting demand is that a set of price variations is usually limited. A subsequent price increase after its reduction can be negatively perceived by the buyers and treated as overpayment and foregone benefits. This makes it impossible to set prices flexibly and monitor changes in demand.

Let’s consider the contribution margin generation problem (M); the contribution margin can be defined as the difference between revenue (Revenue) and costs (Costs): 

\[ M = \text{Revenue} - \text{Costs} = \text{Price} \cdot \text{Num} - \text{Num} \cdot \text{C}_{\text{unit}}. \]  

(13)

where Price is the unit price; Num is the quantity of product sold; C_{unit} is the cost-per-unit.

The cost of the product in question is RUB 19, the price is RUB 100. Then the revenue-costs dependence will be as follows:

\[ \text{Revenue} = \frac{\text{Price} \cdot \text{Costs}}{\text{C}_{\text{unit}}} = 5.263 \cdot \text{Costs} \]  

(14)

Initial values of revenue and costs (per week): \( \text{Revenue} = \text{RUB 3,300}, \text{Costs} = \text{RUB 627} \).

The profit is RUB 2,673. It is necessary to determine the amounts of revenue and costs ensuring a profit of RUB 4,000.

Simultaneous equations are written as:

\[ \Delta \text{Revenue} / \Delta \text{Costs} = 5.263; \]

\[ (\text{Revenue} + \Delta \text{Revenue}) - (\text{Costs} + \Delta \text{Costs}) = 4000. \]

Simultaneous equations solution: \( \Delta \text{Revenue} = 1,638, \Delta \text{Costs} = 311.28 \).

Figure 3 shows the graphical interpretation of the problem: Revenue0(Costs) is the line of initial profit level (RUB 2,673), Revenue1(Costs) is the line of a given profit level (RUB 4,000), Revenue(Costs) is the revenue-costs dependence (2).

Let’s consider next determination of the price and quantity of products sold to achieve the given value of the contribution margin.
The initial value of profit is RUB 4,018, the price is RUB 60, the quantity is 98 pcs. (the cost-per-unit is also RUB 19). It is necessary to determine the values of the price and quantity so that the profit would be equal to RUB 4,900.

The formula for making a level line corresponding to the initial value of profit:

\[ \text{Num0} = \frac{\text{Price}}{4018} \]

The level line corresponding to the given value of profit (RUB 4,200) is made using the following function:

\[ \text{Num1} = \frac{\text{Price}}{4900} \]

The least squares method was used to build the quantity-price equation (data for 2.5 months). The regression equation obtained in case of linear dependence:

\[ y = 148.2 - 1.15 \cdot x \]

The regression coefficients are significant at the 99% interval (the intersection and slope errors are 27.5 and 0.3 respectively).

The price and quantity values determined at the first step of the algorithm described above are 66 and 104, respectively. It is necessary to consider the price values to RUB 60, with a step of RUB 5 and choose the optimal value (\( x_1 = 0.05, k_1 = 0.5, k_2 = 0.5 \)).

Calculation results are provided in Table 2. It can be seen that the optimal problem solution, at which the given value of profit will be achieved, is: Price=RUB 76, Num=86 pcs.

**V. PROGRAM DESCRIPTION**

Based on the developed algorithm, a program was written in Java in IntelliJ IDEA development environment. To run the program on the computer, you must have Java Runtime Environment 1.7 and higher.

The program has a graphical interface, using which the user enters the input data, and then the computation process starts. If the program is successfully executed, output.xlsx Excel file will be generated with the calculation formulas, values computed in the program, as well as the input data and cell description. In this case, there will be a check for incorrect values entered by the user, that can cause erroneous program operation. If any erroneously entered values are found, the computation process will stop, and the user will be notified of the reason for the error with a message.

The program consists of two layers:

- user interface layer;
- work with Excel layer.

**TABLE II. THE OPTIMAL PROBLEM SOLUTION**

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( h_0 )</th>
<th>( h_1 )</th>
<th>( y(x_1) )</th>
<th>( a(x_1) )</th>
<th>( f(x_1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>66.00</td>
<td>104.00</td>
<td>43.60</td>
<td>101.00</td>
<td>0</td>
<td>3.152</td>
<td>0.618</td>
</tr>
<tr>
<td>71.00</td>
<td>94.00</td>
<td>39.75</td>
<td>93.35</td>
<td>125</td>
<td>0.658</td>
<td>0.891</td>
</tr>
<tr>
<td>76.00</td>
<td>86.00</td>
<td>35.56</td>
<td>86.04</td>
<td>424</td>
<td>-0.042</td>
<td>0.934</td>
</tr>
<tr>
<td>81.00</td>
<td>79.00</td>
<td>30.95</td>
<td>79.15</td>
<td>850</td>
<td>-0.145</td>
<td>0.891</td>
</tr>
<tr>
<td>86.00</td>
<td>73.00</td>
<td>25.88</td>
<td>72.72</td>
<td>1361</td>
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<td>0.777</td>
</tr>
<tr>
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<td>20.28</td>
<td>66.82</td>
<td>1921</td>
<td>1.215</td>
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</tr>
<tr>
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<td>64.00</td>
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<td>60.00</td>
<td>7.55</td>
<td>56.55</td>
<td>3161</td>
<td>3.693</td>
<td>0.151</td>
</tr>
</tbody>
</table>

**Fig. 4. Level lines and quantity-price regression dependence**

Figure 4 shows the level lines providing the initial and given values of profit (\( \text{Num0(Price)}, \text{Num1(Price)} \)) and the quantity-price dependence graph (\( \text{Num(Price)} \)).
The second layer contains the basic computing algorithms. To work with Excel document, the layer uses Apache-POI library allowing to perform such operations as:

- create/edit/delete worksheets;
- record values and formulas into cells;
- generate function results;
- set cell styles, etc.

The name of the output document and the model with the input data are transferred to this layer. After this, the output document is filled with the input data, corresponding formulas and cell description. UtilsImpl (filling table cells), Constants (defining cell calculation formulas and text data), MySet (defining a data list model) classes are designed for working with an Excel file.

Output document contains 3 sheets: Inverse Problem, Solution and Predictive Interval. The sheets are generated using ExcelWriterActivator, LinearData, and LinearPredictive classes.

The Inverse Problem sheet contains the inverse problem solving result (step 1 of the algorithm), while the price and quantity values are rounded so that the resulting profit value has a minimum deviation from the set value. Solution sheet (Fig. 8) includes a table with the price (changed at a given step) and quantity, interval boundaries, location indicators relative to the boundaries of the predictive interval and the initial solution, the objective function values. The price corresponding to the largest value of the objective function is marked as a solution to the problem. The last Predictive Interval sheet is auxiliary and designed for calculating the predictive interval boundaries at a given value of the explanatory variable (price).

VI. CONCLUSION

The article deals with solving inverse economic analysis problems where there is dependence between the arguments of the function. It offers a solving algorithm for stochastic dependence between the function arguments. The algorithm is based on calculating the remoteness from the initial solution indicator and the location characteristic relative to the midpoint of the predictive interval. By way of example, the problem of fast food restaurant’s contribution margin generation was considered: determining the necessary values of revenue and costs, the price and quantity of the goods sold.
References


