

Empirical Analysis of Chinese Stock Market Volatility Based on GARCH Models and Markov Switching Models

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Abstract. Volatility has been the focus in the financial field in recent decades. It can be used to measure the uncertainties of yield and represent the risk of assets. In this paper, GARCH models and Markov switching models are used to fit the volatility of the Chinese stock market. Results illustrate that Markov switching models take the regime-switch as an endogenous variable and a random process, which enable it to describe all the remarkable structural change in one united model and help to forecast price. Therefore, it is superior to GARCH models.

1. Introduction

With the increasing financialization of the global economy, stock markets, as an important part of financial markets have begun to play a critical role in economic development. American economists, Fama divided the stock market into strong, semi-strong and weak efficient markets. Most theoretical studies have shown that the Chinese stock market displays weak efficiency. According to the efficient market hypothesis, compared with the strong and semi-strong efficient market, the stock price in the weak efficient market fluctuates more frequently, with larger fluctuation, and higher investment risk. Generally speaking, moderate fluctuations of stock price can promote the prosperity of the stock market, violent fluctuations may lead to distortions in the price regime, declines in the resource allocation function and stagnation of the stock market [1]. The frequent fluctuations in the Chinese stock market have increased investors' investment risks and harmed the interests of them. Therefore, this paper studies the volatility characteristics of the Shanghai and Shenzhen stock markets, analyzes stock market development by creating a reasonable model, which is of practical significance for investors who want to recognize stock market risks and make rational investments.

International research on the volatility of the stock market has been mature. In 1982, Engle [2] proposed the ARCH model, which for the first time simulated the peak and tail of stock market volatility. In 1991, Nelson [3] found that the impact of "leverage" was exponential, so the EGARCH model was proposed to simplify the calculation process. Based on the SV-T model introduced by Harvey [4] (1994) and the joint distribution indicated by Mahieu and Schotman [5](1998), Jacquier[6] showed the parameter posterior distribution of the thick-tailed model. Koopman, S.J. and Uspensky, E.H. [7](2010) proposed an extended model and analyzed the relationship between risks and expected returns. China's research on the volatility of stock market started late, however, with improvement of the capital market, more and more research results have been obtained. In 2007, Wang Yaozhen[8] used the Shanghai Composite Index data to compare the GARCH model with the Risk Metrics standard method. The comparison result was that the GARCH model is more accurate. In 2009, Meng Lifeng and Zhang Shiyong[9] proposed a nonlinear SV model with leverage effect through empirical analysis. In 2011, Hu Shulan, Wei Jie, and Huang Wei[10] used the hidden Markov model to segment the five stock index states of the stock market reasonably, and analyzed the transformation of the stock market between the five states.

Generally, researchers established the GARCH family models or SV family models to study stock volatility. The GARCH family models can describe the characteristics of volatility. Due to its simple

form and convenient operation, the GARCH family models have enjoyed wide use. The SV model is not very widely used in empirical research as a result of its complicated parameter estimation. Markov switching model, derived from the hidden Markov model is represented as a dual stochastic process composed of a Markov process and a general stochastic process. Therefore, we introduce Markov switching model to fit the volatility of the Chinese stock market. Meanwhile, it also solves the problem of the GARCH family model only being able to describe the fluctuation of the stock market in a fixed pattern, as it cannot describe the sudden fluctuation and the amplitude conversion. Markov switching model has made up for the defects in the GARCH family model, as it can capture the sudden fluctuation and amplitude conversions, which reflects the development characteristics of the Chinese stock market.

Based on the above analysis, this article will introduce GARCH family model and Markov switching model to establish the volatility model and fit the yield of the Chinese stock markets.

2. Models

2.1 GARCH family model

Consider the time series $\{x_t\}$ which follows the GARCH process. $\{\varepsilon_t\}$ is residual error, ε is the mean value of the absolute residual, $f(t, x_{t-1}, x_{t-2}, \dots)$ is an autoregression model of $\{x_t\}$, $\omega, \lambda_j, \eta_i, \delta_j, \gamma$ are unknown parameters.

2.1.1 GARCH model

According to the definition of Bollerslov (1986)[11], the GARCH model was structured as follows:

$$\left\{ \begin{array}{l} x_t = f(t, x_{t-1}, x_{t-2}, \dots) + \varepsilon_t \\ \varepsilon_t = \sqrt{h_t} e_t \\ h_t = \omega + \sum_{i=1}^p \eta_i h_{t-i} + \sum_{j=1}^q \lambda_j \varepsilon_{t-j}^2 \end{array} \right. \quad \text{where} \quad \left\{ \begin{array}{l} \omega > 0, \eta_i \geq 0, \lambda_j \geq 0 \\ \sum_{i=1}^p \eta_i + \sum_{j=1}^q \lambda_j < 1 \end{array} \right. \quad (1)$$

and in the formula there exists $e_t \sim iid N(0,1)$.

The conditional variance of time t is h_t . It is one-step prediction based on the time $t-1$ and its previous information. It is related to the degree of deviation from the mean $h_{t-i} (i = 1, 2 \dots p)$, and also depends on the size of the previous conditional variance equation $\varepsilon_{t-j}^2 (j = 1, 2 \dots q)$. Therefore, the conditional variance equations above describe characterizes the clustering and persistence of fluctuations. This model is abbreviated as GARCH (p, q).

2.1.2 EGARCH model

The EGARCH model was proposed by Nelson (1991), it was structured as follows:

$$\left\{ \begin{array}{l} x_t = f(t, x_{t-1}, x_{t-2}, \dots) + \varepsilon_t \\ \varepsilon_t = \sqrt{h_t} e_t \\ \ln(h_t) = \omega + \sum_{i=1}^p \eta_i \ln(h_{t-i}) + \sum_{j=1}^q \lambda_j \left[\frac{\varepsilon_{t-j}}{\sqrt{h_{t-i}}} \right] + \sum_{j=1}^q \delta_j \left[\left| \frac{\varepsilon_{t-j}}{\sqrt{h_{t-i}}} \right| - \varepsilon \right] \end{array} \right. \quad (2)$$

In this formula, $[\frac{\varepsilon_{t-j}}{\sqrt{h_{t-i}}}]$ is ARCH term, $[\frac{\varepsilon_{t-j}}{\sqrt{h_{t-i}}}]$ describes the difference between bull news and bear news. By taking the logarithm of variance equation, the EGARCH model relaxes the non-negative convention of parameters in the GARCH model, and makes the stochastic disturbance term of h_t have different changes when taking positive and negative values. Therefore, the EGARCH model describes the fluctuations of the financial market more accurately.

2.2 Markov switching model

The Markov switching model was proposed by Hamilton[12] (1989) , it was structured as follows:

$$x_t = u_{s_t} + \sum_{i=1}^p \phi_{is_t} x_{t-i} + \varepsilon_t, \varepsilon_t | F_{t-1} \sim iid N(0, \sigma_{s_t}^2) \tag{3}$$

$$\mu_{s_t} = \mu_0(1 - S_t) + \mu_1 S_t \tag{4}$$

$$\phi_{is_t} = \phi_{i0}(1 - S_t) + \phi_{i1} S_t \tag{5}$$

$$\sigma_{s_t}^2 = \sigma_0^2(1 - S_t) + \sigma_1^2 S_t \tag{6}$$

In these formulas, $\{x_t\}$ is a series of observation values, S_t is a state variable, F_t is the information stream before time t (Sigma algebra), and μ_t, σ_t are the mean and the variance respectively. The switching probabilities p_{ij} (switching probability matrix Q), $\mu_t, \sigma_t, \phi_{is_t}$ are unknown parameters, which are necessary to estimate in practicalities.

The changes of Chinese stock market volatility can be divided into two-regime switching model, high volatility and low volatility. It can be supposed that $S_t = 1$ indicates high fluctuation and $S_t = 0$ indicates low fluctuation, when establishing Markov Dual-Regime-switching Model for the volatility of stock market. The states satisfy Markov property:

$$P[S_t = j | S_{t-1} = i, S_{t-2} = k, \dots, x_{t-1}, x_{t-2}, \dots] = P(S_t = j | S_{t-1} = i) = p_{ij} \quad (i, j, k = 1, 0) . \tag{7}$$

The switching probability matrix can be expressed as $Q = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix}$, where $\sum_{j=0}^1 p_{ij} = 1$.

3. Data characteristics and preprocessing

3.1 Data sources

The stock price index is a dynamic indicator reflecting the comprehensive changes in stock price and the fluctuations of the overall price level. In studying the volatility of the stock market, Shanghai Composite Index and Shenzhen Component Index were used. In 1996, Shanghai and Shenzhen stock markets implemented a 10% increase price limit, which had profound effects on the stock market. Therefore, we have selected the closing price from January 2, 1997 to December 28, 2018, a total of 5,330 observations. The data comes from the Resset Financial Research Database.

Suppose P_t is the closing price of the t period and P_{t-1} is the closing price of the previous period, yield can be expressed as $r_t = \ln P_t - \ln P_{t-1}$, we can get the yield series. Figure 1 to Figure 4 are the daily closing price of the Shanghai Composite Index with its yield series and the Shenzhen Component Index with its yield series.

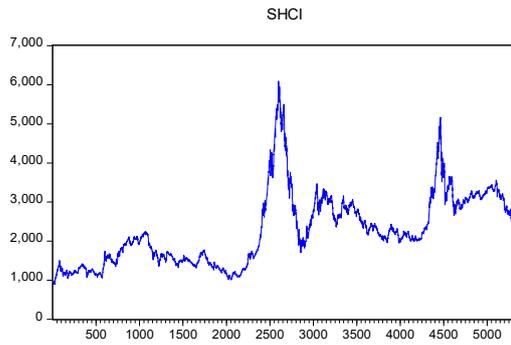


Fig. 1. Shanghai Composite Index(97.01.02-18.12.28)

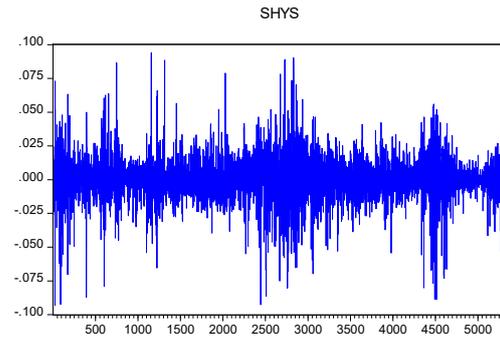


Fig. 2. Shanghai Yield Sequence (97.01.03-18.12.28)



Fig. 3. Shenzhen Component Index(97.01.02-18.12.28)

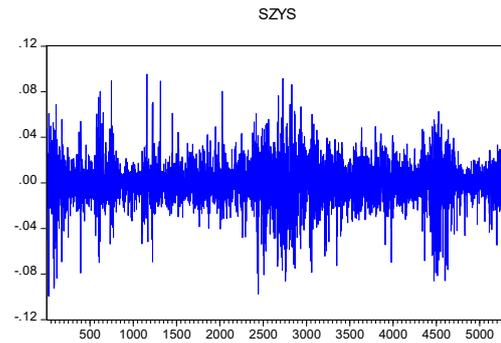


Fig. 4. Shenzhen Yield Sequence (97.01.03-18.12.28)

3.2 Data characteristic

3.2.1 Volatility clustering

It's perceptible from Fig. 2. and Fig. 4. that the volatility of the Shanghai and Shenzhen stocks yields have a characteristic of clustering. The large fluctuations are followed by large fluctuations, and vice-versa. In one period the fluctuations are small, and in another period the fluctuations are large. There is noticeable difference in the different intervals.

3.2.2 High peak and fat tail and non-normality

There is a common assumption in financial theory that the yield series obeys the normal distribution in the empirical distribution, but it might not be.[13] As shown in Fig.5 and Fig.6, distributions of two yield series have obvious characteristics of high peaks and fat tails. The kurtosis of both markets is extremely high, one has reached more than 7.9, the other has reached more than 6.6, and their skewness coefficient are less than 0, indicating that there exists certain negative skewness. The fat-tailed distribution means that the absolute value of yield is larger than expected, and the skewness less than 0 means that large yields tend to be negative. The Jarque-Bera statistics of the two markets are very large, so it's perceptible that the two markets' yields do not obey the normal distribution.

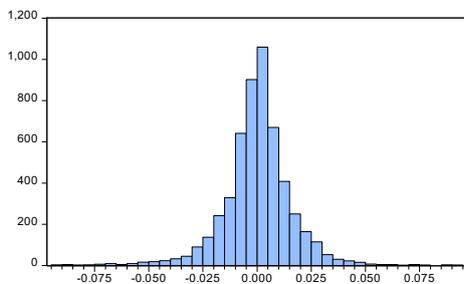


Fig. 5. Basic statistics of Shanghai yield series

Series: SHYS	
Sample	1 5329
Observations	5329
Mean	0.000187
Median	0.000630
Maximum	0.094008
Minimum	-0.093342
Std. Dev.	0.016113
Skewness	-0.398730
Kurtosis	7.901463
Jarque-Bera	5475.610
Probability	0.000000

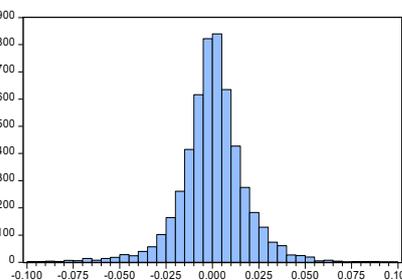


Fig. 6. Basic statistics of Shenzhen yield series

Series: SZYS	
Sample	1 5329
Observations	5329
Mean	0.000149
Median	0.000296
Maximum	0.095299
Minimum	-0.099352
Std. Dev.	0.018106
Skewness	-0.335705
Kurtosis	6.656042
Jarque-Bera	3068.045
Probability	0.000000

3.2.3 Stationary and autocorrelation

It is necessary to conduct a stationarity test and an autocorrelation test while modeling via time series. Considering that the yield series may exhibit heteroskedasticity, using the ADF method to check the stationarity is unreasonable, so the PP test is used to check the stationarity at this time.[14] The test

results show that both stock market yield series are stable and predictable, which are shown in Table1.

Table 1. PP test result

	Statistics	P-value
Shanghai stock market	-72.295 60	0.000 1
Shenzhen stock market	-70.101 13	0.000 1

Meanwhile, the auto-correlogram and partial correlogram of the lag of 15 periods show that the yield of Shanghai Composite Index has strong 3 and more order autocorrelation. Therefore, the yield series of Shanghai and Shenzhen stock markets are not white noise, they are all stationary time series.

4. Empirical analysis of Chinese stock market

4.1 GARCH family model

In this paper, Eviews is used to estimate the parameters of the GARCH model. Based on AIC and SC criterion, EGARCH (1,1) model is selected to fit the fluctuation of stock markets. The results show that fitting effect are reasonable. The impact of the conditional variance on the stock market volatility is long-term and asymmetrical, and both stock market have leverage effect. When there is bull news, the volatility quadratic term coefficient of the Shanghai stock market EGARCH (1,1) model is 0.146, and the coefficient is -0.212 when it is bear news. In EGARCH (1,1) model of the Shenzhen stock market, the coefficient is 0.140 when there is bull news, and the coefficient is -0.198 when it is bear news.

Table 2. EGARCH (1,1) model estimation results

	Mean equation			Variance equation			
	Constant term	AR(1)	AR(2)	Constant term	ARCH term	GARCH term	leverage term
Shanghai market parameters	—	—	-0.026	-0.241	0.179	0.987	-0.033
T-Statistics	—	—	-2.637	-8.362	12.261	350.802	-4.174
log likelihood value	lnL=15 447.16						
Shenzhen market parameters	—	0.042	—	-0.239	0.169	0.986	-0.029
T-Statistics	—	4.241	—	-8.058	11.906	326.900	-3.877
log likelihood value	lnL=14 646.31						

Notes: 1. AR(1) represents the yield of lag-1, and AR(2) represents the yield of the lag-2.
2.“—”indicates that this parameter is not significant or does not exist.

4.2 Markov switching model

Considering there exist the sudden fluctuation and the amplitude conversion in Chinese stock market, in this paper, Matlab is used to fit the Markov switching model. Based on AIC and BIC criterion, we find that the stock market volatility has the first-order lag autoregressive process. The results show that the autoregressive coefficients are not significant, but the residual variance values and switching probabilities of the two states are extremely significant. The difference between two states' residual variance is very large, indicating there are noticeable differences between high and low volatility regime. The data values of the switching probability matrix indicate that the probability of a high or low volatility state maintaining its own state is large, and the probability of switching is significantly small. All the above results show that the structure of the volatility process within the Chinese stock

market has changed from high volatility to low volatility or vice-versa. Compared with the EARCH model, the log-likelihood value of Markov switching model is slightly lower, however, it considers the possibility of regime switching and describes the sudden fluctuation and the amplitude conversion, making it better than the GARCH family model. Therefore, it is better for explaining the stock market volatility than the GARCH model.

Table 3. Estimation results of the Markov switching model for the first-order lag in Shanghai

Parameters	ϕ_0	σ_0^2	ϕ_1	σ_1^2	P_{00}	P_{11}
T-Statistics	3.44E-04	1.07E-04	-2.39E-04	7.08 E-04	0.98	0.97
Standard deviation	0.054 4	0.000 0	0.594 4	0.000 0	0.000 0	0.000 0
log likelihood value	lnL=15244.					

Table 4. Estimation results of the Markov switching model for the first-order lag in Shenzhen

Parameters	ϕ_0	σ_0^2	ϕ_1	σ_1^2	P_{00}	P_{11}
T-Statistics	8.80E-05	1.40E-04	3.63E-04	8.45 E-04	0.97	0.97
Standard deviation	0.614 0	0.000 0	0.516 3	0.000 0	0.000 0	0.000 0
log likelihood value	lnL=14513.74					

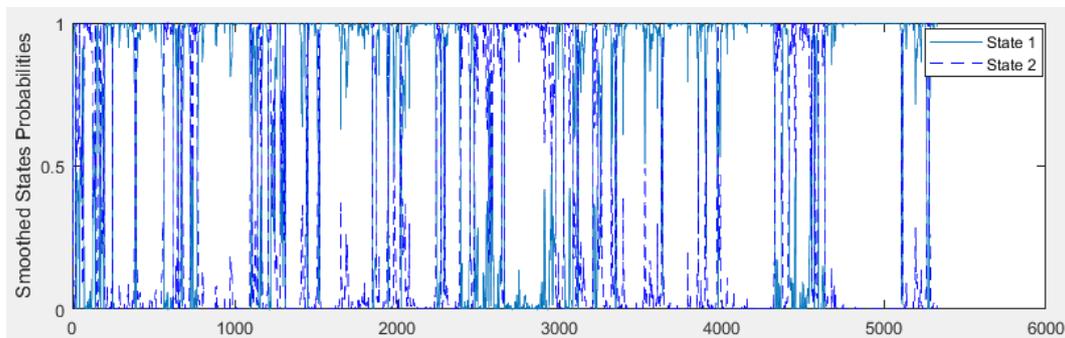


Fig. 7. Smoothed Probability of Markov switching model in Shanghai

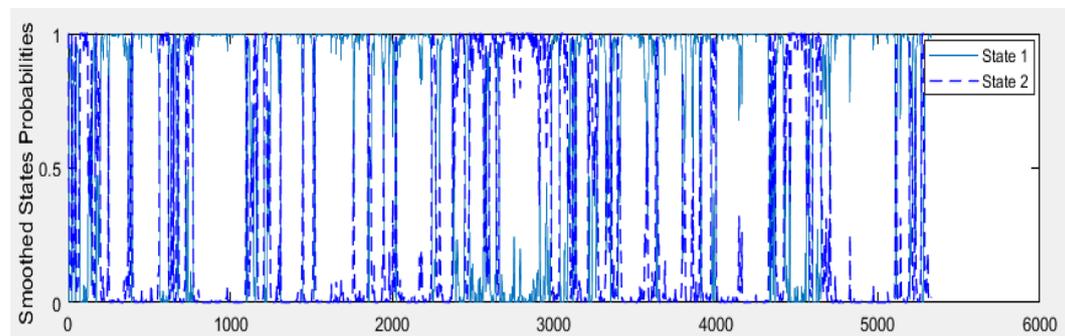


Fig. 8. Smoothed Probability of Markov switching model in Shenzhen

Fig. 7 and Fig. 8 illustrate the smoothed states probability of Markov switching model within the Shanghai and Shenzhen stock market, where the real line indicates the smoothed probability of the low-fluctuation regime (labeled as State1) and the dashed line indicates the smoothed probability of high-fluctuation regime.(labeled as State2). The disparity between these two states' probability in different intervals is relatively large, indicating that there have been many sudden fluctuations in the Chinese stock market. In general, the smoothed probability of the low-volatility regime was large in the early days, which provides an indication that the fluctuation of the stock market in China from

1997 to 2006 (1 to 2400 interval in the figure) was relatively stable. After 2007, the smoothed probability of the high-volatility regime was large, indicating that the stock market volatility was relatively large in this period. In 2009, China's Monetary Policy had changed from “tight” to “loose” in response to the financial crisis, and the newly issued currency increased. So, the stock price had been continuously rising in the whole year of 2009. The corresponding conclusion that China entered the low-volatility regime period is shown in these figures (2900 to 3200 interval). In 2010, due to the external factors such as European debt crisis and the changes of domestic policy, the stock market was more volatile. In this figure, it is shown that the high volatility regime has a higher smoothed probability in the interval from 3200 to 3400. The stock price was becoming stable step by step from 2011, and it is shown in figure (3400 to 4300 interval) that smoothed probability of the low-volatility regime was large at that time. The steady phenomenon continued until the end of 2014. Due to the sharp increase in brokerage stocks and the movement of leverage funds from the end of 2014 to 2015, it is shown in figure (4400 to 4500 interval) that the smoothed probability of high-volatility regimes was large, and the stock market shook heavily in this period. Meanwhile, the government introduced corresponding policies and increased the total amount of funds entering the market. Therefore, from the end of 2015 to the middle of 2016, the stock market gradually became stable. Due to strengthening supervision in the financial market and normalizing issuance of new shares, the entire stock market was relatively stable from 2016 to 2018, it is also shown in the figure (4900 to 5300 interval) that low-volatility regime has a higher smoothed probability. It can be seen that the Markov switching model describes and measures the fluctuations of Chinese stock market completely and accurately.

5. Conclusion

Through the above empirical analysis, it is obvious that, both models have studied the volatility of the Chinese stock market. However, the GARCH family model is simple and portrays the characteristics -- volatility clustering and asymmetry of the Chinese stock market. The Markov switching model not only fits the fluctuation, but also shows that the Chinese stock market has already transferred between the high and the low volatility regimes. Therefore, Markov switching model is able to more accurately fit the development of the Chinese stock market. According to the results of the Markov switching model, the Chinese stock market may be divided into the following stages. In 1997, the Chinese stock market was in a high volatility regime, and it was in a low volatility regime from 1998 to mid-2001, high volatility regime from mid-2001 to 2002. From 2003 to mid-2007, it was in a low-volatility regime, and from the end of 2007 to 2010, it was in a high-volatility regime. In 2011, the Chinese stock market once again entered a low-volatility regime until the end of 2014. From the end of 2014 to the beginning of 2016, the Chinese stock market was in a high volatility regime. In addition, it was in a low volatility regime from 2016 to 2018. In general, the Markov switching model fits the volatility of Chinese stock market more reasonably, providing a new modeling method for us to study the volatility of the Chinese stock market.

Acknowledgments

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