Protrusion Guided 3D Mesh Segmentation

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Abstract. Mesh segmentation, as a crucial step in model analysis and understanding, is vital to the success of the applications, e.g., the 3D reverse engineering and modeling by example. A number of recent works suggested segmenting 3D meshes on the basis of the cognitive psychology concept of “part salience”, which comprises three dominant factors: the protrusion degree, boundary strength, and relative size of the parts of the inspected object. In previous works, the estimation of salient feature such as protrusion degree is computation intensive. In this paper, we proposed a simpler measure of the protrusion degree based on the definition of 2D shape protrusion. With our new approach, the significant features can be successfully identified and extracted.

Keywords: polygon mesh, segmentation, protrusion, visual salience.

1 Introduction

With the popularity of 3D applications such as the 3D animations and 3D games, 3D meshes have gained growing importance in recent years. Among the essential technology associated with the 3D modeling, the model analysis and shape understanding have always been the most prominent study and have gained ever increasing importance. Mesh segmentation, being as a crucial step in model analysis and understanding, is vital for the success of the applications, e.g., the 3D reverse engineering and modeling by example.

The 3D polygon mesh decomposition, or 3D mesh segmentation (or partitioning) is an important means to obtain a meaningful component-wise representation of a given 3D polygon mesh object. With the help of this technique, the components can be individually identified, modified, or analyzed according to the interests of the applications. A variety of applications including the collision detection [1], matching or content-based retrieval [2, 3], metamorphosis [3], simplification [3, 4], compression [5], texture mapping [10] and skeleton extraction [8], etc. are benefited from this technique.

Recently, a number of visual salience-based approach to 3D mesh segmentation has been proposed [6–8]. According to their work, the salience of a part can be determined by three factors: the degree of protrusion, the strength of cut
boundary, and the size of the parts relative to the inspected object. In their works, the estimation of protrusion degree is rather computation intensive and time consuming. In this paper, we proposed a simpler measure of the protrusion degree based on the definition of 2D shape protrusion. With our new approach, the significant features can be successfully identified and extracted.

2 Preliminaries

In this paper, we only deal with orientable polygonal meshes. Some notations and definitions related to our problem is given as follows.

A 3D mesh denoted as $M = V,F$, is defined by two sets: a set of vertices $V$ and a set of faces $F$. The set $V$ comprises the vertices of the mesh defined in a three dimensional Cartesian space $R^3$, in which each vertex $v \in V$, is represented by a 3-tuple of real numbers, say $(v_x, v_y, v_z) \in R^3$. The set $F$ comprises the faces of the mesh, or triangular faces in this context, defined by a subset of different vertices of $V$; in particular, a 3-tuple of indices to the vertices of $V$, say $f = (I_a, I_b, I_c) \in I^3$.

According to [9], the problem of 3D mesh segmentation can be defined as follows.

Let $M$ be a 3D polygon mesh and be a segmentation of $M$. A segmentation $\sum$ of a given mesh $M(V,F)$ is represented by a set of sub-meshes $\sum = \{M_0, ..., M_{k-1}\}$ induced by a partition of $V$ or $F$ into $k$ disjoint sub-sets.

Suggested by [AShamir08], the segmentation problem can also be treated as an optimization problem by defining a criterion function $J$ on the partitioning of $M$ for the interest of each application in the following manner:

Given a mesh $M = V,F$ and a set of elements of $M$, i.e., $S = V$, or $S = F$. The process of finding a segmentation of $M$ can be casted as the problem of finding a disjoint partitioning of $S$ into $S_0, ..., S_{k-1}$ such that the value evaluated by the target function given by the criterion $J = J(S_0, ..., S_{k-1})$ is minimized (or maximized) under a set of constraints $C$.

3 Related Works

According to a previous study [9], the mesh segmentation methods can be roughly classified into two categories according to their purpose and result: namely, the part-type and patch-type segmentation methods. Since our focus is on the improvement of visual salience-based 3D mesh segmentation. We only discuss some recent related works applying similar metric. For the other works, interested readers are suggested to read the survey done in [9].
Hoffman and Singh [10] proposed the theory of part salience. According to their work, the salience of a part can be determined by three factors: the degree of protrusion, the strength of cut boundary, and the size of the parts relative to the inspected object. To characterize the protrusion of a part, [2] suggested quantifying the protrusion factor with an integral function $\mu$:

$$\mu(v) = \int_{p \in S} g(v,p) dS$$

(1)

where $g(v,p)$ represents the geodesic distance between $v$ and $p$ on the surface $S$, which is essentially the sum of geodesic distances from the point $v$ to all the other points on $S$.

Since the calculation of the integral on geodesic distance requires intensive computation, Hilaga et al suggested using Dijkstra’s algorithm to approximate geodesic distance based on the edge lengths of a 3D mesh to trade off the accuracy of approximation for better computational efficiency. In [2] the evaluation of the integral is performed on the dual graph of a given 3D mesh by assuming a vertex in the dual graph is the dual of the center of mass of a face in the input mesh and that a dual edge connecting two dual vertices corresponds to the link between the center-of-mass of two adjacent faces. On the basis of such assumptions, they defined the protrusion degree as follows:

$$\mu(v) = \sum_i g(v,b_i)area(P_i)$$

(2)

where $b_0, b_1, ...$ are the dual-vertices representing the faces $P_0, P_1, ...$. The protrusion degree of $v$ can be given by a normalization suggest by [7] as follows.

$$P(v) = \frac{(u(v) - u_{min})}{u_{max}}.$$  

(3)

A detailed discussion on the quantification of these three factors is given in [7].

4 The Protrusion Guided 3D Mesh Segmentation

In this paper, we proposed a simple metric for protrusion degree estimation. Unlike previous works, our definition of protrusion degree is defined on each vertex of the mesh rather than on the dual vertex of a triangle face. Furthermore, the estimation can be considered as an extension to a well-known definition of the protrusion measure of a 2D shape. Hence, it is more intuitive than that proposed previously in [2,6–8].

Inspired by [7], our algorithm adopt a similar framework that begins with the selection of salient representatives according to the part protrusion, followed by locating and optimizing the part boundaries. Therefore, we start our discussion on the definition of our new metric to the vertex protrusion, then on the process of the part boundaries.
4.1 The salient features extraction based on the simple protrusion metric

According to [10], the protrusion of a part is defined relative to its base. For a viewer-independent representation, an invariant base of the part is assumed to be the minimal surface delimited by its boundary curve. The invariant protrusion of the part is then given by the ratio of its surface area and its base area. To better understand the concept of part protrusion, we may first briefly review a common protrusion measurement to a principle vertex of a planar polygon.

**Protrusion Estimation on a Planar Polygon** Given a polygon $P$ and the set of boundary vertices of $P$, denoted as $\partial P = \{v_1, v_2, ..., v_n\}$ is properly ordered according to their adjacency such that $v_i \uparrow v_{i+1}$ and $v_i \downarrow v_{i-1}$ respectively are the next or previous immediate adjacent vertices to $v_i$. For any principle vertex $v_i \in \partial P$, we define a local measure to the protrusion of $v_i$ by

$$p(v_i) = \frac{|v_{i-1}v_i| + |v_i v_{i+1}|}{v_{i-1}v_{i+1}}. \quad (4)$$

**Protrusion Estimation on a 3D Mesh** Similar to the definition given in [10], we may define the protrusion of a vertex $v_i \in V$ of a 3D triangle mesh $M(V, F)$, denoted as $p(v_i)$, in terms of the invariant protrusion of the part defined by the $k$-th neighborhood of $v_i$, denoted as $N_k(v)$, as follows.

$$p(v_i) = \frac{1}{\text{BaseArea}(\partial N_k(v_i))} \times \frac{\sum_{v_j \in \partial N_k(v_i)} \text{Length}(P_{i,j})}{\text{Perimeter}(\partial N_k(v_i))}. \quad (5)$$

After the evaluations of vertex protrusion, a given number of feature points, or “salient representatives”, which appeared to be local maximals, are then extracted according to the vertex salience $S(v)$, whose value is essentially an accumulation of the differences of vertex protrusion with respect to the vertex $v$ and the vertices in its $k$-th neighborhood, which is given by

$$s(v) = \sum_{v_j \in \partial N_k(v)} p(v_j) - p(v). \quad (6)$$

To be transformation invariant, the vertex salience is normalized to value range $[0, 1]$. Therefore, the degree of protrusion is greater as the salience closer to 1; on the contrary, the vertex is less protrusive as the salience is small.

$$\hat{s}(v) = \frac{s(v) - \min_{u \in V} s(u)}{\max_{u \in V} s(u) - \min_{u \in V} s(u)}. \quad (7)$$
4.2 Locating and Optimizing the Part Boundaries

According to previous works, there are a lots of way to evolve the part boundaries. In [10], a minimal rule states that the part boundary has to be on the edges with locally minimal curvature where the locale of curve turns. On the basis of such concept, [7] proposed a locale-based method to find and optimize the part boundary strength. We follow the their “locale-base approach” to locating and optimizing the boundary of the parts.

4.3 The Computation of Locales

Similar to [7], with respect to a salient representative $r$, the mesh is divided into a set of $m$ locales $L_r = \{L_1^r, L_2^r, ..., L_m^r\}$, where $L_i^r$ is evolved from the boundary of $L_{i-1}^r$ in the following ways.

**Require:** $m > 0$

**Require:** $R = \{r_1, ... r_l\}$

**Require:** $L = \{L_i^j | i = 1 \rightarrow l, j = 1 \rightarrow m\}$

for $j = 1 \rightarrow l$ do
  $L_1^j = \{v | v \in N(r)\}$
  for all $v \in N_1(L_1^j)$ do
    if $\sum_{\forall v \in L_1^j} A(v) < e$ then
      $L_1^j = L_1^j \cup v$
    end if
  end for
for $i = 2 \rightarrow m$ do
  $L_i^j = N_1(L_{i-1}^j)$
  for all $f \in N_1(L_i^j)$ do
    if $\sum_{\forall v \in L_i^j} A(v) < e$ then
      $L_i^j = L_i^j \cup v$
    end if
  end for
end for
return

Note that, the extent of locales is controlled by a factor $e = \frac{\sum_{v \in V} A(v)}{m}$, which divides the mesh surface into $m$ regions of equal weighted surface area. Furthermore, a vertex weighted area $A(v)$ represented the base area with respect to a vertex $v$ is associated with each vertex of the mesh, which is given by

$$A(v) = \delta \cdot \frac{A(v)}{A} + (1 - \delta) \cdot \frac{p(v)}{p},$$

(8)
where \( \bar{A} \) and \( \bar{p} \) respectively represents the *mean vertex weighted area* and *mean vertex protrusion*. To balance the coverage of the locales, a heuristic factor \( \delta \) is used to adjust the influence of vertex area and vertex protrusion.

### 4.4 Locating and Optimizing the Boundaries

For each salient representative, the exact location of corresponding part boundary defines the final segmentation results. On the basis of the locales we have computed so far, we adopt the method proposed by Lin et. al. [7] to properly locate the part boundary by choosing from the boundaries between neighboring locales with respect to its boundary strength defined as follows.

Let \( V_{L_i^r} \) be a set of vertices adjoining the locales \( L_i^r \) and \( L_{i+1}^r \) with respect to a salient representative \( r \) and its corresponding locales \( L_i^r = L_0^r, L_1^r, \ldots, L_m^r \). We may define a function \( f(x) \) as the sum of the weighted areas of the boundary vertices between the locales \( L_i^r \) and \( L_{i+1}^r \) by

\[
f(x) = \sum_{v \in V_{L_i^r}} A(v). \quad (9)
\]

Therefore, the “boundary strength” of the boundary between locales \( L_i^r \) and \( L_{i+1}^r \) can be given by

\[
b(x) = f(x + 1) - f(x). \quad (10)
\]

Following to the evaluation of the boundary strengths, the part boundary \( B(r) \) with respect to the salient representative \( r \) can then be determined by

\[
B_r = k, b(k) = \max \{ b(i) | i = 1 \rightarrow m \}. \quad (11)
\]

Consequently, a segment or part of the mesh with respect to the salient representative \( r \) and its boundary \( B(r) \) is derived by

\[
\text{Segment}(r) = L_0^r \bigcup L_1^r \bigcup \ldots \bigcup L_{B_r}^r. \quad (12)
\]

### 5 Experimental Results

To test our method, we have conducted a series of experiments using a number of commonly used models from public domain. All the experiments were performed on a PC equipped with Intel-Pentium Dual-Core CPU E5300 processor, 4GB RAM, and an ATI Radeon HD 4550 powered display card. The statistics of each test mesh along with their processing time are listed in Table 1 shown as follows.

The segmented results of the test meshes with their corresponding colored segments generated by our method are shown in Fig. 1(a)-1(d). Through a close exam of the segmented results, it is plain to see that the test meshes were properly segmented in accordance with their visual salience features and followed the minimal rule.
Table 1. The attributes and processing time of all the models used in our experiments.

| Model    | |V| | F | Processing Time(Sec.) |
|----------|---|---|---|-----------------------|
| Dinopet  | 4500 | 8996 | 1.413 |
| Octopus  | 4140 | 8276 | 1.040 |
| Cactus   | 620  | 1236 | 0.026 |
| Triceratops | 2832 | 5660 | 0.665 |

Fig. 1. The segmentation results of the (a)Dinopet, (b)Octopus, (c)Cactus, and (d)Triceratops meshes.

In comparison with previous works, the result of the mesh Dinopet from a number of previous well-known works \[7, 11, 12\] along with our result are given in Fig. 2(a) to 1(d), respectively. As can be seen from the figure, the segmented result of the Dinopet mesh of our method is almost as good as those by the other works. However, since we did not apply boundary smoothing, the boundary lines of our result appeared to be more jaggy than the others.

Fig. 2. The segmented Dinopet: (a) fuzzy clustering \[11\]; (b) core extraction \[12\]; (c) visual salience-based \[7\]; (d) our method.
6 Concluding Remarks and Future Works

In this paper, we have proposed a simple heuristic protrusion metric and a novel method on the basis of such metric to segmenting an orientable 3D polygonal mesh. We have adopt a similar framework as that describe in [6, 7] but differs in the heuristic protrusion metric. Our metric, which is based on the original idea from cognitive science work [10], is more intuitive and closer to the concept of protrusion than those describe in [7]. From our experiment, we have shown that our new method has very good computation efficient and is able to yield competitive part-type segmentation results. However, in this work, we do not address the issue of boundary smoothing and the effect of relative size. In the future work, we may encourage the successors to deal with these issues and perform more tests or benchmarks [13] to further proof the reliability of our method.

References