Adaptive PI Control of Piezoelectric Systems Using Takagi-Sugeno Fuzzy Logic

Gwo-Ruey Yu, Wen-Bin Lin and Shih-Wei Liu
Electrical Engineering Department
National Ilan University

Abstract

This paper presents the nano-positioning control of a piezoelectric platform. A Bouc-Wen model is established to describe the nonlinear hysteretic effect of piezoelectric systems. Three kinds of compensators, traditional PI controller, PI-like fuzzy logic controller (FLC) and adaptive PI-FLC controller, are designed for the piezoelectric systems. The adaptive PI controller is tuned on-line by means of Takagi-Sugeno fuzzy logic. Computer simulation demonstrates that the adaptive PI-FLC controller possesses the best robustness against external disturbance.

Keywords: hysteretic effect, piezoelectric systems, adaptive PI-FLC control.

1. Introduction

Nano-positioning control has been an important technology in recent years. Due to flourishing development of nano-positioning control, miniaturization has been a trend in communication, semiconductors, and precision mechanics. The piezoelectric ceramic materials are often used as actuators in nano-positioning control, e.g. diamond turning machines, scanning tunneling microscopy, piezoelectric voltage feedback for grinding tables, and active vibration control of robot bearing system [1], [6], [7], [20].

Piezoelectricity is a fundamental process in electromechanical energy conversion. Piezoelectric actuators use longitudinal/transverse effect to produce displacements of the piezoelectric materials [2], [16], [17]. The displacement amount range of stack type piezoelectric actuator is from $\mu m$ to $10 \mu m$. The advantages of piezoelectric actuators include unlimited resolution, fast frequency response, no friction, high stiffness and no backlash [14]. The main disadvantages of piezoelectric actuators are hysteresis phenomena. The hysteresis phenomena arise from materials polarization and molecule friction. As the input voltage alternates between increasing and decreasing, and its displacement amount have inconsistent phenomenon.

To analyze the hysteresis behavior of piezoelectric actuators, there have been many researches in founding a mathematical model. The asymmetrical types of hysteretic model include polynomial model, Preisach’s model and neural network model. The polynomial model is developed by using piecewise polynomial functions to approximate hysteretic curves [4]. The Preisach’s model is built by hysteretic operating factors [12]. The neural network model is established according to input and out data [5], [15]. The symmetrical types of hysteretic model include Maxwell model, Duhem model and Bouc-Wen model. The Maxwell model is developed according to energy parameter law [11]. The Duhem model is built by one order of differential equation [14]. The Bouc-Wen model is established by piezoelectric coefficients and dynamic equation of piezoelectric actuators [3]. In this paper, the Bouc-Wen model is analyzed and studied.

Three kinds of compensators, traditional PI controller, PI-like fuzzy logic controller (FLC) and adaptive PI-FLC controller, are designed for the piezoelectric systems. The PI controller has been widely used in industry due to small steady-state error and low costs. However, the transient state performance and robustness against disturbance of PI controller are not satisfied with the specifications of nano-positioning systems. This paper proposes two kinds of Takagi-Sugeno fuzzy logic controller (TSFLC) [10]. One is the PI-like FLC, and another is the adaptive PI-FLC. The advantage of PI-like FLC makes system output can track reference command fast. Furthermore, the adaptive PI-FLC is designed to improve the robust performance. Thus, the gains of the adaptive PI-FLC output are not fixed and will be automatically tuned to external changes.

2. Model of Piezoelectric Systems

This paper uses Bouc-Wen model to describe non-linear hysteretic curve of piezoelectric systems. The mathematical equations are as follows [19]:

\[
\begin{align*}
\ddot{x} + c \dot{x} + f(x, \dot{x}, u) &= u \quad (u \leq u_0) \\
\ddot{x} + c \dot{x} + f(x, \dot{x}, u) &= -u \quad (u > u_0)
\end{align*}
\]
\[ \begin{aligned}
  m \ddot{x} + b \dot{x} + kx &= k(\alpha u - h) + \rho \\
  \dot{h} &= \alpha d \dot{u} - \beta |\dot{h}| h - \gamma |h| \dot{h}
\end{aligned} \]  

(1)

where \( \rho = kx_0 \); \( m \), \( b \), \( k \), \( d \), and \( u \) are the tangent mass, damping, stiffness, effective piezoelectric coefficients, and input voltage of piezoelectric actuator. The \( \alpha \), \( \beta \) and \( \gamma \) are constants that control the shapes of the hysteresis curve. The \( \rho \) is a pre-compression strength. The displacement of piezoelectric actuator is \( x \), and the variable \( h \) is from the nonlinear hysteresis equation.

To find these parameters, a voltage signal is inputted to the piezoelectric actuator. The voltage signal is a triangular wave with amplitude 100 voltages and frequency 10Hz. After several experiments and computer simulations, the parameters of piezoelectric actuator are listed in Table I. Figure 1 shows the real hysteresis curve of piezoelectric actuator and simulation response of Bouc-Wen model.

Table I: Parameters of Bouc-Wen Model

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>0.148 kg</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>( b )</td>
<td>129.5 Ns/m</td>
<td>( \beta )</td>
</tr>
<tr>
<td>( k )</td>
<td>3e6 N/m</td>
<td>( \gamma )</td>
</tr>
<tr>
<td>( d )</td>
<td>3.5e-7 m/V</td>
<td>( \rho )</td>
</tr>
</tbody>
</table>

Fig. 1: Hysteresic curves of Bouc-Wen model.

### 3. Control of Piezoelectric Systems

To improve the robustness of PI controller against external disturbance, an adaptive PI-FLC is designed such that the gains are not fixed and will be automatically tuned on-line. Fig. 2 shows the block diagram of adaptive PI-FLC. The control law is

\[ u = u_1' + u_2' = [k_{p_0} e(s) + Fk_p] + [k_{i_0} e(s) + Fk_i] \cdot \frac{1}{s} \]  

(2)

where \( k_{p0} = 0.105 \) and \( k_{i0} = 550 \) are the nominal value of PI controller.

Table II lists these scaling factors \( G_4 \sim G_{10} \). Fig. 3 and Fig. 4 show the membership functions of input and output, respectively.

Table II: Scaling Factors

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_4 )</td>
<td>( 2.4 \times 10^5 )</td>
<td>( G_5 )</td>
<td>36.5</td>
</tr>
<tr>
<td>( G_6 )</td>
<td>( 8 \times 10^4 )</td>
<td>( G_7 )</td>
<td>5</td>
</tr>
<tr>
<td>( G_8 )</td>
<td>2.16</td>
<td>( G_9 )</td>
<td>10</td>
</tr>
<tr>
<td>( G_{10} )</td>
<td>90</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 2: Block diagram of adaptive PI-FLC.

Fig. 3: Input membership functions of adaptive PI-FLC.

Fig. 4: Output membership function of adaptive PI-FLC.
4. Simulation Results

Let the reference displacement command be a step signal with size of 10 \( \mu m \), and the simulation results of three different controllers are shown in Figs. 5, 6 and 7, respectively. Figs. 8 and 9 show the gains of adaptive PI-FLC.

Table III shows the performances of three controllers, where IAE means integral absolute error and ITAE means integral of time multiplied absolute error. Moreover, ISE and ISTE represent integral square error and integral of time multiplied square error, respectively. The equations of IAE, ITAE, ISE and ISTE are defined as

\[
\text{IAE} = \lim_{t \to \infty} \int_0^t |e| \, dt \tag{3}
\]

\[
\text{ITAE} = \lim_{t \to \infty} \int_0^t |e| \cdot t \, dt \tag{4}
\]

\[
\text{ISE} = \lim_{t \to \infty} \int_0^t e^2 \, dt \tag{5}
\]

\[
\text{ISTE} = \lim_{t \to \infty} \int_0^t e^2 \cdot t \, dt \tag{6}
\]

From the table, we can get that both PI-like FLC and adaptive PI-FLC can improve the transient state response. The adaptive PI-FLC owns the best performance.

<table>
<thead>
<tr>
<th></th>
<th>PI</th>
<th>PI-Like FLC</th>
<th>Adaptive PI-FLC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise time (ms)</td>
<td>151</td>
<td>9.6</td>
<td>0.69</td>
</tr>
<tr>
<td>Maximum overshoot</td>
<td>0</td>
<td>0</td>
<td>0.0824 %</td>
</tr>
<tr>
<td>Settling time</td>
<td>25.7</td>
<td>16.1</td>
<td>0.83</td>
</tr>
<tr>
<td>Steady-State Error (nm)</td>
<td>0</td>
<td>0</td>
<td>0.2956</td>
</tr>
<tr>
<td>IAE (nm)</td>
<td>70.78</td>
<td>57.7</td>
<td>4.3</td>
</tr>
</tbody>
</table>

Fig. 7: Step response of adaptive PI-FLC.

Fig. 8: The \( k_p' \) value of adaptive PI-FLC.

Fig. 9: The \( k_i' \) value of adaptive PI-FLC.

Table III: Control Performance
<table>
<thead>
<tr>
<th>ITAE (nm)</th>
<th>0.465</th>
<th>0.244</th>
<th>0.263</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISE (nm)</td>
<td>$3.7 \times 10^{-4}$</td>
<td>$3.59 \times 10^{-4}$</td>
<td>$2.24 \times 10^{-3}$</td>
</tr>
<tr>
<td>ITSE (nm)</td>
<td>$1.30 \times 10^{-6}$</td>
<td>$9.70 \times 10^{-7}$</td>
<td>$4.36 \times 10^{-9}$</td>
</tr>
</tbody>
</table>

5. Conclusion

The Bouc-Wen model of piezoelectric systems has been established to approximate the real hysteresis curve after several experiments and computer simulations. Three types of controllers, PI, PI-like FLC and adaptive PI-FLC, are designed to suppress the nonlinear hysteresis of piezoelectric systems. The gains of adaptive PI-FLC are tuned on-line automatically. Computer simulation demonstrates that the proposed adaptive PI-FLC possesses the best robustness against external disturbance.

6. Acknowledgement

This research is sponsored by means of the National Science Council of the Republic of China, under Grant NSC 95-2221-E-197-008-7.

7. References


