Random Multiple Access Protocol combined Binary Tree Conflict-resolving Algorithm and the 1-persistent CSMA Protocol

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Abstract. This paper presents a new random multiple access control protocol – 1-persistent CSMA control protocol based on improved binary tree conflict resolution algorithm. First, the paper gives an outline of 1-persistent CSMA and the binary tree conflict-resolving algorithm by building a mathematical model and using the average cycle method, and then respectively obtains the throughput and the number of time-slots of the protocol and the algorithm. The simulation results are given, and agree well with theoretical values. But, in a shared channel, each user independently sends information packets randomly, and it is prone to conflict and resulting in the failure. Especially to the 1-persistent CSMA control protocol, when the load is larger, it has a lower throughput and greatly increasing the probability of channel conflict. Therefore, bringing in the binary tree conflict-resolving algorithm is a good method to improve the throughput, especially the improved binary tree conflict resolution algorithm. So at last, the paper introduces the random multiple access protocol combined improved binary tree conflict resolution algorithm and 1-persistent CSMA protocol, obtains the throughput and gives the simulations of the joint protocol. Theoretical calculations and experimental results show that 1-persistent CSMA protocol with conflict-resolving has a better performance.

Keywords: 1-persistent CSMA; binary tree conflict resolving algorithm; throughput; the number of time-slots; random multiple access.

1 Introduction

Random multi-access technology [1, 2] has been developed rapidly after the 1970’s. It has been widely applied in satellite communication, wireless packet network and computer networks due to allowing users to access channel conveniently and flexibly. There are many factors measuring the performance of random multi-access technology, such as throughput, QoS, power consumption and utilization ratio of the channel. But the throughput is very important performance index. If throughput of the web system is too low, the data sent by users may be not transmitted successfully.

In a shared channel, each user independently sends information packets randomly, but it is prone to conflict [3, 4] and resulting in the failure of the sending of the information packets. But 1-persistent CSMA protocol [5, 6] has an inevitable collision in the channel, therefore, we need to introduce a reasonable conflict-resolving algorithm [7, 8] to decompose and retransmit the impacted packets.
2 Analyses of Models

2.1 Analysis of 1-persistent CSMA protocol

It will sense the channel status while each node sends information packets, if the channel is in an idle time slot, then the data will be sent in the beginning of the next TS immediately, but if the channel is sensed busy, the packet will be transmitted immediately until the channel is detected in the idle state. The control model of 1-persistent CSMA is shown in Fig.1.

Before analyze the system performance, the assumption is shown as follows:

The information packets arrival on the channel follows the Poisson distribution with an independent parameter \( G \).
The channel is ideal without noise and interference.
Assume that all packets are of unit length and \( a \) denote the maximum propagation delay of the information packets. The system time is divided into time slots.
Packets arriving at the last time slot during the idle time must wait for the next time slot to transmit. If the number of the packet equals to one, it transmists successfully, otherwise it transmits unsuccessfully.

The probability of no packets on the channel is \( p = e^{-(1+a)G} \), and the probability of no packets in a transmission period, TP, is \( p = e^{-(1+a)G} \).

The probability of that appear \( j \) idle events I continuously in an idle time is
\[
P(N_I = j) = (e^{-(1+a)G})^j e^{-(1+a)G}
\]
The probabilities of that appear \( i \) TP continuously in a busy period is
\[
P(N_{BU} = i) = (1-e^{-(1+a)G})^i (1-e^{-aG})
\]
The joint probability that appears \( i \) composite events BU and \( j \) idle events I continuously in an cycling period is
\[
P(N_I = j, N_{BU} = i) = (1-e^{-(1+a)G})(1-e^{-(1+a)G})^i e^{-(1+a)G}(e^{-aG})^j
\]
The average number of the idle slot \( E[j]\) is
\[
E[j] = \frac{1}{1-e^{-aG}}
\]
So the mean length of the idle time \( E[I]\) is
\[
E[I] = aE[j] = \frac{a}{1-e^{-aG}}
\]
The average number of slot time \( E[i]\) is
\[
E[i] = \frac{1}{e^{-(1+a)G}}
\]
The mean length \( E[BU]\) is
\[
E[BU] = (1+a)E[i] = \frac{1+a}{e^{-(1+a)G}}
\]

\( E[U1] \) denote the mean length of the packet that arriving at the last slot time during the idle time and be transmitted at the first slot time in the transmission period, TP, \( E[U2] \) denote the average length of the packet that arrived in the transmission period and be transmitted successfully.
\[ P(i, j) \text{ can be identically equal to} \]
\[ p(i, j) = \sum_{k=0}^{\infty} C_k^k [(1 + a)Ge^{-aG}]^k \left[ 1 - e^{-(1 + a)G} - (1 + a)Ge^{-(1 + a)G} \right]^{-1 - k} \left( 1 - e^{-aG} \right) \left( e^{-aG} \right)^{-1} \]

\[ E[U_1] = \frac{aGe^{-aG}}{1 - e^{-aG}} \]
\[ E[U_2] = (1 + a)G \]

\[ E[U] = E[U_1] + E[U_2] = E[U] = \frac{aGe^{-aG}}{1 - e^{-aG}} + (1 + a)G \]

Combine (1), (2), (3) and \[ S = \frac{E[U]}{E[BU] + E[I]} \], the throughput of 1-persistent CSMA is

\[ S = \frac{aGe^{-(1 + a)G} + G(1 - e^{-aG})e^{-(1 + a)G}}{(1 + a)(1 - e^{-aG}) + ae^{-(1 + a)G}} \]

2.2 Analysis of the improved binary tree conflict decomposition algorithm \[^9,10\] 

The mathematics processes of decomposition mechanism are:

When there is a collision occurred in the packets sent by two active terminals within a time slot, as shown in Fig. 2. There are the following two situations:

(1) The left and the right slot each has a packet respectively, decompose successfully. The probability is:

\[ P_{11} = \frac{2!C_2^1C_1^1P^2}{2} = C_2^1C_1^1P^2 = 2 \left( \frac{1}{2} \right)^2 = \frac{1}{2} \]

(2) The left or the right slot has two packets; the other slot has no packet, decompose unsuccessfully and need to be re-decomposed. The probability is:

\[ P_{20} = 2!C_2^2C_0^0P^2 = \frac{1}{2} \]

The average number of slots for the successful decomposition is:

\[ \overline{L} = 2P_{11} + (1 + \overline{L})P_{20}, \quad \overline{L} = 3 \]

The decomposition efficiency is:

\[ \eta = \frac{M}{L} = \frac{2}{3} \times 100\% = 66.7\% \]

When there is a collision occurred among three active terminals, making them do Bemoulli trials independently, as shown in Fig. 3, the experimental results will appear the following three situations:
Fig. 3 The tree structure of the improved binary tree conflict decomposition (M=3)

(1) The left or the right slot has three packets, the other has no packet. The probability is:

\[ P_{30} = 2! \binom{3}{3} \binom{0}{0} P^3 = \frac{1}{4} \]

(2) The left or the right slot has two packets, the other has one packet. If the left slot has two packets, the system stop to re-do Bernoulli trials until the packets send successfully, then transmit the right one. If the left has one packet, the system re-do Bernoulli trials after the left transmit successfully until the right slot decompose successfully. The probability is:

\[ P_{21} = P_{12} = \frac{2! \binom{1}{1} \binom{2}{2} P^3}{2} = \binom{3}{2} \binom{1}{1} P^3 = \frac{3}{8} \]

The average number of slots for the successful decomposition is:

\[ L_3 = (1 + L_3) P_{30} + (2 + L_2) P_{21} + (1 + L_2) P_{12} \]

So \( L_3 = 4.833 \)

The decomposition efficiency is:

\[ \eta = \frac{M}{L_3} = \frac{3}{4.833} \times 100\% = 62.073\% \]

When there is a collision occurred among four active terminals, making them do Bernoulli trials independently, if the result is “0”, and send the packets on the left slot; if the result is “1”, and send it on the right slot. As shown in Fig. 4, it will appear the following four situations:

Fig. 4 The tree structure of the improved binary tree conflict decomposition (M=4)

(1) The left or the right slot has four packets, the other has no one. Like M=2, we have

\[ P_{40} = 2! \binom{4}{4} \binom{0}{0} P^4 = \frac{1}{8} \]

(2) The left or the right slot has three packets and the other has only one, Like M=3, we have

\[ P_{31} = P_{13} = \frac{2! \binom{3}{3} \binom{1}{1} P^4}{2} = \binom{3}{1} \binom{3}{1} P^4 = \frac{1}{4} \]

(3) The left and the right slot both have two packets and send unsuccessfully, then the system stop to make them re-do Bernoulli trials, decompose the left packets first, and then to the right, until the right slot decompose successfully.

\[ P_{22} = \frac{2! \binom{2}{2} \binom{2}{2} P^4}{2} = \binom{3}{2} \binom{2}{2} P^4 = \frac{3}{8} \]

The average number of slots for the successful decomposition is:

\[ L_4 = (1 + L_4) P_{40} + (2 + L_3) P_{31} + (1 + L_3) P_{13} + (1 + 2L_2) P_{22} \]

So \( L_4 = 6.762 \)

The decomposition efficiency is:
\[ \eta = \frac{M}{L_\eta} = \frac{4}{6.762} \times 100\% = 59.154\% \]

The average number of time slots of N impacted information packets decompose successfully using the improved binary tree conflict decomposition algorithm is

\[ E(L_\eta) = \frac{p_0 + p'_0(3 + 2E(L_{N-1}))}{1 - p_0} \]

Take \( M = 2, 3, \ldots, 25 \) in the simulation experiments, the theoretical values and simulation values are shown in the following table.

<table>
<thead>
<tr>
<th>M</th>
<th>( \overline{L}^* ) theoretical value</th>
<th>( \overline{L}^* ) simulation value</th>
<th>M</th>
<th>( \overline{L}^* ) theoretical value</th>
<th>( \overline{L}^* ) simulation value</th>
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<td>2</td>
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<td>3.006</td>
<td>3</td>
<td>4.833</td>
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<td>12.600</td>
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<td>47.567</td>
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</tr>
</tbody>
</table>

2.3 Analysis of random multiple access protocol combined binary tree conflict-resolving algorithm and 1-persistent CSMA protocol.

In a shared channel [11, 12], each user independently sends information packets randomly, but it is prone to conflict and resulting in the failure. By now, adding 1-persistent CSMA protocol in the channel, then, each user sense the channel status before sending packets, if the channel is in an idle time slot, then the data will be sent immediately, this makes the probability of occurrence of the channel conflict greatly reduced. However, due to the transmission delay and other factors, 1-persistent CSMA can’t completely avoid the occurrence of the conflict. When the load is larger, 1-persistent CSMA has a lower throughput and greatly increasing the probability of channel conflict. By now, using improved binary tree conflict-resolving algorithm to decompose these collisions, the other terminals stop sending signals during this time until all collisions being decomposed, then the system re-use 1-persistment CSMA protocol. As shown in the Fig.5.
Two or more packets arriving
Decompose the collisions using improved binary tree decomposition protocol
Re-use 1-persistment CSMA protocol

Only one packet arriving
Transmit successfully

Fig.5 The control process of the protocol that combined the improved binary tree conflict decomposition algorithm and 1-persistent CSMA protocol.

The joint probability that appears i composite events BU and j idle events I continuously in an cycling period is

\[ P[N_i = j, N_{BU} = i] = (1 - e^{-Ga}) \left( 1 - e^{-G(1+a)} \right)^{j-1} e^{-G(1+a)} \left( e^{-Ga} \right)^{i-1} \quad i = 1, 2, \ldots; j = 1, 2, \ldots \]

\[ E[j] = \frac{P_0}{P_0(1 - P_0)} \]

\[ E(N_i) = \frac{P_1}{P_0(1 - P_0)}, \quad P_1 = G(1 + a) e^{-G(1+a)} \]

\[ E(N_x) = \frac{P_x}{P_0(1 - P_0)}, \quad P_x = \left[ G(1 + a) \right]^x e^{-G(1+a)} \]

\[ E(B_0^*) = E(N_0) a \]

\[ E(B_i) = E(N_i) \times 1 \quad E(B_i') = E(N_i)(1 + a) \]

\[ E(B_x^*) = E(N_x) \times \bar{E}(B_x^*) = E(N_x) (4 + a) \bar{L} \]

\[ \bar{L}_x \] is the desired average length of time slot that the collisions being decomposed successfully by the improved binary tree conflict resolving algorithm.

From above, we have

\[ S = \frac{E(B_i) + \sum_{x=2}^{\infty} E(B_x^*)}{E(B_0^*) + E(B_1^*) + \sum_{x=2}^{\infty} E(B_x^*)} = \frac{G(1+a)e^{G(1+a)}}{ae^G + (1+a)G \left[ e^{G(1+a)} - 1 \right] + (1+a) \sum_{x=2}^{\infty} \frac{L_x[G(1+a)]^x}{x!}} \]

The following figures are the comparison diagrams of 1-persistent CSMA protocol and 1-persistent CSMA protocol with conflict-resolving.

Fig.6 a=0.01, the throughput for the two protocols

Fig.7 a=0.03, the throughput for the two protocols
From the above, the following can be obtained:

When $a$ take a small value (less than 0.1), 1-persistent CSMA protocol with conflict-resolving reached the maximum while arrival rate is very small and as the delay increases, the maximum value of the throughput gradually decreases.

When the arrival rate gradually increases, 1-persistent CSMA protocol with conflict-resolving reached a stable throughput, and when $a$ increases, the stable value of the throughput gradually decreases.

For 1-persistent CSMA access technology, the variation of the theoretical value of the throughput is to be increased as the increase of the arrival rate and then decreased rapidly, and the throughput reaches a maximum when the arrival rate $G=1$. After $a$ gradually becomes larger, the maximum throughput will gradually becomes small and reaches the maximum value in the arrival rate is less than 1.

Regardless of the value of $a$, the theoretical throughput of 1-persistent CSMA protocol with conflict-resolving is higher than that of 1-persistent CSMA access technology.

### 4 Conclusions

This paper mainly introduce the mechanism of 1-persistent CSMA access technology, the improved binary tree conflict-resolving algorithm and the improved binary tree conflict resolving – 1-persistent CSMA protocol and analyze the throughputs of three protocols by building mathematical models. Theoretical calculations and experimental results show that 1-persistent CSMA protocol with conflict-resolving has a better performance.
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