ATLANTIS PRESS
Advances in Economics, Business and Management Research, volume 81
1st International Scientific Conference "Modern Management Trends and the Digital Economy: from Regional Development to Global Economic Growth" (MTDE 2019)

Economic analysis inversion mechanism taking into account argument inerrelation

Gribanova E.B.
Department of control systems
Tomsk state university of control systems and radioelectronics
Tomsk, Russia
katag@yandex.ru

Logvin I.N.
Department of control systems
Tomsk state university of control systems and radioelectronics
Tomsk, Russia
igor31821788@gmail.com

***Abstract* — The article deals with solving inverse economic analysis problems using inverse calculations where there is dependence between the arguments of the function. It offers a solving algorithm for an inverse problem with stochastic dependence between arguments, which includes the optimal solution. It provides a description of the program for solving a problem of fast food restaurant's profit generation.**

Keywords* — *economic analysis, inverse calculations, regression, contribution margin

I. Introduction

The concept of the inverse problem was first used by A.N. Tikhonov, who defined the content of such problems as the recovery of unknown values by a given consequence [1]. Inverse problems in various statements became widely used in economics [2–4]. B.E. Odintsov offered an inverse calculation instrument for solving the following inverse economic problems: determination of the increments of the function arguments based on initial values of the arguments, given value of the function and expert information, by way of indicator trends and relative importance coefficients.

In case of a two-argument function ($y = f(x_1,x_2)$), solving an inverse problem by inverse calculations involves solving the following simultaneous equations [5]:

$$y \pm \Delta y = f(x_1 \pm \Delta x_1(\alpha), x_2 \pm \Delta x_2(\beta));$$

$$\Delta x_1/\Delta x_2 = \alpha / \beta; \qquad (1)$$

$$\alpha + \beta = 1;$$

where y is the initial value of the function;

x_1, x_2 are the initial values of the arguments;

Δx_1, Δx_2 are argument increments;

α, β are argument priority coefficients, x_1, x_2 respectively.

The sign before the increment shows the indicator trend: increase ("+"), decrease ("–").

Solution to a problem allows to determine how to achieve the desired performance of an economic entity. The obtained information can be used in management decision making. By changing expert information, it is possible to consider various options for achieving the goal.

If any restrictions are imposed on the argument values, which may be caused, for example, by company's limited resources, then finding the solution is reduced to multiple-solving simultaneous equations (1), while changing the resulting indicator by a small value until the restrictions are violated. Further, the work of the algorithm either ends, or the function changes due to other arguments (in case of their interchangeability).

B.E. Odintsov also considered solving the inverse problem taking into account the "golden proportions" of enterprise performance (part-whole ratio is equal to 1.618). For this purpose, the solutions are sequentially adjusted with the specified targets and taking into account the "golden proportions", depending on the priority of the ways to achieve the goal.

II. Inverse Solution in case of a Deterministic Dependence between the Function Arguments

Expert information (relative importance coefficients and indicator trends) determines the dependence between the arguments of the function. In the modified inverse calculations method, a linear connection is built between the arguments (a, b are the parameters):

$$x_1 = a + bx_2. \qquad (2)$$

Equation parameters are determined by the type of dependence between the arguments (direct or inverse), which corresponds to the indicator trend in the classical inverse calculations method.

If there is a given deterministic dependence between the arguments, simultaneous equations will be written as:

$$x_2 + \Delta x_2 = f\,(x_1 \pm \Delta x_1); \tag{3}$$

$$(x_1 + \Delta x_1)\,(x_2 + \Delta x_2) \;=\; y \pm \Delta y. \tag{4}$$

In other words, instead of the importance coefficients, it is necessary to indicate the equation of dependence between the function arguments.

Let's consider the revenue generation problem:

$$r = p \cdot c \tag{5}$$

where r is revenue;

p is quantity;

c is price.

Suppose there is the following hyperbolic dependence between the arguments (Fig. 1):

$$p = 10+10 / c. \tag{6}$$

Initial data: r = 50 standard monetary unit, p = 12.5 standard units, c = 4 standard units. It is necessary to determine the values of price and quantity, which will provide the revenue value of 100.

Figure 1 shows 50 and 100 level lines. This means that any point of the given graph, formed by the quantity and price values, will provide the corresponding revenue value (50 or 100). Therefore, the solution of the inverse problem with the deterministic dependence between the function arguments is reduced to finding the point of intersection of the function argument dependence graph with the line of a given level.

Simultaneous equations for solving the problem are written as:

$$p + \Delta p = 10+10 / (c + \Delta c); \tag{7}$$

$$(p + \Delta p)\,(c + \Delta c) = 100.$$

The following values will be the solution of simultaneous equations (point B, Fig. 1): Δc = 5, Δp = –1.3889, c = 9, p = 1.1111.

In the absence of a solution (there is no point of intersection of the argument dependence graph with the line of a given level), point x_1^* can be found where the distance between the argument dependence function ($x_2^*(x_1^*)$) and the level line $x_2(x_1^*)$ is minimal:

$$(x_2(x_1^*) - \; x_2^*(x_1^*))^2 \rightarrow \min. \tag{8}$$

The obtained value x_1* suggests that:

$x_2^*(x_1^*)$ is the value of the argument, which, under the existing dependence, provides the solution closest to the target.

$x_2(x_1^*)$ is the value of the argument, which, when the target is satisfied, is closest to the existing dependence.

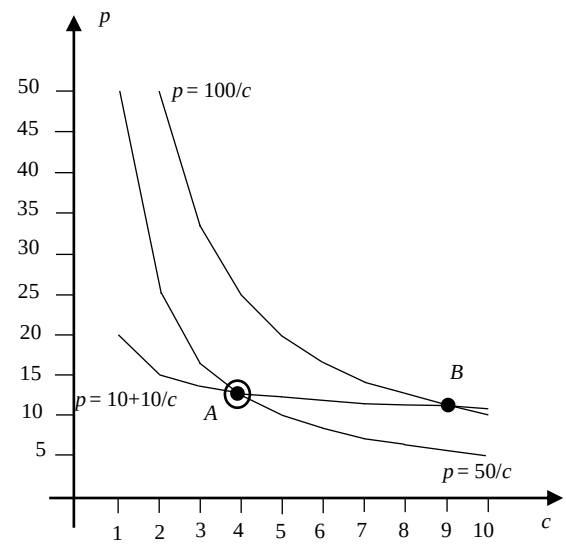


Fig. 1. Hyperbolic dependence between the function arguments

III. Inverse Solution in case of a Stochastic Dependence between the Function Arguments

Let's consider a modification of the inverse calculation instrument to take the stochastic dependence between the function arguments into account. To do that, the argument equation is built: statistics data for the previous periods is collected and θ_0 and θ_1 parameters are determined by the least square method. In case of a linear dependence, the dependence formula will be as follows:

$$x_2 \;=\; \theta_0 + \theta_1 x_1, \tag{9}$$

where x_2 is the dependent variable;

x_1 is the independent variable.

Upon the given value of the explanatory variable, the value of the explained variable can belong to an interval. The lower and upper boundaries of such a predictive interval are determined by:

$$l_t \;=\; x_2 + s \cdot t_\alpha,$$

$$l_b \;=\; x_2 - s \cdot t_\alpha, \tag{10}$$

where l_t is the upper interval boundary;

l_b is the lower interval boundary;

s is the standard deviation of the individual value of the dependent variable forecast error;

x_2 is the point estimation of the dependent variable forecast;

t_α is the table value of the t-test at α significance level.

Solution, taking into account the possible values of the explained variable by way of one of the function arguments $y = f(x_1, x_2)$, is determined by the following steps:

Step 1. Using the initial x_1^0, x_2^0 data as the starting point, find x_1^1, x_2^1 point to reach the specified y^1 function value. The point can be found, for example, by specifying the relative importance coefficients (1), or by minimizing the argument increments.

Step 2. Build the predictive interval for x_2^1 with $x_1 = x_1^1$. If x_2^1 belongs to the obtained interval, the solution is considered found and the algorithm work is completed, otherwise we move to step 3 ($x_1^* = x_1^1$).

Step 3. Change x_1^* value by the specified step h. Determine the corresponding x_2^* value (expressed in terms of $f(x_1, x_2)$ function), the upper l_t and the lower l_b boundaries of the predictive interval for x_2^*.

Step 4. Checking the break condition: if $x_1^* \geq x_{max}$, the value enumeration stops and we move to step 5, otherwise we move to step 3 (with a negative step, the minimum x_{min} value is set).

Step 5. x_1^* point selection by solving the optimization problem:

$$f(x_1^*) = k_1\gamma_{norm}(x_1^*) + k_2\lambda_{norm}(x_1^*) \rightarrow \min, \tag{11}$$

$$\gamma(x_1^*) = (x_1^* - x_1^1)^2 + (x_2(x_1^*) - x_2^1)^2,$$

$$\gamma_{norm}(x_1^*) = (\max(\gamma(x_1^*)) - \gamma(x_1^*)) / (\max(\gamma(x_1^*)) - \min(\gamma(x_1^*))), \tag{12}$$

$$\lambda(x_1^*) = \square(l_t - x_2(x_1^*))\,(x_2(x_1^*) - l_b) / (l_t - l_b),$$

$$\lambda_{norm}(x_1^*) = (\max(\lambda(x_1^*)) - \lambda(x_1^*)) / (\max(\lambda(x_1^*)) - \min(\lambda(x_1^*))),$$

where k_i is i indicator importance coefficient (is equal to zero if the indicator is disregarded);

$\gamma(x_1^*)$ is the location indicator relative to the initial solution;

$\lambda(x_1^*)$ is the location indicator relative to the midpoint of the predictive interval value;

$\gamma_{norm}(x_1^*)$, $\lambda_{norm}(x_1^*)$ are standardized value of $\gamma(x_1^*)$ and $\lambda(x_1^*)$ indicators, respectively.

The objective function has two parts. $((x_1^* - x_1^1)^2 + (x_2(x_1^*) - x_2^1)^2$ value characterizes the remoteness from the solution found at step 1 corresponding to the given targets. At constant boundaries $(l_t - x_2(x_1^*))\,(x_2(x_1^*) - l_b) / (l_t - l_b)$, expression will take the maximum value when $x_2(x_1^*)$ value is in the middle of the predictive interval (corresponds to the point estimation). The value is standardized according to the size of the predictive interval (($l_t - l_b$) difference in the denominator is the size of predictive interval). Also, a penalty for increasing the predictive interval value may be introduced. In this case, the expression must be multiplied by the interval value and divided by the value of the maximum interval.

Table 1 shows a problem solution example. The starting point $x_1^1 = 5$, $x_2^1 = 10$ (point A), solution found at the first step: $x_1^1 = 8.215$, $x_2^1 = 12.172$ ($x_2 = 100 / x_1$), the step is 0.5, the maximum value $x_{\max} = 0.937$ ($\alpha = 0.05$, $k_1 = 0.3$, $k_2 = 0.7$).

TABLE I. A PROBLEM SOLUTION EXAMPLE

Values of indicators						
x_1^*	x_2^*	l_b	l_t	$\gamma(x_1^*)$	$\lambda(x_1^*)$	$f(x_1^*)$
8.215	12.172	10.072	12.072	0	0.105	0.489
8.715	11.474	10.238	12.238	0.737	-0.472	0.962
9.215	10.852	10.405	12.405	2.742	-0.347	0.747
9.715	10.293	10.572	12.572	5.781	0.318	0.000

The increments in the first step are determined using the following formula:

$\Delta x_1 = x_1^* - x_1^1 = 8.215 - 8.215 = 0,$

$\Delta x_2 = x_2^* - x_2^1 = 12.172 - 12.172 = 0.$

Finally, the objective function is:

$\gamma(8.215) = 0^2 + 0^2 = 0;$

$\gamma_{norm}(8.215) = (0.737 - 0) / (0.737 - 0) = 1,$

$\lambda(8.215) = -(12.072 - 12.172)\,(12.172 - 10.072) / (12.072 - 10.072) = 0.105,$

$\lambda_{norm}(8.215) = (0.318 - 0.105) / (0.318 - (-0.472)) = 0.269,$

$f(8.215) = 0.3 \cdot 1 + 0.7 \cdot 0.269 = 0.489.$

According to the obtained results, the solution to the problem with the highest value of objective function will be: $x_1^* = 8.715$, $x_2^* = 11.474$ (Fig. 2, point B).

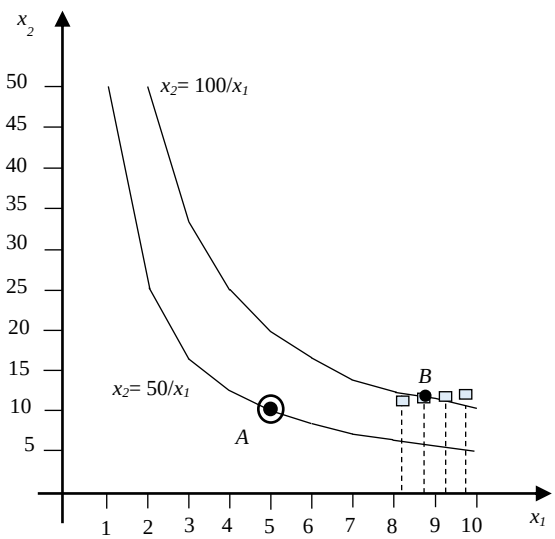


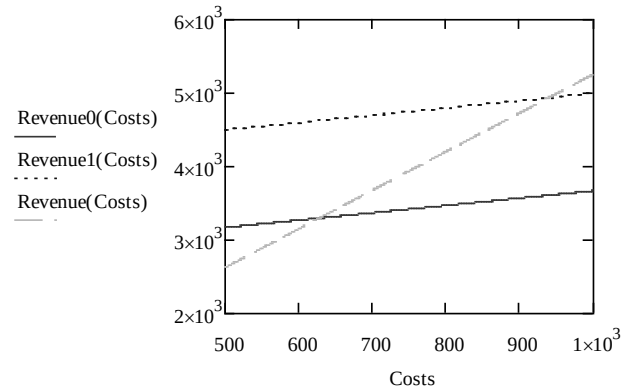
Fig. 2. Finding the optimal value

IV. Generation of Fast Food Restaurant's Contribution Margin

Let's consider the use of the presented algorithm for generating organization's contribution margin. To solve the inverse problem, fast food restaurant data on 'wheat noodles' product were used: cost, price, total demand for a week. Demand is unevenly distributed over the week: its weekend values are higher the weekday values. The relationship between the demand for food and its price for consumers is called own price elasticity of demand, i.e. percentage change in quantity sold, with an increase in price by 1%. Generally, own price elasticity of demand is negative, since demand almost always decreases with price increases. However, the elasticity for different products may be higher or lower, depending on the availability of substitutes, the degree of necessity of the product in the diet and consumer income. For example, the elasticity coefficient for confectionery products will be higher than its value for bread (article [6] presents the elasticity values for different types of products). For the product in question, the elasticity value turned out to be high (–2.3), since food in fast food restaurants is not necessary for most buyers (unlike products consumed daily).

Therefore, setting the price for the products sold, the enterprises determine the amount of their revenue, hence, the profit. You need to forecast the demand to choose the optimal price. The forecasting model is chosen depending on the nature of the demand: seasonality (dependence on the season of the year, the day of the week, etc.), regularity (how often there is a need for products), etc.Moreover, purchases can be made offline or online via the Internet. Article [7] presents a solution to the price optimization problem: revenue maximization at the existing price-demand dependence. Regression models and decision trees were considered to demonstrate the dependence. The integer programming problem was solved using the branch and bound method. The articles by authors present cars [8], designer clothes [9] and accessories [10].

The main problem in forecasting demand is that a set of price variations is usually limited. A subsequent price increase after its reduction can be negatively perceived by the buyers and treated as



overpayment and foregone benefits. This makes it impossible to set prices flexibly and monitor changes in demand.

Let's consider the contribution margin generation problem (*M*); the contribution margin can be defined as the difference between revenue (*Revenue*) and costs (*Costs*):

$$M = Revenue - Costs = Price \cdot Num - Num \cdot C_{unit} \quad (13)$$

where *Price* is the unit price;

Num is the quantity of product sold;

C_{unit} is the cost-per-unit.

The cost of the product in question is RUB 19, the price is RUB 100.Then the revenue-costs dependence will be as follows:

$$Revenue = Price \cdot Costs / C_{unit} = 5.263\text{Costs} \quad (14)$$

Initial values of revenue and costs (per week): *Revenue* = RUB 3,300, *Costs* = RUB 627.

The profit is RUB 2,673. It is necessary to determine the amounts of revenue and costs ensuring a profit of RUB 4,000.

Simultaneous equations are written as:

$\Delta Revenue / \Delta Costs = 5.263$;

$(Revenue + \Delta Revenue) - (Costs + \Delta Costs) = 4000$.

Simultaneous equations solution: $\Delta Revenue$=1,638, $\Delta Costs$ =311.28.

Figure 3 shows the graphical interpretation of the problem: *Revenue0*(*Costs*) is the line of initial profit level (RUB 2,673), *Revenue1*(*Costs*) is the line of a given profit level (RUB 4,000), *Revenue*(*Costs*) is the revenue-costs dependence (2).

Let's consider next determination of the price and quantity of products sold to achieve the given value of the contribution margin.

The initial value of profit is RUB 4,018, the price is RUB 60, the quantity is 98 pcs. (the cost-per-unit is also RUB 19). It is necessary to determine the values of the price and quantity so that the profit would be equal to RUB 4,900.

The formula for making a level line corresponding to the initial value of profit:

$Num0(Price) = 4018 / (Price - 19)$.

The level line corresponding to the given value of profit (RUB 4,200) is made using the following function:

$Num1(Price) = 4900 / (Price - 19)$.

The least squares method was used to build the quantity-price equation (data for 2.5 months). The regression equation obtained in case of linear dependence:

$Num = 148.2 - 1.15 \cdot Price$.

Fig. 3. Graphic view of the problem

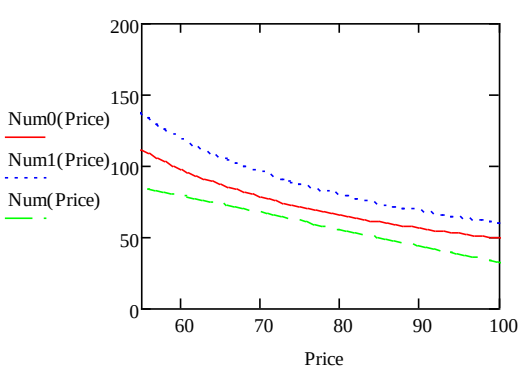
The regression coefficients are significant at the 99% interval (the intersection and slope errors are 27.5 and 0.3 respectively).

The price and quantity values determined at the first step of the algorithm described above are 66 and 104, respectively. It is necessary to consider the price values to RUB 106 with a step of RUB 5 and choose the optimal value ($\alpha = 0.05$, $k_1 = 0.5$, $k_2 = 0.5$).

Calculation results are provided in Table 2. It can be seen that the optimal problem solution, at which the given value of profit will be achieved, is: *Price*=RUB 76, *Num*=86 pcs.

TABLE II. THE OPTIMAL PROBLEM SOLUTION

Values of indicators						
x_1^*	x_2^*	l_b	l_t	$\gamma(x_1^*)$	$\lambda(x_1^*)$	$f(x_1^*)$
66.00	104.00	43.60	101.00	0	3.152	0.618
71.00	94.00	39.75	93.35	125	0.658	0.891
76.00	86.00	35.56	86.04	424	-0.042	0.934
81.00	79.00	30.95	79.15	850	-0.145	0.891
86.00	73.00	25.88	72.72	1361	0.278	0.777
91.00	68.00	20.28	66.82	1921	1.215	0.596
96.00	64.00	14.17	61.43	2500	2.707	0.349
101.00	60.00	7.55	56.55	3161	3.693	0.151



Values of indicators						
x_1^*	x_2^*	l_b	l_t	$\gamma(x_1^*)$	$\lambda(x_1^*)$	$f(x_1^*)$
106.00	56.00	0.48	52.12	3904	4.172	0.000

Figure 4 shows the level lines providing the initial and given values of profit (*Num0*(*Price*), *Num1*(*Price*)) and the quantity-price dependence graph (*Num*(*Price*)).

Fig. 4. Level lines and quantity-price regression dependence

V. PROGRAM DESCRIPTION

Based on the developed algorithm, a program was written in Java in IntelliJ IDEA development environment. To run the program on the computer, you must have Java Runtime Environment 1.7 and higher.

The program has a graphical interface, using which the user enters the input data, and then the computation process starts. If the program is successfully executed, output.xlsx Excel file will be generated with the calculation formulas, values computed in the program, as well as the input data and cell description. In this case, there will be a check for incorrect values entered by the user, that can cause erroneous program operation. If any erroneously entered values are found, the computation process will stop, and the user will be notified of the reason for the error with a message.

The program consists of two layers:

- user interface layer;
- work with Excel layer.

Figure 5 shows a diagram of the program classes, where the first layer is marked on the right, and the second layer is marked on the left.

Work with the user interface is implemented in the first layer: language switch (Russian and English), filling text fields and tables with source data (statistics data can be imported from another Excel document), clicking (StartWindow class, fig.). Also, a Validator class (Validator) was implemented as part of this layer, responsible for calculations security. Various options for entering incorrect values in the fields and table are checked in this class. In addition, the layer includes InputDataDto class, which fills the model and passes it to the second layer. Figures 6 and 7 show the program interface.

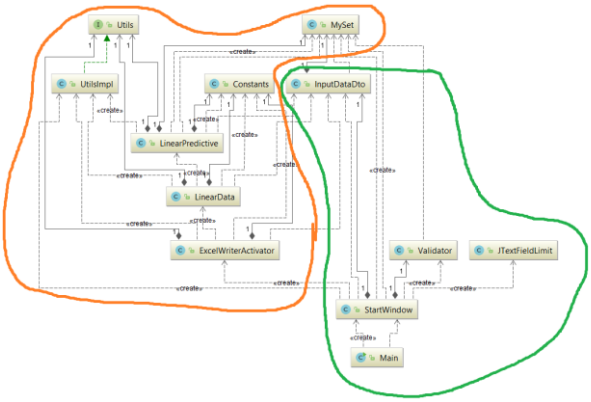


Fig. 5. Program classes diagram

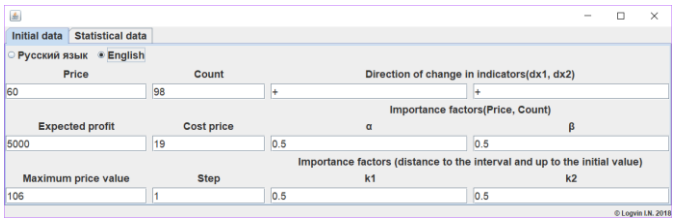


Fig. 6. Program interface (Initial Data tab)

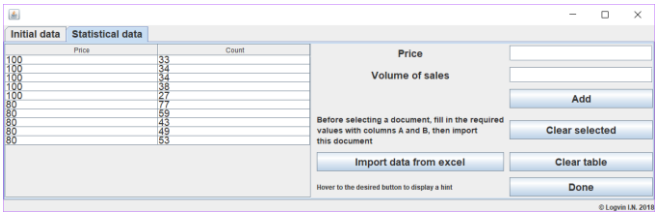


Fig. 7. Program interface (Statistics Data tab)

The second layer contains the basic computing algorithms. To work with Excel document, the layer uses Apache-POI library allowing to perform such operations as:

- create/edit/delete worksheets;
- record values and formulas into cells;
- generate function results;
- set cell styles, etc.

The name of the output document and the model with the input data are transferred to this layer. After this, the output document is filled with the input data, corresponding formulas and cell description. UtilsImpl (filling table cells), Constants (defining cell calculation formulas and text data), MySet (defining a data list model) classes are designed for working with an Excel file.

Output document contains 3 sheets: Inverse Problem, Solution and Predictive Interval. The sheets are generated using ExcelWriterActivator, LinearData, and LinearPredictive classes.

The Inverse Problem sheet contains the inverse problem solving result (step 1 of the algorithm), while the price and quantity values are rounded so that the resulting profit value has a minimum deviation from the set value. Solution sheet (Fig. 8) includes a table with the price (changed at a given step) and quantity, interval boundaries, location indicators relative to the boundaries of the predictive interval and the initial solution, the objective function values. The price corresponding to the largest value of the objective function is marked as a solution to the problem. The last Predictive Interval sheet is auxiliary and designed for calculating the predictive interval boundaries at a given value of the explanatory variable (price).

VI. Conclusion

The article deals with solving inverse economic analysis problems where there is dependence between the arguments of the function. It offers a solving algorithm for stochastic dependence between the function arguments. The algorithm is based on calculating the remoteness from the initial solution indicator and the location characteristic relative to the midpoint of the predictive interval. By way of example, the problem of fast food restaurant's contribution margin generation was considered: determining the necessary values of revenue and costs, the price and quantity of the goods sold.

A	B	C	F	G	L	M
1	Expected profit	5000	Maximum price value	106	Importance coefficients	
2					k1	k2
3	Step	1	Cost per unit	19	0.5	0.5
4	Price(x1)	Count(x2)	Lower bound	Upper bound	Objective function	
5	67.00	104.00	42.851	99.449	0.598	
6	68.00	102.00	42.094	97.906	0.653	
7	69.00	100.00	41.326	96.374	0.708	
8	70.00	98.00	40.544	94.856	0.763	
9	71.00	96.00	39.750	93.350	0.817	
10	72.00	94.00	38.941	91.859	0.870	
11	73.00	93.00	38.118	90.382	0.810	
12	74.00	91.00	37.281	88.919	0.865	
13	75.00	89.00	36.428	87.472	0.919	
14	76.00	88.00	35.558	86.042	0.863	
15	77.00	86.00	34.673	84.627	0.918	
16	78.00	85.00	33.770	83.230	0.865	
17	79	83.00	32.850	81.850	0.922	Max
18	80.00	82.00	31.911	80.489	0.871	
19	81.00	81.00	30.954	79.146	0.822	
20	82.00	79.00	29.978	77.822	0.883	
21	83.00	78.00	28.983	76.517	0.837	
22	84.00	77.00	27.968	75.232	0.793	

Fig. 8. Solution sheet

Based on the offered algorithm, a program was implemented to determine the parameters providing the given value of the contribution margin when there is stochastic dependence between the price and quantity of goods. The offered algorithm can be used in management decision making regarding economic entities management.

References

[1] J. L. Fernandez-Martinez, J. L. Pallero, Z. Fernandez-Muniz, L. M. Pedruelo-Gonzalez, "The effect of noise and Tikhonov's regularization in inverse problems. Part I: The linear case", Journal of applied geophysics, vol. 108, 2014, pp.176–185.

[2] I. Ekeland, N. Djitte, "An inverse problem in the economic thery of demand", Annales de l'Institut Henri Poincare (C) non linear analysis, vol. 23, 2006, pp.269–281.

[3] A. Barbagallo, P. Mauro, "An inverse problem for the dynamic oligopolistic market equilibrium problem in presence of excesses", Procedia – social and behavioral sciences, vol. 108, 2014, pp. 270–284.

[4] Z. Lili, "An inverse optimization model for human resource allocation problem considering", Procedia computer science, vol. 112, 2017, pp.1611–1622.

[5] E. Gribanova, "Stochastic algorithm to solve the problem of linear programming with backward calculations", Applied methods of statistical analysis. Nonparametric methods in cybernetics and system analysis, pp. 196-203, September 2017.

[6] R. Green., L. Cornelsen, A. Dangour, R. Turner, B. Shankar, M. Mazzocch, R. Smith, "The effect of rising food prices on food consumption: systematic review with meta-regression", BMJ, vol. 346, 2013, pp. 1-9.

[7] T. Qu, J. H. Zhang, F. Chan, R. S. Srivastava, M. K. Tiwari, W. Park, "Demand prediction and price optimization for semi-luxury supermarket segment", Computers & industrial engineering, vol. 113, 2017, pp. 91-102.

[8] T. Noparumpa, B. Kazaz, S. Webste, "Wine futures and Advance Selling UnderQuality Uncertainty", Manufacturing & service operations management, vol. 3, 2015, pp. 411-426.

[9] S. Berry, J. Levinsohn, A. Pakes, "Automobile prices in market equilibrium steven", Econometrica, vol. 63, 2007, pp. 841–890.

[10] K. Johnson, B. Hong, A. Lee, D. Simchi-levi, "Analytics for an online retailer: demand forecasting and price optimization", Manufacturing & Service Operations Management, vol. 18, 2016, pp. 69-85.