Quasi-Fractal Model of the Semantic Knowledge Network as the Basis for the Formation of a Pedagogical Test

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Abstract: It is substantiated that one of the problematic issues of pedagogical testing is the inconsistency (or incomplete compliance) of the structure and content of test tasks, on the one hand, and the information model of knowledge representation about the academic discipline implemented in the learning process, on the other hand. The approach to resolving this issue is set forth where the semantic knowledge network is considered as an information model for representing knowledge about a subject area. The definition of the quasi-fractal graph is introduced, it is shown that the semantic network of knowledge about the subject area, as well as the information model of the pedagogical test, can be represented as quasi-fractal graphs. It is shown that the quasi-fractal graph is one of the forms for recording the semantic knowledge network. The conditions under which the structure and content of the test tasks will correspond to the information model of knowledge representation about the academic discipline being implemented in the learning process.

1. Introduction

Monitoring learning outcomes can be carried out by different methods. None of them allows to determine the true value of the level of knowledge of students in the academic discipline. In this regard, it is desirable that the conditions for monitoring learning outcomes be the same for all students, so that those who exercise control cannot change these conditions to someone’s benefit.

The method of pedagogical testing is one of the methods that satisfy this requirement. At the same time, inconsistency (or incomplete correspondence) between the information model of knowledge representation about the academic discipline implemented in the learning process, on the one hand, and the information model of the pedagogical test (structure and content of its test tasks), on the other hand, is a problematic issue of pedagogical testing.

This problem question is relevant for schoolchildren, for example. In accordance with the Federal Law of December 29, 2012 No273-FZ “On Education in the Russian Federation”, each educational institution of general education trains students in its educational program, which must meet the requirements of the Federal State Educational Standard (GEF). Consequently, various educational institutions of general education (and there are tens of thousands of them in the country) will train students in educational programs that meet the GEF requirements but differ from each other. Upon completion of training, students pass the Unified State Examinations (USE). These exams are conducted by the method of pedagogical testing according to test tasks, the structure and content of which is developed by another department. As a result, these tests may not fully correspond to the educational programs of an educational institution of general education. And there are many such examples.

Let’s consider one of the possible ways to resolve this issue.
2. Materials and Methods

In order to ensure that the objectives of testing are consistent with the objectives of training, both in the academic discipline as a whole and in its sections, topics, educational issues, methods for solving educational problems, it is necessary that [1]:

- The same information model of knowledge representation about the academic discipline (subject area) is used as the basis of training and knowledge control;
- The information model of knowledge representation about the subject area allows for the possibility of its decomposition, based on learning objectives, sections of educational disciplines, topics of studies, training questions, their fragments (including setting and solving typical educational tasks, etc.) into information models of knowledge of relevant parts of the subject area;
- Information models of knowledge representation about the relevant parts of the domain can be synthesized into more general models of knowledge representation at various levels, up to the model of knowledge representation about the domain.

Semantic knowledge networks are considered as information models for the representation of knowledge about the subject area and its components. In the history of science, the beginning of the formation of the theory of semantic knowledge networks is usually associated with the work of C. Linnaeus [2]. In the most general form, the information model of the semantic knowledge network is a directed graph [3; 4]:

\[ G = \langle \{V_\alpha | \alpha \in A\}, \{u_{\alpha,\beta} | \alpha, \beta \in A\} \rangle, \] (1)

where \(\{V_\alpha | \alpha \in A\}\) is the set of vertices of the graph \(G\), \(\{u_{\alpha,\beta} | \alpha, \beta \in A\}\) is the set of edges of the graph \(G\).

Analysis of the graph (1) showed that all its components can be considered as fractals [5; 6]. At the same time, a fractal is a geometric shape that can be divided into parts, where each of the parts is a smaller version of the whole. B. Mandelbrot gave a stricter definition of the fractal given above. According to this definition, a fractal is an object, the Hausdorff-Besicovitch dimension of which is greater than its topological dimension [6].

For the description of test items, the language of narrow predicate calculus is used (NPC) [7].

3. Results

The following definitions of graphs 1, 2, ..., \(k + 1\) levels, where \(k\) is a positive integer, are introduced, and definitions of quasi-fractal and fractal graphs are given.

Definition of a graph 1 level. Consider the graph:

\[ \Gamma_1 = \langle \{V_{\alpha_1}^1 | \alpha_1 \in A_1\}, \{u_{\alpha_1,\beta_1}^1 | \alpha_1, \beta_1 \in A_1\} \rangle, \] (2)

where \(\{V_{\alpha_1}^1 | \alpha_1 \in A_1\}\) – the set of vertices of the graph \(\Gamma_1\);
\(\{u_{\alpha_1,\beta_1}^1 | \alpha_1, \beta_1 \in A_1\}\) – the set of edges of the graph \(\Gamma_1\).

Then the graph \(\Gamma_1\) will be called the first level graph.

Definition of a graph 2 level. Let, in turn, the vertices \(V_{\alpha_1}^2, \alpha_1 \in A_1\), are graphs,

\[ \Gamma_2 = \langle \{V_{\alpha_2}^2 | \alpha_2 \in A_2\}, \{u_{\alpha_2,\beta_2}^2 | \alpha_2, \beta_2 \in A_2\} \rangle, \] (3)

where \(\{V_{\alpha_2}^2 | \alpha_2 \in A_2\}\) is the set of vertices of the graph \(\Gamma_2\);
\(\{u_{\alpha_2,\beta_2}^2 | \alpha_2, \beta_2 \in A_2\}\) is the set of edges of the graph \(\Gamma_2\).

Then the graph \(\Gamma_2\) will be called the graph of the second level.

We will continue the consideration of the process of introducing definitions of graphs 1, 2 and subsequent levels of graphs by induction.
Definition of \( k + 1 \) level graph. If there is a \( k \)-level graph

\[
I_k = \langle \{V^k_{\alpha_k} | \alpha_k \in \Lambda_k \}, \{u^k_{\alpha_k, \beta_k} | \alpha_k, \beta_k \in \Lambda_k \} \rangle,
\]

(4)

where \( \{V^k_{\alpha_k} | \alpha_k \in \Lambda_k \} \) is the set of vertices of the graph \( I_k \);
\( \{u^k_{\alpha_k, \beta_k} | \alpha_k, \beta_k \in \Lambda_k \} \) is the set of edges of the graph \( I_k \),

and the vertices of this graph \( V^k_{\alpha_k} \), \( \alpha_k \in \Lambda_k \), are in turn graphs

\[
I_{k+1} = \langle \{V^{k+1}_{\alpha_{k+1}} | \alpha_{k+1} \in \Lambda_{k+1} \}, \{u^{k+1}_{\alpha_{k+1}, \beta_{k+1}} | \alpha_{k+1}, \beta_{k+1} \in \Lambda_{k+1} \} \rangle,
\]

(5)

where \( \{V^{k+1}_{\alpha_{k+1}} | \alpha_{k+1} \in \Lambda_{k+1} \} \) is the set of vertices of the graph \( I_{k+1} \);
\( \{u^{k+1}_{\alpha_{k+1}, \beta_{k+1}} | \alpha_{k+1}, \beta_{k+1} \in \Lambda_{k+1} \} \) is the set edges of the graph \( I_{k+1} \),

then the graph \( I_{k+1} \) will be called the level graph \( k + 1 \).

Remarks. All vertices of the graph are divided into initial, intermediate, and final [4].

Definition of quasi-fractal graph:

\[
I_1 = \langle \{V^1_{\alpha_1} | \alpha_1 \in \Lambda_1 \}, \{u^1_{\alpha_1, \beta_1} | \alpha_1, \beta_1 \in \Lambda_1 \} \rangle,
\]

(6)

where each vertex in turn is a graph of 2, 3 and subsequent levels, excluding finite ones will be called a quasi-fractal graph.

In the particular case, we obtain the definition of a fractal graph from this definition.

Definition of a fractal graph. If in a quasi-fractal graph all the graphs

\[
I_k = \langle \{V^k_{\alpha_k} | \alpha_k \in \Lambda_k \}, \{u^k_{\alpha_k, \beta_k} | \alpha_k, \beta_k \in \Lambda_k \} \rangle, k = 1, ..., n, ...
\]

are isomorphic to each other, then the graph

\[
I_1 = \langle \{V^1_{\alpha_1} | \alpha_1 \in \Lambda_1 \}, \{u^1_{\alpha_1, \beta_1} | \alpha_1, \beta_1 \in \Lambda_1 \} \rangle
\]

is called fractal.

Consequently, the definition of a quasi-fractal graph can be viewed as a generalization of the definition of a graph, which, in turn, suggests that a quasi-fractal graph is one of the forms for recording the semantic knowledge network (1).

Let the information model of knowledge representation about the domain (its part) is a quasi-fractal graph \( I_A \), consisting of graphs 1, 2 and subsequent levels:

\[
I_1 = \langle \{V^1_{\alpha_1} | \alpha_1 \in \Lambda_1 \}, \{u^1_{\alpha_1, \beta_1} | \alpha_1, \beta_1 \in \Lambda_1 \} \rangle,
\]

\[
I_2 = \langle \{V^2_{\alpha_2} | \alpha_2 \in \Lambda_2 \}, \{u^2_{\alpha_2, \beta_2} | \alpha_2, \beta_2 \in \Lambda_2 \} \rangle,
\]

...

(7)

Then, in order to resolve the problem issue under consideration, it is necessary that the information model of the pedagogical test matches the model (7), i.e. it would have the following form \( T_A \):

\[
T_1 = \langle \{t^1_{\alpha_1} | \alpha_1 \in \Lambda_1 \}, \{r^1_{\alpha_1, \beta_1} | \alpha_1, \beta_1 \in \Lambda_1 \} \rangle,
\]

\[
T_2 = \langle \{t^2_{\alpha_2} | \alpha_2 \in \Lambda_2 \}, \{r^2_{\alpha_2, \beta_2} | \alpha_2, \beta_2 \in \Lambda_2 \} \rangle,
\]

...

(8)

where \( \{t^i_{\alpha_i} | \alpha_i \in \Lambda_i \} \) is the set of vertices of the graph \( T_i \), \( i = 1, 2, ...; \)
\( \{ u_{\alpha, \beta}^i | \alpha, \beta \in A_1 \} \) is the set of edges of the graph \( T_1 \).

Each of the set of graphs \( T_m, m \leq k \), is the basis for the development of test tasks 1, 2 and subsequent levels. At the same time, each test task can be represented in the database of test tasks in several versions.

In this regard, the pedagogical test can be defined as follows.

**Definition.** The pedagogical test consisting of test tasks that do not exceed the level \( k \) on the set of graphs \( \Gamma_m, m \leq k \), is represented by the formula of the NPC (that is, a narrow predicate calculus)

\[
F(x_1, x_2, ..., x_n),
\]

where variables

\( x_1, x_2, ..., x_n \)

take values in the domains \( D_m, m \leq k \), modeled by the graphs \( T_m, m \leq k \); \( n \) - the number of test tasks in the pedagogical test.

Thus, the concept of a fractal (respectively, quasi-fractal) graph defines, in essence, the structure of the system being modeled, which defines the process of decomposition of the original system into subsystems.

4. **Discussion**

We begin with the interpretation of the concept of a quasi-fractal graph. Probably, each of us from childhood remembers a toy in the form of an optical tube called the “Kaleidoscope”. With its help, the child could observe patterns, the number of which was limited and was determined by the inclination of the location of the mirrors in the optical tube. All this is somewhat reminiscent of a set of graphs \( \Gamma_m, m \leq k \), each of them can be interpreted as a kind of “pattern”.

The pedagogical test developed in accordance with the information model (8) allows to solve the problem question, namely to ensure consistency between the information model of knowledge presentation about the academic discipline implemented in the learning process and the information model of the pedagogical test.

As an example of the occurrence of a possible discrepancy between the model of knowledge representation about the academic discipline implemented in the learning process and the information model of the pedagogical test, earlier, in section 1, the Unified State Examination was specified. In relation to this case, we can offer the following solution to this problem issue.

Suppose that in the process of teaching an academic discipline, the information model of knowledge representation on an academic discipline \( \Gamma_A \), presented earlier by relations, (7) was implemented. The model of the pedagogical test \( T_A \), presented earlier by relations (8), corresponds to this model. Suppose that for the Unified State Exam in this discipline, the pedagogical test \( E_A \) was prepared based on a quasi-fractal graph representing the knowledge of the subject area different from the graph \( \Gamma_A \). Therefore, the case when \( T_A \neq E_A \) will take place.

Then, the monitoring of learning outcomes should be carried out using the pedagogical test \( T_A \), or the pedagogical test \( T_A \cap E_A \). In any case, this problematic issue should be resolved in the interests of schoolchildren, who conscientiously treated their academic duties in the process of studying this academic discipline.

5. **Conclusion**

The problem question of pedagogical testing is put in work. It lies in the possible inconsistency (or incomplete compliance) of the structure and content of the test tasks and the information model of knowledge representation about the academic discipline. The relevance of this issue is justified by the example of the EGE. One of the possible ways of resolving it is considered, which os based on the presentation of an information model of knowledge about an academic discipline in the form of a quasi-fractal graph.
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