Abstract—This paper discusses ruin problems in the renewal risk model with interest force. By using inductive method, the recursive expressions of the distribution of the ruin probability, the distribution of maximum surplus before the ruin and distribution of minimum surplus before the ruin are obtained, and the corresponding integral equation for the distributions are obtained.

Keywords—risk model; Interest force; renewal risk model; ruin probability; maximum surplus; minimum surplus

I. INTRODUCTION

In recent years, the classic risk model has received a remarkable amount of attention and there have been many generalizations. Sundt and Teugels (1995,1997) considered a compound Poisson model with a constant interest force, by using renewal techniques, upper and lower bounds for the ruin probability and the integral equation satisfied by the ruin probability were obtained. Yang and Zhang (2001a, 2001b, 2001c) used the techniques of Sundt and Teugels (1995), some related problems were obtained. Yang (1998) considered a discrete time risk model with a constant interest force, both Lundberg-type inequality and non-exponential upper bounds for ruin probabilities were obtained by using martingale inequalities. Renewal risk model with interest force as a generalization of the classic risk model was considered in Wu and Du (2002), the problem was translated discrete, then used Markov property and transition probability to derive the explicit expression for the ruin probability. Lin and Wang (2005) adopted a different discrete techniques, derived the distribution of surplus immediately before ruin and that of deficit at ruin, further the integral equations of these distributions were obtained.

In this paper, we consider renewal risk model with interest force, by using the techniques of Lin and Wang (2005), the ruin probability, the minimum surplus before the ruin, the maximum surplus before the ruin and its corresponding integral equations for those distributions are obtained.

The paper is organized as follows: we define the model in section II; then, in section III, IV, V, we discuss the distribution of the ruin probability, the minimum surplus before the ruin, the maximum surplus before the ruin, the recursive expressions and integral equation for the distributions are obtained; finally conclude is in section VI.

II. DEFINITION OF THE MODEL

Let \((\Omega, F, P)\) be a complete probability space. We consider the renewal risk model with interest force. Suppose \(S(t)\) denote the amount of claim in the time interval \((0,t]\), i.e. \(S(t) = \sum_{i=1}^{N(t)} X_i\), where \(\{X_i, i \geq 1\}\) is independent and identically distributed (i.i.d.) random variables with common distribution function \(F(x)\), denotes the amount of the \(i\) th claim. The counting process \(\{N(t), t \geq 0\}\) denotes the number of claims up to time \(t\) and is defined as \(N(t) = \max\{k: W_1 + W_2 + \cdots + W_k \leq t\}\), where the inter-claim times \(\{W_i, i \geq 1\}\) are assumed to be i.i.d. random variables with commom distribution function \(K(w)\). Further, we assume the sequences \(\{W_i, i \geq 1\}\) and \(\{X_i, i \geq 1\}\) are independent, and that \(cE(W_i) > E(X_i)\), providing a positive safety loading factor.

Let \(U_\delta(t)\) denotes the insurance company’s surplus at time \(t\). From the above assumption, it follows that
\[
\frac{dU_\delta(t)}{dt} = cdt + U_\delta(t)\exists t - dS(t)
\]
From (1), we know that
\[
U_\delta(t) = \text{e}^{\alpha t} + cS_\delta\gamma + \int_0^t e^{\delta(t-v)} dS(v)
\]
where
\[
S_\delta = \int_0^t e^{\delta v} dv = \begin{cases} t & \text{if} \quad \delta = 0 \\ \frac{e^{\delta t} - 1}{\delta} & \text{if} \quad \delta > 0 \end{cases}
\]
\(u > 0\) is initial surplus of insurance company, \(c > 0\) is the premium income of unite time, \(\delta\) is constant interest force.

Definition 1

If \(T = \inf\{t > 0 : U_\delta(t) < 0\}\) (\(T = \infty\) if the set is empty), \(T\) is the ruin time. Obviously, it’s a stopping time.

Definition 2
Let $\Psi_{\delta}(u)$ denote the ultimate ruin probability with initial reserve $u$. That is,
\[
\Psi_{\delta}(u) = P_{\cap_{t\geq 0}}(U_{\delta}(t) < 0 \mid U_{\delta}(0) = u)
\]

When $t = T_n$, we have
\[
U_{\delta}(T_n) = e^{\delta t} + c e^{\delta t} - 1 - \sum_{i_1=1}^{n_1} X_i e^{\delta (t - T_{i_1})}
\]

when $t = T_n$, we have
\[
U_{\delta}(T_n) = e^{\delta t} + c e^{\delta t} - 1 - \sum_{i=1}^{n} X_i e^{\delta (t - T_i)}
\]

where $Y_i = X_i - c e^{\delta t} - 1 / \delta$, $i \geq 1$.

Obviously, $\{Y_i, W_i, i \geq 1\}$ are independent and have the same distribution $G(y, w)$, $G(y, w) = P(Y_i = X_i - c e^{\delta t} - 1 / \delta \leq y, W_i \leq w)
\]

Theorem 1 Let $\Psi_{\delta}(u)$ be defined as (3), then we have
\[
\Psi_{\delta}(u) = P[T < \infty] = \sum_{n=1}^{\infty} f_n(u)
\]

where
\[
f_1(u) = \int_0^{\infty} F(u e^{\delta t} + c S_{\delta t}) dK(t)
\]

\[
f_n(u) = \int_0^{\infty} e^{\delta t} F(u e^{\delta t} + c S_{\delta t} - y) dF(y) dK(t)
\]

Proof
\[
\Psi_{\delta}(u) = P[T < \infty] = \sum_{n=1}^{\infty} P[T = T_n]
\]

\[
= \sum_{n=1}^{\infty} P(U_{\delta}(T_n) \geq 0, U_{\delta}(T_2) \geq 0, \ldots, U_{\delta}(T_{n+1}) \geq 0, U_{\delta}(T_n) < 0)
\]

According to the definition (3), when $n = 1$
\[
f_1(u) = P(U_{\delta}(T_1) < 0)
\]

\[
= P[e^{\delta t} - Y_1 < 0]
\]

\[
= \int_0^{\infty} \int_0^{\infty} P[e^{\delta t} - Y_1 < 0 \mid Y_1 = y, W_1 = t] dG(y, t)
\]

\[
= \int_0^{\infty} \int_0^{\infty} dF(y + c S_{\delta t}) dK(t)
\]

\[
= \int_0^{\infty} \int_0^{\infty} F(e^{\delta t} + c S_{\delta t}) dK(t)
\]

\[
f_2(u) = P(U_{\delta}(T_1) \geq 0, U_{\delta}(T_2) < 0)
\]

\[
= P[e^{\delta t} - Y_1 \geq 0, e^{\delta t} - e^{\delta (t - T_1)} Y_1 - Y_2 < 0]
\]

\[
= \int_0^{\infty} \int_0^{\infty} P[e^{\delta t} - Y_1 \geq 0, e^{\delta t} - e^{\delta (t - T_1)} Y_1 - Y_2 < 0] dG(y, t)
\]

\[
= \int_0^{\infty} \int_0^{\infty} F(e^{\delta t} + c S_{\delta t} - y) dF(y) dK(t)
\]

By inductive assumption, when $n \geq 3$, we have
\[
f_n(u) = P(U_{\delta}(T_1) \geq 0, U_{\delta}(T_2) \geq 0, \ldots, U_{\delta}(T_{n+1}) \geq 0, U_{\delta}(T_n) < 0)
\]

\[
= P[e^{\delta t} - Y_1 \geq 0, e^{\delta t} - e^{\delta (t - T_1)} Y_1 - Y_2 \geq 0, \ldots, U_{\delta}(T_{n+1}) \geq 0, U_{\delta}(T_n) < 0]
\]

\[
= \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} P((e^{\delta t} - y) e^{\delta t} - Y_2 < 0] dG(y, t)
\]

\[
= \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \ldots \int_0^{\infty} P((e^{\delta t} - y) e^{\delta t} - Y_2 \geq 0]
\]

\[
= \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \ldots \int_0^{\infty} P((e^{\delta t} - y) e^{\delta t} - Y_2 < 0] dG(y, t)
\]

\[
= \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \ldots \int_0^{\infty} P((e^{\delta t} - y) e^{\delta t} - Y_2 \geq 0]
\]

\[
= \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \ldots \int_0^{\infty} P((e^{\delta t} - y) e^{\delta t} - Y_2 < 0] dG(y, t)
\]

\[
= \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \ldots \int_0^{\infty} P((e^{\delta t} - y) e^{\delta t} - Y_2 \geq 0]
\]

\[
= \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \ldots \int_0^{\infty} P((e^{\delta t} - y) e^{\delta t} - Y_2 < 0] dG(y, t)
\]

\[
= \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \ldots \int_0^{\infty} P((e^{\delta t} - y) e^{\delta t} - Y_2 \geq 0]
\]

\[
= \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \ldots \int_0^{\infty} P((e^{\delta t} - y) e^{\delta t} - Y_2 < 0] dG(y, t)
\]

\[
= \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \ldots \int_0^{\infty} P((e^{\delta t} - y) e^{\delta t} - Y_2 \geq 0]
\]

\[
= \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \ldots \int_0^{\infty} P((e^{\delta t} - y) e^{\delta t} - Y_2 < 0] dG(y, t)
\]

\[
= \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \ldots \int_0^{\infty} P((e^{\delta t} - y) e^{\delta t} - Y_2 \geq 0]
\]

\[
= \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \ldots \int_0^{\infty} P((e^{\delta t} - y) e^{\delta t} - Y_2 < 0] dG(y, t)
\]
\[
= \int_{-\infty}^{\infty} \int_{c-\varepsilon}^{-\infty} f_{n-1}(ue^{\delta} - y) dF(y + c\bar{S}_n) dK(t)
\]
\[
= \int_{0}^{\infty} \int_{0}^{\infty} f_{n-1}(ue^{\delta} + c\bar{S}_n - y) dF(y) dK(t)
\]

Obviously \(\sum f_n(u)\) is convergence, that \(\Psi_\delta(u)\) has the following theorem 2

**Theorem 2** Let \(\Psi_\delta(u)\) be defined as (3), then \(\Psi_\delta(u)\) satisfies the following integral equation

\[
\Psi_\delta(u) = \int_{0}^{\infty} F(u e^\delta + c\bar{S}_n) dK(t)
\]
\[
+ \int_{0}^{\infty} \int_{0}^{\infty} \psi_\delta(u e^\delta + c\bar{S}_n - y) dF(y) dK(t)
\]

**Proof**

\[
\Psi_\delta(u) = \sum_{n=1}^{\infty} f_n(u) = f_1(u) + \sum_{n=2}^{\infty} f_n(u)
\]
\[
= f_1(u) + \sum_{n=2}^{\infty} \int_{0}^{\infty} \int_{c-\varepsilon}^{-\infty} f_{n-1}(ue^{\delta} + c\bar{S}_n - y) dF(y) dK(t)
\]
\[
= f_1(u) + \int_{0}^{\infty} \int_{0}^{\infty} \psi_\delta(u e^\delta + c\bar{S}_n - y) dF(y) dK(t)
\]
\[
= \int_{0}^{\infty} F(u e^\delta + c\bar{S}_n) dK(t)
\]
\[
+ \int_{0}^{\infty} \int_{0}^{\infty} \psi_\delta(u e^\delta + c\bar{S}_n - y) dF(y) dK(t)
\]

**IV. THE DISTRIBUTION OF MAXIMUM SURPLUS BEFORE THE RUIN**

Denote the distribution of maximum before the ruin with the initial reserve \(u\) by

\[
H(u,x) = P\{\sup_{0 \leq t \leq T} U_\delta(t) \leq x, T < \infty | U_\delta(0) = u\}
\]

**Theorem 3** Let \(H(u,x)\) be defined as (4), then we can get

1. When \(x < u\), we have
   \[
   H(u,x) = 0;
   \]
2. When \(x \geq u\), we have
   \[
   H(u,x) = \sum_{n=1}^{\infty} h_n(u,x)
   \]
where \(n = 1\), we have

\[
h_1(u,x) = \int_{0}^{\infty} F(u e^\delta + c\bar{S}_n) dK(t)
\]
and \(n \geq 2\), we have

\[
h_n(u,x) = \int_{0}^{\infty} \int_{c-\varepsilon}^{-\infty} f_{n-1}(ue^{\delta} + c\bar{S}_n - y,x) dF(y) dK(t)
\]

**Proof**

(1) when \(x < u\), according to definition (4), we easily get

\[
H(u,x) = 0
\]

(2) when \(x \geq u\), we have

\[
H(u,x) = P\{\sup_{0 \leq t \leq T} U_\delta(t) \leq x, T < \infty \}
\]
\[
= \sum_{n=1}^{\infty} P\{\sup_{0 \leq t \leq T} U_\delta(t) \leq x, T < \infty \}
\]
\[
= \sum_{n=1}^{\infty} P\{0 \leq U_\delta(T_1) \leq x, 0 \leq U_\delta(T_2) \leq x, \cdots, 0 \leq U_\delta(T_{n-1}) \leq x, U_\delta(T_n) > x\}
\]
\[
= \sum_{n=1}^{\infty} h_n(u,x)
\]

According to definition (4), when \(n = 1\)

\[
h_1(u) = P\{0 \leq u \leq x, U_\delta(T_1) < 0\}
\]
\[
= P\{ue^{\delta t_1} - Y_1 < 0\}
\]
\[
= \int_{0}^{\infty} F(u e^\delta + c\bar{S}_n) dK(t)
\]
\[
h_2(u) = P\{0 \leq U_\delta(T_1) \leq x, U_\delta(T_2) < 0\}
\]
\[
= P\{0 \leq ue^{\delta t_1} - Y_1 \leq x, ue^{\delta t_2} - e^{\delta(t_2-t_1)}Y_1 < Y_2 < 0\}
\]
\[
= \int_{0}^{\infty} \int_{0}^{\infty} P\{0 \leq u e^{\delta t_1} - Y_1 < x, u e^{\delta(t_1+t_2)} - Y_2 < 0\}
\]
\[
= \int_{0}^{\infty} \int_{0}^{\infty} P\{(|u e^{\delta t_1} - Y_1| e^{\delta t_1} - Y_2 < 0\}
\]
\[
= \int_{0}^{\infty} \int_{0}^{\infty} P\{(|u e^{\delta t_1} - Y_1| e^{\delta t_1} - Y_2 < 0\}
\]
\[
= \int_{0}^{\infty} \int_{0}^{\infty} h_1(u e^{\delta t_1} + c\bar{S}_n - y,x) dF(y) dK(t)
\]

By inductive assumption, when \(n \geq 3\), we have

\[
h_n(u) = P\{0 \leq U_\delta(T_1) \leq x, 0 \leq U_\delta(T_2) \leq x, \cdots, 0 \leq U_\delta(T_{n-1}) \leq x, U_\delta(T_n) < 0\}
\]
\[
= P\{0 \leq u e^{\delta t_1} - Y_1 \leq x, 0 \leq u e^{\delta(t_1+t_2)} - e^{\delta(t_1-t_2)}Y_1 - Y_2 \leq x, 
\]
\[
\cdots, ue^{\delta t_{n-1}} - \sum_{j=1}^{n-1} Y_je^{\delta t_{n-1}} < 0\}
\]
\[
= \int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} h_{n-1}(u e^{\delta t_1} + c\bar{S}_n - y,x) dF(y) dK(t)
\]

388
\[ \cdots, (ue^\delta - y)e^{-\sum_{j=1}^{n} W_j} - \sum_{j=1}^{n} Ye^{-\sum_{j=1}^{n} W_j} < 0 \] \]

\[ = \int_0^\infty \int_{ue^\delta} y h_{n-1}(ue^\delta - y, x)dF(y + c\overline{S}_n^\delta)dK(t) \]

\[ = \int_0^\infty \int_{ue^\delta + c\overline{S}_n^\delta} h_{n-1}(ue^\delta + c\overline{S}_n^\delta - y, x)dF(y)dK(t) \]

We know that \( \sum_{n=1}^{\infty} h_n(u, x) \) is convergence. Then, we can get the follow theorem

**Theorem 4** Let \( H(u, x) \) be defined as (4), then \( H(u, x) \) satisfies the following integral equation

\[ H(u, x) = \sum_{n=1}^{\infty} h_n(u, x) = h_1(u) + \sum_{n=2}^{\infty} h_n(u) \]

\[ = h_1(u) + \sum_{n=2}^{\infty} \int_{ue^\delta + c\overline{S}_n^\delta}^\infty \int_{ue^\delta + c\overline{S}_n^\delta} h_{n-1}(ue^\delta + c\overline{S}_n^\delta - y, x)dF(y)dK(t) \]

\[ = h_1(u) + \int_{ue^\delta + c\overline{S}_n^\delta}^\infty \int_{ue^\delta + c\overline{S}_n^\delta} h_{n-1}(ue^\delta + c\overline{S}_n^\delta - y, x)dF(y)dK(t) \]

\[ = \int_{0}^{\infty} \int_{ue^\delta + c\overline{S}_n^\delta}^\infty H(ue^\delta + c\overline{S}_n^\delta - y, x)dF(y)dK(t) \]

\[ = \int_{0}^{\infty} F(ue^\delta + c\overline{S}_n^\delta)dK(t) \]

\[ + \int_{0}^{\infty} \int_{ue^\delta + c\overline{S}_n^\delta}^\infty H(ue^\delta + c\overline{S}_n^\delta - y, x)dF(y)dK(t) \]

V. THE DISTRIBUTION OF MINIMUM SURPLUS BEFORE THE RUIN

Denote the distribution of minimum before the ruin with the initial reserve \( u \) by

\[ K(u, x) = P\{ \inf_{t \geq 0} U_\delta(t) \geq x, T < \infty \left| U_\delta(0) = u \right\} \] (5)

**Theorem 3** Let \( K(u, x) \) be defined as (4), then we can get

(3) When \( x < u \), we have

\[ K(u, x) = 0; \]

(4) When \( x \geq u \), we have

\[ K(u, x) = \sum_{n=1}^{\infty} k_n(u, x) \]

where \( n = 1 \), we have

\[ k_1(u, x) = \int_{0}^{\infty} F(ue^\delta + c\overline{S}_n^\delta)dK(t) \]

and \( n \geq 2 \), we have

\[ k_n(u, x) = \int_{0}^{\infty} \int_{ue^\delta + c\overline{S}_n^\delta}^\infty k_{n-1}(ue^\delta + c\overline{S}_n^\delta - y, x)dF(y)dK(t) \]

**Proof**

(1) when \( x < u \), according to definition (5), we easily get

\[ K(u, x) = 0 \]

(2) when \( x \geq u \), we have

\[ K(u, x) = P\{ \inf_{t \geq 0} U_\delta(t) \leq x, T < \infty \} \]

\[ = \sum_{n=1}^{\infty} P\{ \inf_{t \geq 0} U_\delta(t) \leq x, T = T_n \} \]

\[ = \sum_{n=1}^{\infty} P\{ U_\delta(T_n) \geq x, U_\delta(T_2) \geq x, \ldots, U_\delta(T_{n-1}) \geq x \} \]

\[ = \sum_{n=1}^{\infty} k_n(u, x) \]

According to definition (5), when \( n = 1 \)

\[ k_1(u, x) = P\{ u \geq x, U_\delta(T_1) < 0 \} \]

\[ = P\{ue^\delta - Y_1 < 0 \} \]

\[ = \int_{0}^{\infty} F(ue^\delta + c\overline{S}_n^\delta)dK(t) \]

\[ k_2(u, x) = P\{ U_\delta(T_1) \geq x, U_\delta(T_2) < 0 \} \]

\[ = P\{ue^{\delta_1} - Y_1 \geq x, ue^{\delta_2} - e^{\delta(T_2 - T_1)}Y_1 - Y_2 < 0 \} \]

\[ = \int_{0}^{\infty} \int_{0}^{\infty} P\{ue^{\delta_1} - Y_1 \geq x, ue^{\delta_2} - e^{\delta(T_2 - T_1)}Y_1 - Y_2 < 0 \}dG(y, t) \]

\[ = \int_{0}^{\infty} \int_{0}^{\infty} P\{ue^{\delta - x} - y \leq 0 \}dF(y + c\overline{S}_n^\delta)dK(t) \]

\[ = \int_{0}^{\infty} \int_{0}^{\infty} P\{ue^{\delta} + c\overline{S}_n^\delta - y \leq 0 \}dF(y)dK(t) \]

\[ = \int_{0}^{\infty} \int_{0}^{\infty} k_1(ue^{\delta} + c\overline{S}_n^\delta - y, x)dF(y)dK(t) \]

By inductive assumption, when \( n \geq 3 \), we have

\[ k_n(u, x) = P\{U_\delta(T_1) \geq x, U_\delta(T_2) \geq x, \ldots, U_\delta(T_{n-1}) \geq x, U_\delta(T_n) < 0 \} \]

\[ = P\{0 \leq ue^{\delta_1} - Y_1 \leq x, 0 \leq ue^{\delta(T_2 - T_1)} - e^{\delta(T_2 - T_1)}Y_1 - Y_2 \leq x, \ldots, \}

\[ \leq \sum_{i=1}^{n} ye^{\delta(W_i - W_{i-1})} < 0 \}

\[ = \int_{0}^{\infty} \int_{0}^{\infty} k_1(ue^{\delta} - y)ye^{\delta(W_i - W_{i-1})} - e^{\delta(W_i - W_{i-1})}Y_1 - Y_2 < 0 \}

\[ \geq x, \]

389


...\((ue^\theta - y)e^{-\delta \sum_{j=1}^n w_j} - \sum_{i=2}^{n}Ye^{-\delta \sum_{i=1}^n w_i} < 0)\ dG(y,t)\)

\[= \int_0^\infty \int_{-\infty}^{\infty} k_{n-1}(ue^\theta - y, x)dF(y + cS\bar{t}_1)dK(t)\]

\[= \int_0^\infty \int_{0}^{\infty} k_{n-1}(ue^\theta + cS\bar{t}_1 - y, x)dF(y)dK(t)\]

We know that \(\sum_{n=1}^{\infty} k_n(u, x)\) is convergence. Then, we can get the follow theorem

**Theorem 4** Let \(K(u, x)\) be defined as (5), then \(K(u, x)\) satisfies the following integral equation

\[K(u, x) = \int_0^\infty F(ue^\theta + cS\bar{t}_1)dK(t)\]

\[+ \int_0^\infty \int_{0}^{\infty} K(u, x + cS\bar{t}_1 - y, x)dF(y)dK(t)\]

**Proof**

\[K(u, x) = \sum_{n=1}^{\infty} k_n(u) = k_1(u) + \sum_{n=2}^{\infty} k_n(u)\]

\[= k_1(u) + \sum_{n=2}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} k_{n-1}(ue^\theta + cS\bar{t}_1 - y, x)dF(y)dK(t)\]

\[= k_1(u) + \int_{0}^{\infty} \int_{0}^{\infty} k_{n-1}(ue^\theta + cS\bar{t}_1 - y, x)dF(y)dK(t)\]

\[= k_1(u) + \int_{0}^{\infty} \int_{0}^{\infty} K(ue^\theta + cS\bar{t}_1 - y, x)dF(y)dK(t)\]

\[= \int_0^\infty F(ue^\theta + cS\bar{t}_1)dK(t)\]

\[+ \int_0^\infty \int_{0}^{\infty} K(ue^\theta + cS\bar{t}_1 - y, x)dF(y)dK(t)\]

VI. CONCLUSION

In this paper, we have studied the renewal risk model with interest force. Via inductive method technique, some important distributions are obtained. Main results are:

1. The recursive expression of the distribution of the ruin probability is obtained, and its corresponding integral equation for the distribution is obtained.
2. The distribution of maximum surplus before the ruin is obtained, and its corresponding integral equation for the distribution is obtained.
3. The distribution of minimum surplus before the ruin is obtained, and its corresponding integral equation for the distribution is obtained.

ACKNOWLEDGMENT

The authors are most grateful to the anonymous referees for helpful suggestions, and this paper was supported by the Youth Foundation of Hubei Educational Commission (No. Q20122603).

REFERENCES