Modified Algorithm for Medium Length Irregular Low Density Parity Check Code with Low Computational Complexity and Error Floor Rate

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Abstract— Performance of medium length LDPC codes is directly proportional to error floor. In these codes, error floor can be mainly improved through elimination of short cycles. Hence, for effective elimination of short cycles, it is customary to use a girth conditioning in random construction process. Various accurate techniques, such as, density evolution and EXIT chart are developed for performance prediction of large block length, iteratively decoded LDPC codes. However, for medium block length, this method evolved with erroneous results. Even though, the medium block length LDPC codes have many application, short cycles degrades the performance and adds computational complexity in decoding process.

Keywords- Error correcting codes, an irregular LDPC codes, Tanner graph, short cycles, additive white Gaussian noise channel (AWGNC).

I. INTRODUCTION

Low Density Parity Check codes (LDPC) were first introduced in 1962 by R. G. Gallager in his PhD thesis at MIT [1]. The error-correcting performance of irregular low-density parity check (LDPC) codes, when decoded using practical iterative decoding algorithms, is known to be close to Shannon limits for codes with suitably large block lengths. Recently, Luby et al. extended Gallager's results to consider irregular codes, i.e. codes with non constant row weight and column weights and showed that these codes are capable of outperforming over regular codes. One of the main hurdles in the implementation of LDPC codes is the computational complexity (CC) of the encoding algorithm. Encoding is in general performed by matrix multiplication and so complexity is quadratic in the code length.

Long codes with length larger than 10^6 are of asymptotically good [2]. However, it is not very practical to implement such long codes in many applications, due to hardware complexity, as well as, the incurred time-delay. To construct short-length LDPC codes with large girth (shortest cycle length), various methodologies are in existence, i.e. progressive edge growth (PEG), progressive edge growth approximate cycle set extrinsic message degree (PEG-ACE), progressive edge growth approximate minimum cycle set extrinsic message degree (PEG-ACSE), Heuristic approach, an efficient algorithm, and genetic based algorithm. However, such codes may still suffer from the existence of

error floors. High error floors cause a major problem for potential applications of LDPC codes such as data storage, deep-space communications which requires a Bit Error Rate (BER) less than 10^{-15} .

It was recognized at an early stage that cycles of length - 4 in the bipartite/Tanner graph associated to the parity check matrix 'H' had a very negative effect on decoding performance of LDPC codes. Hence, the focus of the work carried out here was to formulate an algorithm which generates the optimistic parity check matrix with less short cycles.

II. RELATED WORK

The work on LDPC codes was initiated in early seventies. The varied utilities of these codes in real life applications tempted researcher to add impetus to make these codes more efficient and effective.

Eran Sharon et.al, proposed a Progressive Edge Growth algorithm to generate the edges in the graph one by one which resulted reduction of error floor [2]. Xia Zheng, proposed an algorithm for short length irregular LDPC codes to avoid major error-contributing trapping sets (TSs) during the code construction process [3]. Reza Asvadi, worked with structured and random codes over binary symmetric channels (BSC) and suggested the methods to eliminate short cycles in trapping sets which ultimately contribute for low error floor [4]. Jaehong Kim, proposed a new class of irregular LDPC codes, a simple rate compatible, puncture structure for finite block length with simplified decoding. The proposed codes are efficiently encodable and have a simple rate-compatible puncturing structure [5]. Chad A., worked on moderate length codes and adopts probabilistic approach for error occurrence [6]. Lara Dolecek, proposed a method based on high SNR and used to absorbing sets of structured LDPC codes [7]. Sae-Young, suggested density evolution methodology which predicts the decoder behavior [8]. Olga Milenkovic, large deviation theory was applied and explored the problem of trapping and stopping sets [9]. Jinghu Chen, proposed an approach based on quasi-cyclic extension and constructed irregular LDPC codes with low error floor in high SNR region [10]. Sang Hyun Lee, use trellis search for the construction of LDPC codes [11]. Benjamin Smith, suggested a numerical approach for estimation of density evolution, achieved

reduction in decoding complexity [12]. In continuation, author suggested heuristic approach [13], genetic algorithm [14], an efficient algorithm [15] for random generation of parity check matrix.

Major focus of all the researcher was to generate the codes with low error floor and less decoding complexity. Hence, effective modified algorithm is proposed here for generation of irregular LDPC codes. This paper is organized as, section III describes construction of codes using proposed approach, section IV presents results and section V concludes the work.

III. CONSTRUCTION OF IRREGULAR LOW DENSITY PARITY CHECK (LDPC) CODES

The LDPC code is defined by a sparse parity check matrix 'H'. It is called low density because number of 1's are very less compared to number of 0's. It is represented by $m \times n$ matrix, where 'm' is the number of rows (parity nodes or check nodes) and 'n' is the number of columns (variable nodes or message nodes). It is also represented as (d_v, d_c) , where ' d_v ' is the maximum number of 1's in at least one of columns and ' d_c ' is the maximum number of 1's in at least one of rows. If ' d_v ' is not same for each column and ' d_c ' is not same for each rows in a sparse parity check matrix then the designed codes are called as irregular LDPC code. However if the density of 1's in each column and each row are constant then the designed codes are called as regular LDPC codes.

The parity check matrix 'H' of size 4x8 is an example given below. All 1's in the columns are the position indices of the matrices and generated randomly for construction of H matrix.

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 1, shows the Tanner graph representation for the given H matrix.

Figure 1. Tanner graph with representation of short cycle of length-4

In Tanner graph the upper v-nodes are called variable nodes or message nodes and the bottom c-nodes are called parity nodes or check nodes. The messages are exchanged between v-nodes and c-nodes with the edges of the graph which act as the information pathways. The term 'bipartite' means the c-node should not be connected to another c-node. The 1's in the check node and variable nodes i.e. C0 & V0 are connected by information pathways. The example of typical short cycle of length 4 is represented in graph. The

performance of LDPC code formed with the process can be evaluated with Bit Error Rate, Frame Error Rate. The expression for the same are;

A. Equations

$$BER = \alpha = \frac{\text{Total bit in error}}{\text{Blocks} \times \text{No. of bit per block}} \quad \dots(1)$$

$$FER = \beta = \frac{\text{Total block in error}}{\text{Blocks transmitted}} \quad \dots(2)$$

$$\text{Fitness value of code matrix} = \gamma = \frac{1}{\text{Sum}(\alpha + \beta)} \quad \dots(3)$$

B. Proposed Algorithm

1. Decide no. of columns (message node: n)
2. Decide no. rows (check node: m)
3. Initialize zero matrix of $m \times n$
4. Construct 'H' matrix (proposed method)
 - random generation of column size m
 - consolidation of index position
 - append index position by 1
 - replace the column in $m \times n$ matrix
 - repeat the process for n columns
5. Performance evaluation BER, FER and CC
6. Evaluate the matrix for best fitness

IV. SIMULATIONS RESULTS

Figure 2. Tanner Graphs for code length (40, 20) using various methods (i) Proposed Algorithm (ii) Heuristic approach (iii) Genetic approach (iv) An efficient algorithm (v) PEG-ACSE

TABLE I. COMPARATIVE PERFORMANCE FOR BER OF IRREGULAR LDPC CODE WITH DIFFERENT METHODS

Code Length	PEG-ACSE	Efficient heuristic approach	Genetic based approach	An efficient algorithm	Proposed algorithm
400,200	0.107	0.107	0.107	0.107	0.057
300,150	0.107	0.107	0.107	0.107	0.057
200,100	0.106	0.107	0.107	0.107	0.055
160,80	0.103	0.106	0.106	0.106	0.057
120,60	0.106	0.106	0.104	0.103	0.057
100,50	0.105	0.106	0.102	0.102	0.060
96,48	0.105	0.106	0.101	0.100	0.063
80,40	0.102	0.104	0.101	0.094	0.057
60,30	0.098	0.102	0.096	0.089	0.060
40,20	0.100	0.101	0.081	0.066	0.061
32,16	0.101	0.097	0.077	0.066	0.063

Figure 3. Bit Error Rate v/s SNR for code length (120, 60)

TABLE II. COMPARATIVE PERFORMANCE FOR FER OF IRREGULAR LDPC CODE WITH DIFFERENT METHODS

Code Length	PEG-ACSE	Efficient heuristic approach	Genetic based approach	An efficient algorithm	Proposed algorithm
400,200	1	1	1	1	0.993
300,150	1	1	1	1	0.057
200,100	1	1	1	1	0.981
160,80	1	1	1	0.987	0.987
120,60	0.106	0.106	0.104	0.943	0.943
100,50	0.987	0.834	0.918	0.895	0.926
96,48	0.987	0.981	0.890	0.858	0.950
80,40	0.941	0.964	0.868	0.787	0.873
60,30	0.921	0.935	0.767	0.714	0.832
40,20	0.897	0.834	0.579	0.505	0.715
32,16	0.764	0.724	0.562	0.465	0.670

Figure 4. Frame Error Rate v/s SNR for code length (120, 60)

TABLE III. COMPARATIVE PERFORMANCE FOR CC IN SECONDS WITH DIFFERENT METHODS

Code Length	PEG-ACSE	Efficient heuristic approach	Genetic based approach	An efficient algorithm	Proposed algorithm
400,200	7100.4	480.3	410.1	115.7	7.6
300,150	2899.3	177.1	174.0	51.2	3.8
200,100	574.8	53.9	53.7	16.6	2.3
160,80	446.1	30.5	31.3	9.7	2.0
120,60	188.8	14.0	13.6	4.7	1.3
100,50	111.2	7.9	8.1	2.9	1.0
96,48	92.8	6.9	6.9	2.6	0.7
80,40	58.0	4.5	4.5	1.8	0.6
60,30	22.5	1.9	1.9	0.9	0.5
40,20	7.1	0.9	0.9	0.5	0.4
32,16	4.2	0.5	0.6	0.5	0.3

Figure 5. Computational time v/s various code length

The performance parameters are computed for the proposed method and compared with the existing methods. BER is 0.057 which is 47% improvement over the existing methods. Improvement in FER is 0.7% for code length 400 bits and 43% improvement with code length 300 bits. Computational complexity is enhanced by 99.89% over PEG-ACSE, 98.41% over efficient heuristic approach, 98.14% over genetic based algorithm and 93.43% over efficient algorithm.

V. CONCLUSION

Irregular LDPC codes with various lengths are simulated with MATLAB platform. Performance parameters such as BER, FER and CC are computed for proposed algorithm.

Results are compared with the existing methods such as PEG-ACSE, an Efficient Heuristic approach, an efficient genetic based algorithm and efficient algorithm. It is observe from the simulated result that the results with proposed algorithm are better over the existing methods.

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