Precise large deviation of claim surplus process in a nonstandard renewal risk model with constant premium rate

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Abstract:

In this paper, we consider a nonstandard renewal risk model in which the claim sizes and their inter-arrival times form a sequence of independent and identically distributed random variables, respectively. The claim size and corresponding inter-arrival time satisfy a certain dependence structure. In addition, the premium rate is a constant, and the number of insurance policies is described by a renewal process. When the distribution of claim sizes belongs to the consistent variation class, we obtain precise large deviation of claim surplus process.

Keywords: Precise large deviation, claim surplus process, nonstandard renewal risk model, constant premium rate, consistent variation class.

1. Introduction

We consider the following nonstandard renewal risk model: claim sizes \( \{X_i, i = 1, 2, \cdots\} \) form a sequence of independent, identically distributed (i.i.d) and nonnegative random variables with common distribution \( F \). The inter-claim times \( \{\theta_i, i = 1, 2, \cdots\} \) form another sequence of i.i.d and nonnegative random variables. Let \( \tau_k = \sum_{i=1}^{k} \theta_i \) denote the arrival time of the \( k \)th claim. Then \( 0 = \tau_0 \leq \tau_1 \leq \cdots \), constitute a renewal counting process

\[
N(t) = \sup\{n \geq 1: \tau_n \leq t\}, \quad t \geq 0.
\]

Write \( EN(t) = \lambda t \).

The amount of aggregate claims up to \( t \) can be expressed as

\[
S(t) = \sum_{i=1}^{N(t)} X_i.
\]

Denote by \( X \) and \( \theta \) the generic random variables of the claim sizes and their corresponding inter-claim times. We make use of the same dependent structure as in [1]. That is to say, \( X \) and \( \theta \) satisfy the following assumption.

**Assumption 1:** There is some \( x_0 > 0 \) such that it holds for all \( x \geq x_0 \) and \( t > 0 \) that

\[
P(\theta > t | X > x) \leq P(\theta^* > t).
\]

In addition, we assume that an insurer receives insurance policies in a discrete-time way. The arrival times of the successive insurance policies constitute a renewal process \( \{M(t), t \geq 0\} \). Let \( c \) denote...
the premium of each insurance policy. The amount of aggregate premiums up to
time \( t \) can be represented by \( cM(t) \), where \( EM(t) = \lambda_1(t) \). Let \( u > 0 \) be
the initial reserve of the insurer. The risk reserve process is given by

\[
R(t) = u + cM(t) - \sum_{i=1}^{N(t)} X_i , \quad t \geq 0.
\]

The claim surplus process can be expressed as

\[
Y(t) = \sum_{i=1}^{N(t)} X_i - cM(t) , \quad t \geq 0.
\]

(1)

The amount of aggregate claims can be denoted by

\[
S(t) = \sum_{i=1}^{N(t)} X_i , \quad t \geq 0.
\]

(2)

We assume \( M(t) \) and \( \sum_{i=1}^{N(t)} X_i \) are
mutually independent. [1] presented precise large deviation of the amount of
aggregate claims \( \{S(t), t \geq 0\} \). [2] showed precise large deviation for the compound Poisson risk model. In the present paper, when the claim-size
distribution belongs to the consistent variation class, we give precise large
deviation of the claim surplus process \( \{Y(t), t \geq 0\} \) for the above nonstandard
renewal risk model.

2. preliminaries

Denote

\[
\bar{F}(x) = 1 - F(x) = P(X > x) .
\]

We first introduce some related heavy-tailed distribution class, which can be found in
[3] and [4].

A distribution \( F \) on \([0, \infty)\) is said to belong to
the long-tailed class, if

\[
\bar{F}(x-y) \sim \bar{F}(x),
\]

for any \( y \in (-\infty, \infty) \). In this case, we write

\[
F \in L .
\]

In addition, we say that a distribution \( F \) belongs to the dominated variation class
and write \( F \in D \), if

\[
\bar{F}(xy) = O(1)\bar{F}(x) , \quad \text{for all } 0 < y < 1.
\]

Compared with \( D \), the consistent variation class \( C \) is a smaller class. We say \( F \in C \), if

\[
\lim\liminf_{y \downarrow 1 \atop x \to \infty} \frac{\bar{F}(xy)}{\bar{F}(x)} = 1 .
\]

It is well known that \( C \) belongs to \( D \cap L \).

By the definition of consistent variation class \( C \), we have the following lemma.

Lemma 1. If \( F \in C \), then

\[
\lim_{x \to \infty} \frac{\bar{F}(x + o(1)x)}{\bar{F}(x)} = 1 .
\]

The following lemma is due to [5].

Lemma 2. For a distribution \( F \) on \([0, \infty)\), if \( F \in D \), then for any
\( p > J_F^+ \), there is some positive number \( c_1 \) and \( d_1 \) such that

\[
\frac{\bar{F}(y)}{\bar{F}(x)} \leq c_1 \left( \frac{x}{y} \right)^p ,
\]

for all \( x \geq y \geq d_1 \).

The following is from [6].

Lemma 3. For a renewal process \( \{M(t), t \geq 0\} \), the generic inter-renewal
distance $T$ has distribution $F$ and expectation $\lambda_1 < \infty$. If $F(\infty) = 1$, then
\[
\lim_{t \to \infty} \frac{\lambda_1(t)}{t} = \frac{1}{\lambda_1}.
\]

The following lemma is from [1].

**Lemma 4.** Consider the amount of aggregate claims (2). Suppose that Assumption is satisfied, and that $F \in C$ and $E(X) = \mu \in (0, \infty)$. Then, for any fixed $\gamma > 0$, it holds uniformly for all $x \geq \gamma t$ that
\[
P(S(t) - \mu t > x) \sim \lambda t F(x),
\]
\[
t \to \infty.
\]

The following lemma is referred to [7].

**Lemma 5.** If $\{M(t), t \geq 0\}$ is a renewal process, then
\[
M(t) \overset{p}{\to} \lambda_1(t).
\]

3. Main result

**Theorem.** Consider the claim surplus process (1). In addition to Assumption 1, suppose that $F \in C$, $E(X) = \mu \in (0, \infty)$. Then, for any fixed $\gamma > 0$, it holds uniformly for all $x \geq \gamma t$ satisfying $c \geq \gamma \lambda_1$ that
\[
P(Y(t) - \mu t > x) \sim \lambda t \overline{F}(x),
\]
\[
t \to \infty.
\]

**Proof.** $P(Y(t) - EY(t) > x) = P(S(t) - cM(t) - ES(t) + c\lambda_1(t) > x) = \sum_{k-\lambda_1(t) < \varepsilon(t) \lambda_1(t)} (P(S(t) - ES(t) > x - c\lambda_1(t) + ck) \times$
\[
P(M(t) = k))
\]
\[
+ \sum_{k-\lambda_1(t) > \varepsilon(t) \lambda_1(t)} (P(S(t) - ES(t) > x - c\lambda_1(t) + ck) \times$
\[
P(M(t) = k))
\]
\[
= I_1(t) + I_2(t) + I_3(t).
\]

First of all, we deal with $I_1(t)$. For $x \geq \gamma t$, as $t \to \infty$, $\frac{\lambda_1(t)}{t} \leq \frac{\lambda_1(t)}{\gamma t} \to \frac{1}{\gamma \lambda_1}$. When $k - \lambda_1(t) \leq \varepsilon(t) \lambda_1(t), x - c\lambda_1(t) + ck = x + o(1) \lambda_1(t) = x + o(1)x$.

By Lemma 4, Lemma 1 and Lemma 5, it holds uniformly all $x \geq \gamma t$,
\[
I_1(t) \sim \sum_{k-\lambda_1(t) \leq \varepsilon(t) \lambda_1(t)} \lambda t \overline{F}(x - c\lambda_1(t) + ck) P(M(t) = k) = \lambda t \overline{F}(x) \sum_{k-\lambda_1(t) \leq \varepsilon(t) \lambda_1(t)} \frac{\overline{F}(x + o(1)x)}{\overline{F}(x)}
\]
\[
P(M(t) = k))
\]
\[
~ \sim \lambda t \overline{F}(x).
\]

Next we discuss $I_2(t)$. By Lemma 2, there is some positive number $D_2$ such that it holds uniformly for all $x \geq \gamma t$ satisfying $x \geq \delta \lambda_1(t)$,
\( I_2(t) = \sum_{k-\lambda(t) < -\varepsilon(t) \lambda(t)} (P(S(t) - ES(t) > x - c\lambda(t)) \times \sum_{k-\lambda(t) > \varepsilon(t) \lambda(t)} P(S(t) - ES(t) > x) P(M(t) = k)) \)

\[ P(M(t) = k)) \]

\[ = \sum_{k-\lambda(t) < -\varepsilon(t) \lambda(t)} (P(S(t) - ES(t) > x - c\lambda(t)) \times \sum_{k-\lambda(t) > \varepsilon(t) \lambda(t)} P(S(t) - ES(t) > x) P(M(t) = k)) \]

\[
\sim \lambda t \bar{F}(x) \sum_{k-\lambda(t) < -\varepsilon(t) \lambda(t)} \frac{\bar{F}(x - c\lambda(t))}{\bar{F}(x)} P(M(t) = k)
\]

\[ \leq D_2 \lambda t \bar{F}(x) P(M(t) - \lambda(t) < -\varepsilon(t) \lambda(t)) \]

\[ = o(1) \lambda t \bar{F}(x). \quad (6) \]

According to (4)-(7), we obtain (3). This ends the proof of the theorem.

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Reference:


Stochastic Processes for Insurance and Finance.