

The Distributed Detection Arithmetic with Fading Channel Based on The Minimum Error Probability

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Abstract

In the actual detection, due to bandwidth, channel fading and channel noise, the ideal channel isn't assured from the local detector to fusion center, so the optimal detection arithmetic is accordingly rectified. In this paper, we study the global optimal detection algorithm based on the two kinds of non-ideal channel schemes, firstly, channel state information is known; secondly, channel statistics characteristics is known. The configuration of the system is mixed with parallel and serial structure. The decision rules of the every local detector and every local processor must be optimized jointly based on the minimum error probability. At last the stimulation favors the analysis.

Keywords: Detection fusion, Non-ideal channel, Global optimization

1. Introduction

The study of distributed detection has traced back to 1960s, the effort significantly intensified since the publication of [1]. In [1], Tenney and Sandell formulated the distributed detection problem using a Bayesian setting. Later the distributed detection developed very quickly, This work was later generalized to multiple sensors by Reibman and Nolte [2] and by Hoballah, Varshney [3]. Similarly, under the Neyman-Pearson (NP) criterion, the optimality of the local LRT has been established in [4]-[6]. In the above mentioned papers, all of them have an assumption: the local decisions are transmitted to fusion center with no error. Yet we must face to non-ideal channel during the actual detection process owing to bandwidth, channel fading and channel noise, the non-ideal channel results in the error during the transmission, based on the error decision received in the fusion center, the performance using the above arithmetic[1]-[6] is degraded distinctly, so the optimal detection arithmetic is rectified accordingly. A few papers[7]-[9] have discussed the arithmetic involving of non-ideal channel in distributed detection, in [7][8]

the optimal detection arithmetic of parallel and serial configuration have been studied based on binary symmetry channel, in [9] it is shown that the fusion center and local detectors are jointly optimized based on two non-ideal channel states in parallel configuration.

In order to enrich the system info of distributed detection, an optimized detection arithmetic is studied based on two kinds of non-ideal channel schemes in this paper, firstly, channel state information is known; secondly, channel statistics characteristics is known. In addition, the configuration of the system is the mixture of parallel and serial structure. According to the minimum error probability rule, the optimum decision rule of the local detectors and local processors are derived. At last, stimulation result shows that the performance of the second channel schemes suffer small performance loss compared with the first channel schemes, but its calculation is simplified and it is applied much more practically.

2. System model

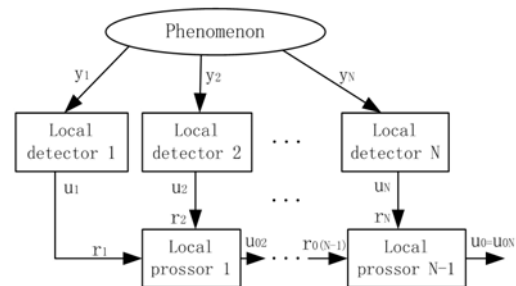


Fig.1: the system figure of the distributed detection.

Consider the problem of testing two hypotheses, denoted by H_0 (existing target) and H_1 (no target), with the prior probabilities π_0 and π_1 respectively. The distributed detection system comprises parallel N local detectors, serial $N-1$ local processors and independent non-ideal channels, the system figure is shown in Fig.1, each local detector makes a decision $u_i (i = 1, 2, \dots, N)$ based on its own observation y_i , throughout this paper we assume that the observations

$y_i(i=1,2,\dots,N)$ are conditionally independent, then this decision u_i is transmitted to the corresponding local processor through a non-ideal channel, moreover the local processor makes a decision $u_{0i}(i=2,\dots,N)$ based on the received decision $r_{0(i-1)}$ and r_i , then this decision u_{0i} is transmitted to the next local processor through a non-ideal channel, the output u_{0N} of local processor $N-1$ is the final decision u_0 .

In this paper, the optimal decision rule of local detectors and local processors are derived based on minimized the average error probability defined as:

$$P_{e0} = \int_g \sum_{j=0}^1 \pi_j P(u_0 = 1 - j | H_j, g) p(g) dg$$

$$= \int_g (\pi_0 P(u_0 = 1 | H_0, g) - \pi_1 P(u_0 = 1 | H_1, g) + \pi_1) p(g) dg \quad (1)$$

Where $g = \{g_1, \dots, g_N, g_{02}, \dots, g_{0(N-1)}\}$ is channel state information, $p(g)$ is probability density function of g . The remainder of the paper studies the optimized decision rule in following two kinds non-ideal channel schemes.

3. The optimal decision rule based on channel state information

We adopt a person-by-person optimization approach, namely we optimize the decision rule of a local detector or a local processor giving decision rules at all other local detectors and local processors fixed.

3.1. The optimal decision rule of local processor

In [10] the author testify that in serial topology configure if each local processor make an optimal decision then the system can obtain the optimal detection property. We can expand the average error probability in (1) with respect to the output u_{0i} and input $r_{0(i-1)}$ 、 r_i of local processor $i-1(2 < i \leq N-1)$, and we get:

$$P_{e0} = \int_g \int_{r_i} \int_{r_{0(i-1)}} \{ [\pi_0 P(u_0 = 1 | u_{0i} = 0, H_0, g) p(u_{0i} = 0 | r_i, r_{0(i-1)}) P(r_i | H_0) p(r_{0(i-1)} | H_0) + \pi_0 P(u_0 = 1 | u_{0i} = 1, H_0, g) p(u_{0i} = 1 | r_i, r_{0(i-1)}) P(r_i | H_0) p(r_{0(i-1)} | H_0)] - [\pi_1 P(u_0 = 1 | u_{0i} = 0, H_1, g) p(u_{0i} = 0 | r_i, r_{0(i-1)}) P(r_i | H_1) p(r_{0(i-1)} | H_1) + \pi_1 P(u_0 = 1 | u_{0i} = 1, H_1, g) p(u_{0i} = 1 | r_i, r_{0(i-1)}) P(r_i | H_1) p(r_{0(i-1)} | H_1)] + \pi_1 \} p(g) dr_i dr_{0(i-1)} dg \quad (2)$$

Observing that $p(u_{0i} = 0 | r_i, r_{0(i-1)}) = 1 - p(u_{0i} = 1 | r_i, r_{0(i-1)})$,

substitute into (2) we have:

$$P_{e0} = \int_g C_1 + \int_{r_i} \int_{r_{0(i-1)}} \{ \pi_0 A(u_0, u_{0i}, H_0, g) P(u_{0i} = 1 | r_i, r_{0(i-1)}) P(r_i | H_0) - \pi_1 A(u_0, u_{0i}, H_1, g) P(u_{0i} = 1 | r_i, r_{0(i-1)}) P(r_i | H_1) p(r_{0(i-1)} | H_1) \} p(g) dr_i dr_{0(i-1)} dg \quad (3)$$

where:

$$A(u_{0N}, u_{0i}, H_1, g) = P(u_{0N} = 1 | u_{0i} = 1, H_1, g) - P(u_{0N} = 1 | u_{0i} = 0, H_1, g) \quad (4)$$

According to [6], we obtain:

$$\begin{cases} A(u_{0N}, u_{0i}, H_j, g) = \prod_{k=i+1}^N A(u_{0k}, u_{0(k-1)}, H_j, g) \\ A(u_{0N}, u_{0i}, H_j, g) > 0, \quad k = 1, 2, \dots, N-1, j = 0, 1 \end{cases}$$

Otherwise C_1 in (3) can be expressed:

$$C_1 = \pi_0 P(u_0 = 1 | u_{0i} = 0, H_0, g) - \pi_1 P(u_0 = 1 | u_{0i} = 0, H_1, g) + \pi_1$$

obviously C_1 is a constant with regard to the decision rule of local processor $i-1(2 < i \leq N-1)$.

To minimize P_{e0} , from (3) the optimal decision rule for local processor $i-1(2 < i \leq N-1)$ is:

$$\frac{P(r_i | H_1) p(r_{0(i-1)} | H_1)}{P(r_i | H_0) p(r_{0(i-1)} | H_0)} \stackrel{H_1}{>} \frac{\pi_0 A(u_0, u_{0i}, H_0, g)}{\pi_1 A(u_0, u_{0i}, H_1, g)} \quad (5)$$

where

$$\begin{cases} p(r_i | H_j) = \int_{g_i} [\sum_{u_i} p(r_i | u_i, g_i) P(u_i | H_j)] p(g_i) dg_i \\ p(r_{0(i-1)} | H_j) = \int_{g_{(i-1)}} [\sum_{u_{0(i-1)}} p(r_{0(i-1)} | u_{0(i-1)}, g_{(i-1)}) P(u_{0(i-1)} | H_j)] p(g_{(i-1)}) dg_{(i-1)} \end{cases} \quad (6)$$

Since there is no local processor before local processor 1, the input of local processor 1 are the received decision of local detector 1 and 2, similarly, we can get the optimal decision rule for local processor 1,

$$\frac{P(r_1 | H_1) p(r_2 | H_1)}{P(r_1 | H_0) p(r_2 | H_0)} \stackrel{H_1}{>} \frac{\pi_0 A(u_0, u_{02}, H_0, g)}{\pi_1 A(u_0, u_{02}, H_1, g)} \quad (7)$$

3.2. The optimal decision rule of local processor

While we design the optimal decision rule of one local processor, at first we assume that all of the local processors and the other local detectors fix the decision rules.

According to probability theory, we have:

$$P(r_i | H_j) = \int_{g_i} \int_{y_i} \sum_{u_i} P(r_i | u_i, g) P(u_i | y_i) p(y_i | H_j) p(g) dy_i dg, \quad j = 0, 1 \quad (8)$$

Substituting (8) into (3),

$$P_{e0} = \int_g C_1 + \int_{r_i} \int_{r_{0(i-1)}} \int_{y_i} \{ \pi_0 A(u_0, u_{0i}, H_0, g) p(u_{0i} = 1 | r_i, r_{0(i-1)}) p(r_i | u_i, g) P(u_i | y_i) p(y_i | H_0) p(r_{0(i-1)} | H_0) - \pi_1 A(u_0, u_{0i}, H_1, g) P(u_{0i} = 1 | r_i, r_{0(i-1)}) p(r_i | u_i, g) P(u_i | y_i) p(y_i | H_1) p(r_{0(i-1)} | H_1) \} p(g) dr_i dr_{0(i-1)} dy_i dg \quad (9)$$

obviously C_1 is a constant for local detector $i(i > 2)$. To minimize P_{e0} , one can see from (9) that the optimal decision rule for local detector $i(i > 2)$ is:

$$\frac{p(y_i | H_1)}{p(y_i | H_0)} \stackrel{H_1}{>} \frac{\pi_0 A(u_0, u_{0i}, H_0, g) p(r_{0(i-1)} | H_0)}{\pi_1 A(u_0, u_{0i}, H_1, g) p(r_{0(i-1)} | H_1)} \quad (10)$$

Due to the decisions of local detector 1、2 through non-ideal channel is the input of local processor 1,

similarly the optimal decision rule for local detector 1、2 are:

$$\frac{P(y_1 | H_1)}{P(y_1 | H_0)} \underset{u_1=0}{\overset{u_1=1}{>}} \frac{\pi_0 A(u_{0N}, u_{02}, H_0, g) P(r_2 | H_0)}{\pi_1 A(u_{0N}, u_{02}, H_1, g) P(r_2 | H_1)} \quad (11)$$

$$\frac{P(y_2 | H_1)}{P(y_2 | H_0)} \underset{u_2=0}{\overset{u_2=1}{>}} \frac{\pi_0 A(u_{0N}, u_{02}, H_0, g) P(r_1 | H_0)}{\pi_1 A(u_{0N}, u_{02}, H_1, g) P(r_1 | H_1)} \quad (12)$$

From (10)-(12), we can see that the likelihood ratio test threshold of local detector $i(i > 2)$ 、1、2 has relation to $r_{0(i-1)}$ 、 r_2 、 r_1 respectively.

4. The optimal decision rule on channel statistical property

Although the optimal decision rule for each sensor is explicitly formulated in (5)(7)(10)(11)(12), the likelihood ratio thresholds are intractable, as they involve highly nonlinear function g , The only possible way of finding the optimal decision rules of local processors and detectors appears to be an exhaustive search, whose complexity becomes prohibitive when N is large.

Instead of directly minimizing the average error probability as in (3), an alternative approach is to first average the channel transition probability with respect to the fading channel. That is, we compute $p(r_i | u_i)$, ($i = 1, \dots, N$) and $p(r_{0i} | u_{0i})$, ($i = 2, \dots, N$) by marginalizing out the channel density function $p(g_i)$, namely,

$$p(r_i | u_i) = \int_{g_i} p(r_i | u_i, g_i) p(g_i) dg_i, \quad i = 1, \dots, N \quad (13)$$

$$p(r_{0i} | u_{0i}) = \int_{g_i} p(r_{0i} | u_{0i}, g(i)) p(g(i)) dg(i), \quad i = 2, \dots, N-1 \quad (14)$$

According to (13)(14), we can obtain the

$$P(u_0 = 1 | u_{0i} = j, H_k), \quad (j, k = 0, 1; i = 2, \dots, N-1)$$

by marginalizing out the channel g :

$$P(u_0 = 1 | u_{0i} = j, H_k, g) = \int_g P(u_0 = 1 | u_{0i} = j, H_k, g) p(g) dg$$

Therefore we gain $A(u_0, u_{0i}, H_k)$. Similarly we have a result resembling that of knowing the channel state scheme, the optimal decision rules of local processor $i-1$ ($2 < i \leq N-1$)、local processor 1、local detector i ($i > 2$)、local detector 2、local detector 1 are respectively:

$$\frac{P(r_i | H_1) p(r_{0(i-1)} | H_1)}{P(r_i | H_0) p(r_{0(i-1)} | H_0)} \underset{u_{0i}=1}{\overset{u_{0i}=0}{>}} \frac{\pi_0 A(u_0, u_{0i}, H_0)}{\pi_1 A(u_0, u_{0i}, H_1)} \quad (15)$$

$$\frac{P(r_1 | H_1) p(r_2 | H_1)}{P(r_1 | H_0) p(r_2 | H_0)} \underset{u_{02}=0}{\overset{u_{02}=1}{>}} \frac{\pi_0 A(u_0, u_{02}, H_0)}{\pi_1 A(u_0, u_{02}, H_1)} \quad (16)$$

$$\frac{p(y_i | H_1)}{p(y_i | H_0)} \underset{u_i=0}{\overset{u_i=1}{>}} \frac{\pi_0 A(u_0, u_{0(i-1)}, H_0) p(r_{0(i-1)} | H_0)}{\pi_1 A(u_0, u_{0(i-1)}, H_1) p(r_{0(i-1)} | H_1)} \quad (17)$$

$$\frac{P(y_1 | H_1)}{P(y_1 | H_0)} \underset{u_1=0}{\overset{u_1=1}{>}} \frac{\pi_0 A(u_{0N}, u_{02}, H_0) P(r_2 | H_0)}{\pi_1 A(u_{0N}, u_{02}, H_1) P(r_2 | H_1)} \quad (18)$$

$$\frac{P(y_2 | H_1)}{P(y_2 | H_0)} \underset{u_2=0}{\overset{u_2=1}{>}} \frac{\pi_0 A(u_{0N}, u_{02}, H_0) P(r_1 | H_0)}{\pi_1 A(u_{0N}, u_{02}, H_1) P(r_1 | H_1)} \quad (19)$$

5. Simulation

In this section, a two-local-processor and three-local-detector example is used to evaluate the detection property with two kinds channel schemes. Consider the detection of a known signal S in zero-mean Gaussian noises that are independent and identically distributed (i.i.d.) for the three local detectors.

$$H_0: \quad y_i = N_i \quad (i = 1, 2, 3)$$

$$H_1: \quad y_i = S + N_i$$

Where $N_1, N_2, N_3 \sim CN(0, \sigma_1^2)$, Without loss of generality, we assume $S = 1, \sigma_1^2 = 2$.

Each sensor makes a binary decision u_i based on its observation y_i , and then transmits it through a Rayleigh fading channel to the corresponding local processor. The channel output is

$$r_i = g_i u_i + W_i$$

where g_1, g_2, g_3 are i.i.d. and zero-mean Gaussian distributed $g_1, g_2, g_3 \sim CN(0, \sigma_g^2)$, $\sigma_g^2 = 1$, W_1, W_2, W_3 are i.i.d. and zero-mean Gaussian noises with distribution $W_1, W_2, W_3 \sim CN(0, \sigma_w^2)$, $\sigma_w^2 = 1$.

In addition, the decision of local processor 1 is transmitted to local processor 2 through a Rayleigh fading channel, the channel output is:

$$r_{02} = g_{02} u_{02} + W_{02}$$

where $g_{02}, W_{02} \sim CN(0, \sigma_{02}^2)$ are i.i.d. we assume $\sigma_{02}^2 = 1$.

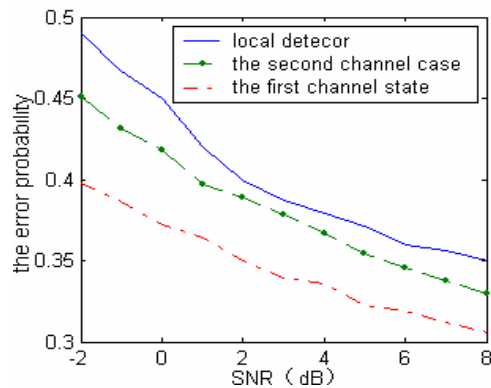


Fig.2: the error probability versus SNR.

If prior probabilities $\pi_1 = \pi_0$, the average error probabilities is a function of the average signal-to-noise ratio (SNR) of the received signal, Fig.2 shows that the error probability of local detectors and local

processors are affected by the SNR. From Fig.2, we can see the property of local processor is better than local detector, the property of knowing channel state information is better than that of knowing channel statistical performance, but the calculation of the second case becomes simple, this is very important especially to large N .

6. Conclusions

In this paper, the topology configuration is mixture of parallel and serial structure, in two channel cases, we derive the optimal decision rule of all sensors based on minimized the globe error probability. By stimulation we can know that though the second channel scheme suffer small performance loss compared with the first scheme, it overcome the problem of calculation intricacy, to more important, the second case has more practicability: firstly, the exact instant channel state can not be obtained accurately, however the statistical property of channel can gained by quantified sampling approximately; secondly if the channel state changes, but the channel statistical property remains unchanged, the optimal decision rule of the second channel case keeps applicable.

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