A Maximum Achievement Model for a Interdependent Multi-location Investment Problem Using Goal Programming and Piecewise-Linear Approximation

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Abstract

Consider an enterprise that wants to expand its business to multiple cities in global and to be one of well-known multinational enterprises (MNEs). Suppose the MNE hopes each planning investment subsidiary gleans a specific target return within a constant time horizon. Under this premise, this paper proposes two optimization models to find the optimal allocation policy of capital investment, which maximize the degree of realization of the MNE's concerned objective. Due to the nonlinear characteristics of the proposed models, a solution procedure developed upon the piecewise-linear approximation and goal programming techniques is used for resolving the models.

Keywords: multi-location investment, goal programming, piecewise-linear

1. Introduction

To invest multiple international cities successfully with the mode of wholly-owned, the choice of resource allocation and/or transfer strategy is one of key issues to enterprises. If investing multiple cities, resolving the budget constrained scheduling of investments is equivalent to resolving a multi-project resource allocation problem. Traditional models corresponding to such issue is on the aspect of selecting and/or scheduling projects limited to the total amount of resource (see for example [1,2]). Not incorporating the soft factors, such as control competence of an organization, into the models of multi-project resource allocation problems to predict all projects' performance is the main gap of the conventional models mentioned above. Chang and Chen [3] proposed an alternative insight, termed Project Advancement (PA), to extend the view of point of project selection and/or scheduling. PA suggests a seven-step list to model a resource allocation problem of multiple projects. Not concerning here with the novel introduction of PA, but we would like to focus attention on the application of project advancement strategies defined in PA. We will introduce four types of project advancement strategies defined in PA in he following section. When one plans to do multiple projects, PA suggests choosing an appropriate project advancement strategy to avoid or decline the influence of ill noises resulting from internal and external environment. Owing to the high complexity of choosing an appropriate project advancement strategies, this paper will focus on modeling two DSAS-based and budget-constrained multi-city investment problems. DSAS refers to expanding business by decentralizing the available amount of capital budget into each planning investment location and investing them concurrently. Owing to the nonlinear characteristics of the proposed models, we present a solution procedure developed upon the goal programming and piecewise-linear approximation approaches to resolve the models.

2. Market Advancement Strategy

When a MNE plans to operate in a wholly-own-based multi-location investment environment for expanding its operational scale of globalization, the decision-makers have to further select a suitable resource-allocation and/or transfer strategy. In this paper we slightly revise the idioms of four types of project advancement strategies defined in PA to be more suitable in terms of the context here. Also, we termed them the market advancement strategies.

2.1. Centralized sequential advancement strategy (CSAS):

It refers to centralizing the available amount of capital budget into a planning investment location, and then transferring a specific portion of the reward, gleaned by investing in this location, onto another location once the target return of this location has been gleaned or a scheduled time limit has been run out. The investment continues by such a rule and gradually expands the MNE's globalization.

2.2. Decentralized synchronized advancement strategy (DSAS):

It refers to expanding the MNE's globalization by the means that decentralizes the available amount of capital budget into all planning investment locations and concurrently invests them at the beginning of implementing the investment program.

2.3. Mixed advancement strategy:

It represents a mode of consisting of both CSAS and DSAS. Consider the investment locations: Cities A, B, C and D, and divide the four investment locations into two groups: {A & B} and {C & D}, which are referred to as "X" and "Y" respectively. Type I MAS means that deploy the CSAS within Groups X and Y, while going ahead between Group X and Y with the DSAS. Whereas, Type II MAS is deploying the DSAS within the Groups X and Y, while going ahead between Group X and Y with the CSAS.

3. The problem

Considering an enterprise intends to expand its business to several international cities and to be one of famous multinational enterprises (MNEs). In order to reach this goal, suppose the MNE determines to adopt the market enter strategy of wholly-owned, as well as the market advancement strategy of DSAS. Assume the demand rate of city j ($j = 1,2,...,J$) will increase in a large amount after time horizon T_j elapsed, and then the potential competitors will competitively enter the market at that time. So the MNE should expand its investment and reach a certainly substantial capital scale within time horizon T_j for guaranteeing competition advantages. Letting that $s_j(T_j)$ represents the necessary capital scale of city j before time horizon T_j elapsed, $\tilde{c}_j$ the total capital invested at the beginning of investing city j, and R_j^{target} the target return (after tax) when T_j elapsed. Accordingly, the cost relationship under adopting DSAS is stated as follows.

$$R_j^{target} = s_j(T_j) - \tilde{c}_j, \; j = 1,2,...,J \qquad (1)$$

Moreover, we consider two cost drivers of capital investment. They are respectively "environment investment for sale (EIFS)" and "environment investment for production (EIFP). Let $EIFS_j$ be the environment investment for sale on city j, $EIFP_j$ the environment investment for production on city j, then

$$\tilde{c}_j = EIFS_j + EIFP_j, \; j = 1,2,...,J \qquad (2)$$

Here we assume that $EIFS_j$ for any j has been planned and therefore is a constant. Also, suppose that infinite alternatives for each $EIFP_j$ are available. Each alternative refers to a specific quality standard for production. Based on this premise, the MNE should first find the optimal $EIFP_j$ and then budget the total amount of $EIFP_j$ for all j, saying it $EIFP^*$. When the available amount of capital investment budget is less than $EIFP^*$, the MNE has to further make an allocation decision. Upon the available amount of budget being scarce corresponding to achieving the concerned objective of the MNE, it has to further find the optimal portfolio of $EIFP_j$ ($\forall j$) to maximize the degree of realization of this concerned objective. Such objective means that every planning investment city j needs to glean its target return R_j^{target} when T_j elapsed. However, when available amount of budget is sufficient enough corresponding to achieving the concerned objective, the MNE necessitates obtaining the optimal portfolio of $EIFP_j$ ($\forall j$) to minimize the time required to glean the target returns of all planning investment cities. For convenience, we term the former the DSAS-MA (maximum achievement) model and the latter the DSAS-MM (minimum makespan) model. A DSAS-MA model associated with commodity pricing policy will consider in the following section, but any DSAS-MM model will be out of

the scope here.

4. DSAS-MA Model

Let $A(\mathbf{c})$ be the degree of realization of a specific fundamental objective (FO) under budget allocation policy $\mathbf{c}$. Then the purpose of formulating DSAS-MA model is to find an optimal budget allocation policy $\mathbf{c}^*$ to maximize $A(\mathbf{c})$. In accordance with the scenario described in Section 3, the FO considered here is that every planning investment city j needs to glean its target return R_j^{target} when T_j has elapsed in order to guarantee the competition advantages of whole company. Let V is the value function with respect to investment projects contributing to the concerned FO here, $z_j(\mathbf{c})$ the expected percentage of realization of capital investment $s_j(T_j)$ within time horizon T_j on adopting budget allocation policy $\mathbf{c}$. Then DSAS-MA model can be formulated simply as below.

$$\max_{\mathbf{c}\in\Omega} A(\mathbf{c}) = \frac{V(z_1(\mathbf{c}),\cdots,z_j(\mathbf{c}),\cdots,z_J(\mathbf{c}))}{V(1,\cdots,1\cdots,1)} \tag{3}$$

Where Ω is the feasible solution space of budget allocation. Clearly, Problem (3) is equivalent to solving the following:

$$\max_{\mathbf{c}\in\Omega} V(z_1(\mathbf{c}),\cdots,z_j(\mathbf{c}),\cdots,z_J(\mathbf{c})) \tag{4}$$

Letting $V_j(x_j)$ is the individual value to the FO when the expected degree of realization of capital cost $s_j(T_j)$ within time horizon T_j is at level x_j; $\beta(x_1,x_2,...,x_J)$ the value to the FO being generated when n investment projects have interactive effects. Then $V(x_1,\cdots,x_j,\cdots,x_J)$ can be shown as follows:

$$V(x_1,x_2,...,x_J) = \sum_j V_j(x_j) + \beta(x_1,x_2,...,x_J) \tag{5}$$

In order to resolve Problem (3), we first borrow one of simple interdependent problems defined in the theory of project advancement (PA). It is stated as follows: If there exists a lower bound x_j^l ($\forall j$) so that

$$\beta(1,1,...,1) - \beta(x_1^l,x_2^l,...,x_J^l) \le \varepsilon$$

(ε is an extremely small positive real number), then Problem (2) can be rewritten as follows:

$$\max_{\mathbf{c}\in\tilde{\Omega}} V(z_1(\mathbf{c}),\cdots,z_j(\mathbf{c}),\cdots,z_J(\mathbf{c})) \tag{6}$$

Where $\tilde{\Omega}$ is the subspace of Ω, which any budget allocation policy $\mathbf{c}$ is capable of being full of lower bound x_j^l ($\forall j$).

Model 1: V_j is a linear function

Suppose $V_j(z_j)$ is a linear function and w_j the relative weight of project j. Then we have

$$V_j(z_j(\mathbf{c})) = \sum_k w_j z_j(\mathbf{c}), \forall j \tag{7}$$

If we let $r_j(c_j)$ be the reward rate of city j on investing capital c_j in each production-line, L_j the number of the production-lines investing in city j, then

$$z_j(\mathbf{c}) = \frac{r_j(c_j)\cdot T_j + L_j c_j + EIFS_j}{S_j(T_j)} \tag{8}$$

Also, if we let

$$c_j^{\min} = \inf\left\{c_j : \frac{r_j(c_j)T_j + L_j c_j + EIFS_j}{S_j(T_j)} \ge x_j^l\right\}$$

Then, DSAS-MA model will be rewritten as follows:

$$\max \sum w_j \frac{r_j(c_j)\cdot T_j + L_j c_j + EIFS_j}{S_j(T_j)} \tag{9}$$

Subject to

$$\sum_{j=1}^{J} L_j \cdot c_j \le B_0 \tag{10}$$

$$r_j(c_j)T_j + L_j c_j \le S_j(T_j) - EIFS_j, \forall j \tag{11}$$

$$c_j^{\min} \le c_j \le c_j^*,\ j = 1,2,\cdots,J \tag{12}$$

Where c_j^* is city j's optimal amount of environment investment for production, which maximize $r_j(c_j)$, and B_0 the available amount of budget at the beginning of whole investment program.

If there is no feasible solution for Constraints (10)-(12), that is

$$\sum_{j=1}^{J} L_j c_j^{\min} > B_0 \tag{13}$$

Then we can use the goal programming approach to formulate DSAS-MA model. Indeed, under such a case, we can formulate the

DSAS-MA model as below:

$$\min\ \delta = \max\{\delta_j^+, \forall j\} \tag{14}$$

Subject to

$$\frac{r_j(c_j)\cdot T_j + c_j + EIFS_j}{S_j(T_j)} + \delta_j^+ - \delta_j^- = x_j^l, \forall j \tag{15}$$

$$\sum_{j=1}^{J} L_j \cdot c_j \le B_0 \tag{16}$$

$$c_j^l \le c_j \le c_j^{\min},\ j = 1,2,\cdots,J \tag{17}$$

$$\delta_j^+, \delta_j^- \ge 0 \tag{18}$$

Where c_j^l is amount of capital investment needed for survival at the beginning of investing city j. If we rewrite Objective Function (14) as

$$\min\ \delta \tag{19}$$

And add the constraints:

$$\delta \ge \delta_j^+, \forall j \tag{20}$$

Then the latter case of Model 1 is easy to resolve.

Model 2: V_j is a nonlinear concave function

Supposing that $V_j(z_j)$ is a nonlinear and nondecreasing concave function. Then by embedding the variables a_j for all j, the former case of Model 1 will be rewritten as

$$\max \sum V_j(a_j) \tag{21}$$

Subject to

$$a_j \le \frac{r_j(c_j)\cdot T_j + L_j c_j + EIFS_j}{S_j(T_j)}, \forall j \tag{22}$$

Other constraints are the same as (10)-(12).

Another approach to resolve Model 2 is using the goal programming method. That is

$$\max\ V = \sum_j \tilde{V}_j(s_j^+) \tag{23}$$

Subject to

$$\frac{Z_j(c_j)\cdot T_j + L_j c_j + EIFS_j}{S_j(T_j)} + s_j^+ - s_j^- = 1, \forall j \tag{24}$$

$$s_j^+, s_j^- \ge 0 \tag{25}$$

Other constraints are the same as (10)-(12), where $\tilde{V}_j(s_j^+) = V_j(1 - s_j^+)$.

The goal programming approach would be more useful when evaluating $\tilde{V}_j$ is easier than evaluating V_j. When Inequality (13) is right, there is no difference between Model 1 and Model 2.

5. Separable Convex Programming

Clearly, both Model 1 and Model 2 can be resolved by using separable convex programming. Do not loss generality, we only examine the former case of Model 2. Let us take K_j grid points from interval $[c_j^l, c_j^*]$, noted by $m_{j(k)}, \forall k$, then $r_j(c_j)$ will be rewritten as (13).

$$r_j(c_j) \approx r_j(m_{j(0)}) + \sum_{k=1}^{K_j} \rho_{j(k)} \cdot c_{j(k)},\ \forall j \tag{26}$$

Where

$0 \le c_{j(k)} \le m_{j(k)} - m_{j(k-1)},\ m_{j(0)} = c_j^{\min}$,

$m_{j(K_j)} = c_j^*$,

$$\rho_{j(k)} = \frac{r_j(m_{j(k)}) - r_j(m_{j(k-1)})}{m_{j(k)} - m_{j(k-1)}}.$$

Consider V_j is strictly decreasing in s_j^+ and is convex. Also, suppose the parameter $s_j^{+(u)}$ so that V_j approaches zero if $s_j^{+(u)} \ge s_j^+$. By the same token, if we take $\tilde{K}_j + 1$ grid points from interval $[0,\ s_j^{+(u)}]$, noted by $\tilde{m}_{j(k)}, k = 0,1,\cdots,\tilde{K}_j$ then $\tilde{V}_j$ will be rewritten as (27).

$$\tilde{V}_j(s_j^+) \approx \tilde{V}_j(\tilde{m}_{j(0)}) - \sum_{k=1}^{\tilde{K}_j} \tilde{\rho}_{j(k)} s_{j(k)}^+,\ \forall j \tag{27}$$

Where

$0 \le s_{j(k)}^+ \le \tilde{m}_{j(k)} - \tilde{m}_{j(k-1)},\ \tilde{m}_{j(0)} = 0$,

$$\tilde{m}_{j(\tilde{K}_j)} = s_j^{+(u)},\ \tilde{\rho}_{j(k)} = \frac{V_j(s_{j(k-1)}^+) - V_j(s_{j(k)}^+)}{\tilde{m}_{j(k)} - \tilde{m}_{j(k-1)}},$$

According to (27), the former case of Model 2 can be rewritten as (28)-(35).

$$\max\ V = \sum_j \tilde{V}_j(0) - \sum_k \tilde{\rho}_{j(k)} s_{j(k)}^+ \tag{28}$$

Subject to

$$\frac{f_j T_j + L_j \left[m_{j(0)} + \sum_k c_{j(k)} \right]}{S_j(T_j)} + \sum_{k=1}^{\tilde{K}_j} s^+_{j(k)} - s^-_j = 1, \forall j \tag{29}$$

$$f_j = r_j(m_{j(0)}) + \sum_{k=1}^{K_j} \rho_{j(k)} \cdot c_{j(k)} \tag{30}$$

$$\sum_{j=1}^{J} L_j \cdot \left[m_{j(0)} + \sum_{k=1}^{K_j} c_{j(k)} \right] \le B_0 \tag{31}$$

$$f_j T_j + L_j \left[m_{j(0)} + \sum_k c_{j(k)} \right] \le S_j(T_j) - EIFS_j, \forall j \tag{32}$$

$$0 \le c_{j(k)} \le m_{j(k)} - m_{j(k-1)}, \ \forall j, \ k \tag{33}$$

$$0 \le s^+_{j(k)} \le \tilde{m}_{j(k)} - \tilde{m}_{j(k-1)}, \forall j, k \tag{34}$$

$$s^-_j \ge 0, \forall j \tag{35}$$

By the same token, for the latter cases of Model 1 and Model 2, we are also able to take a list of grid points from interval $[\tilde{c}^l_j, \ c^l_j]$, and then using separable convex programming to resolve the models.

6. Concluding Remark

A DSAS-MM model for multi-location investment is developed in this paper. The proposed model assume the enterprise utilizes the wholly owned market entry strategy and the decentralized synchronized (market) advancement strategy (DSAS) to invest the locations (cities) to want. DSAS refers to expanding business by decentralizing the available amount of capital budget into each planning investment location and investing them concurrently. Due to the nonlinear characteristics of the proposed DSAS-MM model, we propose a linear transform developed upon the piecewise-linear approximation, fraction programming and weighted method.

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