

Three-Generation Distler-Kachru Models

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Abstract

Distler-Kachru models which yield three generations of chiral fermions with gauge group $SO(10)$ are found. These models have mirror partners.

1 Introduction

Distler-Kachru models [1] are special cases of (0,2) Landau-Ginzburg orbifolds [2]. To construct these models methods of algebraic geometry are being used. These models make it possible to determine the number of generations of chiral fermions. For this purpose it suffices to compute the Euler characteristic of the corresponding vector bundle.

In the present work we use a practical method of computing the Euler characteristic suggested by Kawai and Mohri [3] for a large class of Distler-Kachru models. By means of this method we provide some examples of models which yield three generations of chiral fermions with the gauge group $SO(10)$. These models have mirror partners [4].

2 A class of Distler-Kachru models

The Lagrangian for Distler-Kachru models [3] is given by

$$\mathcal{L} = 2i \int d\theta^+ d\bar{\theta}^+ \Phi^{\bar{i}} \partial_- \Phi^i - \int d\theta^+ d\bar{\theta}^+ \Lambda^k \Lambda^{\bar{k}} + \int d\theta^+ F_k \Lambda^k + \int d\bar{\theta}^+ F_{\bar{k}} \Lambda^{\bar{k}}, \quad (1)$$

where bosonic superfields Φ^i and fermionic ones Λ^k have the component expansions

$$\begin{aligned} \Phi^i &= \phi^i + \theta^+ \psi^i + i \bar{\theta}^+ \theta^+ \partial_+ \phi^i, & 1 \ll i \ll N, \\ \Lambda^k &= \lambda^k - \theta^+ l^k + i \bar{\theta}^+ \theta^+ \partial_+ \lambda^k, & 1 \ll k \ll M, \end{aligned}$$

and superpotentials F_k are homogeneous polynomials of Φ^i satisfying

$$F_k(x^{\omega_1} \Phi^1, x^{\omega_2} \Phi^2, \dots, x^{\omega_N} \Phi^N) = x^{1-\rho_k} F_k(\Phi^1, \Phi^2, \dots, \Phi^N),$$

with ω_i and ρ_k being rational numbers.

Following [3], let us reformulate superconformal model (1) into the language of algebraic geometry. Suppose that X is a $D = N - M + r$ dimensional complete intersection defined by

$$X = \{p \in \mathbf{WP}_{w_1, \dots, w_N}^{N-1} \mid F_{r+2}(p) = \dots = F_M(p) = 0\},$$

where F_{r+1+j} is a polynomial of degree d_j in the coordinates of the weighed projective space $\mathbf{WP}_{w_1, \dots, w_N}^{N-1}$. Let E be the stable vector bundle of rank r over X defined by the following exact sequence

$$0 \rightarrow E \rightarrow \bigoplus_{a=1}^{r+1} \mathcal{O}(n_a) \longrightarrow \mathcal{O}(m) \rightarrow 0,$$

where n_a and m being positive integers. Here $r = 3, 4, 5$ yields the gauge group $E_6, SO(10), SU(5)$. The vector bundle E and tangent bundle T of X must satisfy constraints on their Chern classes

$$\begin{aligned} c_1(T) &= 0, \\ c_1(E) &= 0, \\ c_2(T) &= c_2(E), \end{aligned} \tag{2}$$

which are the anomaly cancellaton conditions. Conditions (2) are tantamount to the system of Diophantine equations

$$\begin{aligned} \sum_i w_i - \sum_j d_j &= 0, \\ \sum_a n_a - m &= 0, \\ \sum_i w_i^2 - \sum_j d_j^2 &= \sum_a n_a^2 - m^2. \end{aligned} \tag{3}$$

Bosonic superfields Φ^i can be interpreted as coordinates of X and fermionic ones Λ^k as sections of E .

3 Orbifoldized elliptic genus

To compute the Euler characteristic $\chi(E)$, it is convenient to start from an orbifoldized elliptic genus [3]

$$Z(\tau, z) = \frac{1}{m} \sum_{\alpha, \beta=0}^{m-1} (-1)^{r(\alpha+\beta+\alpha\beta)} \beta \begin{matrix} \square \\ \alpha \end{matrix} (\tau, z),$$

where

$$\beta \begin{matrix} \square \\ \alpha \end{matrix} (\tau, z) = (-1)^{r\alpha\beta} e^{\pi i r(\alpha^2\tau + 2\alpha z)} \begin{matrix} \square \\ 0 \end{matrix} (\tau, z + \alpha\tau + \beta),$$

with

$$0 \begin{matrix} \square \\ 0 \end{matrix} (\tau, z) = \frac{\prod_{k=1}^M P(\tau, (1 - \rho_k)z)}{\prod_{i=1}^N P(\tau, \omega_i z)},$$

$$P(\tau, z) = \vartheta_1(\tau, z)/\eta(\tau),$$

$\vartheta_1(\tau, z)$ is the Jacobi theta function, $\eta(\tau)$ is the Dedekind eta function.

The modular and double quasi-periodicity properties of the orbifoldized elliptic genus lead to the following relations

$$\begin{aligned} (\omega_1, \dots, \omega_N) &= \frac{1}{m} (w_1, \dots, w_N), \\ (\rho_1, \dots, \rho_M) &= \frac{1}{m} (n_1, \dots, n_{r+1}, m - d_1, \dots, m - d_{M-r-1}). \end{aligned}$$

An expansion

$$Z(\tau, z) = i^{M-N} q^{\frac{M-N}{12}} y^{-r/2} [\chi_y(E) + O(q)]$$

connects the orbifoldized elliptic genus with the Hirzebruch genus

$$\chi_y(E) = \sum_{\alpha=0}^{m-1} \chi_y^{(\alpha)},$$

which consists of contributions from twisted sectors

$$\chi_y^{(\alpha)} = (-1)^{r\alpha} \frac{\prod_{k=1}^M (-1)^{[\alpha\nu_k]} \left[y^{\nu_k} q^{\frac{\{\alpha\nu_k\}-1}{2}} \right]^{\{\alpha\nu_k\}} (1 - y^{\nu_k} q^{\{\alpha\nu_k\}})(1 - y^{-\nu_k} q^{1-\{\alpha\nu_k\}})}{\prod_{i=1}^N (-1)^{[\alpha\omega_i]} \left[y^{\omega_i} q^{\frac{\{\alpha\omega_i\}-1}{2}} \right]^{\{\alpha\omega_i\}} (1 - y^{\omega_i} q^{\{\alpha\omega_i\}})(1 - y^{-\omega_i} q^{1-\{\alpha\omega_i\}})} \Big|_*, \tag{4}$$

where $y = e^{2\pi iz}$, $q = e^{2\pi i\tau}$, $\nu_k = 1 - \rho_k$, $\{x\} = x - [x]$, $[x]$ denotes the greatest integer less than x , $|_*$ means that we extract only terms of the form $q^0 y^{\text{integer}}$ in the expansion (4).

4 $SO(10)$ models

We now use the formula (4) to construct some three generation models with $r = 4$, $j = 1$, $D = 3$. The Euler characteristic for these models is given by

$$\chi(E) = -\chi_y(E)/y(y + 1)(y - 1). \tag{5}$$

Consider the models defined by the following quantum numbers

$$\begin{aligned} (w_1, \dots, w_5; d_1) &= (n_1, \dots, n_5; m) : \\ (3, 4, 6, 13, 13; 39), \\ (5, 8, 9, 11, 12; 45), \\ (10, 12, 13, 15, 25; 75). \end{aligned} \tag{6}$$

The system of equations (3) is satisfied identically by the data (6). The quantum numbers (6) (and (7) below) was used by Klemm and Schimrigk [5] to obtain the three generation (2,2) superconformal models. But whereas in the (2,2) case the gauge group is E_6 , in the (0,2) case the gauge group is $SO(10)$ for $r = 4$. In Tables 1–3 we display the contributions from twisted sectors into the Hirzebruch genera of models (6).

Table 1

$(w_i; d_1) = (n_a; m) = (3, 4, 6, 13, 13; 39)$					
α	$\chi_y^{(\alpha)}$	α	$\chi_y^{(\alpha)}$	α	$\chi_y^{(\alpha)}$
0	$1 + 25y - 25y^3 - y^4$	13	$-6y^2 + 6y^3$	26	$-6y + 6y^2$
1	$-y^3 + y^4$	14	$-y^2 + y^3$	27	$-2y^2 + 2y^3$
2	$-y^2 + y^3$	15	$-2y^2 + 2y^3$	28	$-y^2 + y^3$
3	$-2y^2 + 2y^3$	16	$-y^2 + y^3$	29	$-y + y^2$
4	$-y^2 + y^3$	17	$-y + y^2$	30	$-2y^2 + 2y^3$
5	$-y - y^2$	18	$-2y + 2y^2$	31	$-y^2 + y^3$
6	$-2y + 2y^2$	19	$-y + y^2$	32	$-y + y^2$
7	$-y^2 + y^3$	20	$-y^2 + y^3$	33	$-2y^2 + 2y^3$
8	$-y + y^2$	21	$-2y^2 + 2y^3$	34	$-y^2 + y^3$
9	$-2y + 2y^2$	22	$-y^2 + y^3$	35	$-y + y^2$
10	$-y^2 + y^3$	23	$-y + y^2$	36	$-2y + 2y^2$
11	$-y + y^2$	24	$-2y + 2y^2$	37	$-y + y^2$
12	$-2y + 2y^2$	25	$-y + y^2$	38	$-1 + y$
$\chi_y(E) = -3y(1+y)(1-y)$					

Table 2

$(w_i; d_1) = (n_a; m) = (5, 8, 9, 11, 12; 45)$					
α	$\chi_y^{(\alpha)}$	α	$\chi_y^{(\alpha)}$	α	$\chi_y^{(\alpha)}$
0	$1 + 12y - 12y^3 - y^4$	15	$-y + y^2$	30	$-y^2 + y^3$
1	$-y^3 + y^4$	16	$-y + y^2$	31	$-y^2 + y^3$
2	$-y^2 + y^3$	17	$-y^2 + y^3$	32	$-y + y^2$
3	$-y + y^2$	18	0	33	$-y + y^2$
4	$-y + y^2$	19	$-y^2 + y^3$	34	$-y^2 + y^3$
5	0	20	0	35	0
6	$-y^2 + y^3$	21	$-y^2 + y^3$	36	0
7	$-y + y^2$	22	$-y + y^2$	37	$-y^2 + y^3$
8	$-y + y^2$	23	$-y^2 + y^3$	38	$-y^2 + y^3$
9	0	24	$-y + y^2$	39	$-y + y^2$
10	0	25	0	40	0
11	$-y + y^2$	26	$-y + y^2$	41	$-y^2 + y^3$
12	$-y^2 + y^3$	27	0	42	$-y^2 + y^3$
13	$-y^2 + y^3$	28	$-y + y^2$	43	$-y + y^2$
14	$-y + y^2$	29	$-y^2 + y^3$	44	$-1 + y$
$\chi_y(E) = -3y(1+y)(1-y)$					

Table 3

$(w_i; d_1) = (n_a; m) = (10, 12, 13, 15, 25; 75)$					
α	$\chi_y^{(\alpha)}$	α	$\chi_y^{(\alpha)}$	α	$\chi_y^{(\alpha)}$
0	$1 + 8y - 8y^3 - y^4$	25	$-y^2 + y^3$	50	$-y + y^2$
1	$-y^3 + y^4$	26	$-y^2 + y^3$	51	0
2	$-y^2 + y^3$	27	0	52	$-y^2 + y^3$
3	0	28	$-y + y^2$	53	$-y^2 + y^3$
4	$-y + y^2$	29	$-y + y^2$	54	0
5	0	30	$2y - 2y^3$	55	0
6	0	31	$-y^2 + y^3$	56	$-y + y^2$
7	$-y^2 + y^3$	32	$-y^2 + y^3$	57	0
8	$-y^2 + y^3$	33	0	58	$-y^2 + y^3$
9	0	34	$-y + y^2$	59	$-y + y^2$
10	0	35	0	60	$2y - 2y^3$
11	$-y + y^2$	36	0	61	$-y^2 + y^3$
12	0	37	$-y + y^2$	62	$-y + y^2$
13	$-y^2 + y^3$	38	$-y^2 + y^3$	63	0
14	$-y + y^2$	39	0	64	$-y^2 + y^3$
15	$2y - 2y^3$	40	0	65	0
16	$-y^2 + y^3$	41	$-y^2 + y^3$	66	0
17	$-y + y^2$	42	0	67	$-y + y^2$
18	0	43	$-y + y^2$	68	$-y + y^2$
19	$-y^2 + y^3$	44	$-y + y^2$	69	0
20	0	45	$2y - 2y^3$	70	0
21	0	46	$-y^2 + y^3$	71	$-y^2 + y^3$
22	$-y + y^2$	47	$-y^2 + y^3$	72	0
23	$-y + y^2$	48	0	73	$-y + y^2$
24	0	49	$-y + y^2$	74	$-1 + y$
$\chi_y(E) = -3y(1+y)(1-y)$					

From the results of Tables 1–3 and formula (5), it is clear that $\chi(E) = 3$. Hence, these models yield three generations of chiral fermions with the $SO(10)$ gauge group.

The models (6) have mirror partners defined by the quantum numbers

$$\begin{aligned}
 (w_1, \dots, w_5; d_1) &= (n_1, \dots, n_5; m) : \\
 &(3, 4, 12, 17, 19; 55), \\
 &(4, 7, 9, 10, 15; 45), \\
 &(4, 4, 5, 5, 7; 25).
 \end{aligned} \tag{7}$$

For these mirror partners $\chi(E) = -3$. The contributions from twisted sectors into the Hirzebruch genera of mirror partners (7) are shown in Tables 4–6.

Table 4

$(w_i; d_1) = (n_a; m) = (3, 4, 12, 17, 19; 55)$					
α	$\chi_y^{(\alpha)}$	α	$\chi_y^{(\alpha)}$	α	$\chi_y^{(\alpha)}$
0	$1 + 28y - 28y^3 - y^4$	19	$-y^2 + y^3$	38	$-y^2 + y^3$
1	$-y^3 + y^4$	20	$-y^2 + y^3$	39	$-y^2 + y^3$
2	$-y^2 + y^3$	21	$-y^2 + y^3$	40	$-y + y^2$
3	$-y^2 + y^3$	22	$-y + y^2$	41	$-y + y^2$
4	$-y^2 + y^3$	23	$-y^2 + y^3$	42	$-y^2 + y^3$
5	$-y^2 + y^3$	24	$-y^2 + y^3$	43	$-y^2 + y^3$
6	$-y^2 + y^3$	25	$-y + y^2$	44	$-y^2 + y^3$
7	$-y^2 + y^3$	26	$-y + y^2$	45	$-y + y^2$
8	$-y + y^2$	27	$-y + y^2$	46	$-y^2 + y^3$
9	$-y + y^2$	28	$-y^2 + y^3$	47	$-y^2 + y^3$
10	$-y^2 + y^3$	29	$-y^2 + y^3$	48	$-y + y^2$
11	$-y + y^2$	30	$-y^2 + y^3$	49	$-y + y^2$
12	$-y + y^2$	31	$-y + y^2$	50	$-y + y^2$
13	$-y + y^2$	32	$-y + y^2$	51	$-y + y^2$
14	$-y^2 + y^3$	33	$-y^2 + y^3$	52	$-y + y^2$
15	$-y^2 + y^3$	34	$-y + y^2$	53	$-y + y^2$
16	$-y + y^2$	35	$-y + y^2$	54	$-1 + y$
17	$-y + y^2$	36	$-y + y^2$		
18	$-y + y^2$	37	$-y^2 + y^3$		
$\chi_y(E) = 3y(1+y)(1-y)$					

Table 5

$(w_i; d_1) = (n_a; m) = (4, 7, 9, 10, 15; 45)$					
α	$\chi_y^{(\alpha)}$	α	$\chi_y^{(\alpha)}$	α	$\chi_y^{(\alpha)}$
0	$1 + 15y - 15y^3 - y^4$	15	0	30	0
1	$-y^3 + y^4$	16	$-y^2 + y^3$	31	$-y + y^2$
2	$-y^2 + y^3$	17	$-y + y^2$	32	$-y + y^2$
3	0	18	$-y + y^2$	33	0
4	$-y + y^2$	19	$-y + y^2$	34	$-y^2 + y^3$
5	0	20	0	35	0
6	0	21	0	36	$-y^2 + y^3$
7	$-y^2 + y^3$	22	$-y + y^2$	37	$-y^2 + y^3$
8	$-y + y^2$	23	$-y^2 + y^3$	38	$-y + y^2$
9	$-y + y^2$	24	0	39	0
10	0	25	0	40	0
11	$-y + y^2$	26	$-y^2 + y^3$	41	$-y^2 + y^3$
12	0	27	$-y^2 + y^3$	42	0
13	$-y^2 + y^3$	28	$-y^2 + y^3$	43	$-y + y^2$
14	$-y^2 + y^3$	29	$-y + y^2$	44	$-1 + y$
$\chi_y(E) = 3y(1+y)(1-y)$					

Table 6

$(w_i; d_1) = (n_a; m) = (4, 4, 5, 5, 7; 25)$					
α	$\chi_y^{(\alpha)}$	α	$\chi_y^{(\alpha)}$	α	$\chi_y^{(\alpha)}$
0	$1 + 19y - 19y^3 - y^4$	9	$-y + y^2$	18	$-y + y^2$
1	$-y^3 + y^4$	10	$-4y + 4y^2$	19	$-y^2 + y^3$
2	$-y^2 + y^3$	11	$-y^2 + y^3$	20	$-4y^2 + 4y^3$
3	$-y + y^2$	12	$-y + y^2$	21	$-y^2 + y^3$
4	$-y + y^2$	13	$-y^2 + y^3$	22	$-y^2 + y^3$
5	$-4y + 4y^2$	14	$-y + y^2$	23	$-y + y^2$
6	$-y + y^2$	15	$-4y^2 + 4y^3$	24	$-1 + y$
7	$-y^2 + y^3$	16	$-y^2 + y^3$		
8	$-y^2 + y^3$	17	$-y + y^2$		
$\chi_y(E) = 3y(1+y)(1-y)$					

5 Remark

While this work was completed, there appeared on the hep-th net a paper of Kachru [6], where some three generation (0,2) superconformal models with the gauge groups E_6 and $SU(5)$ had been found.

References

- [1] Distler J. and Kachru S., (0,2) Landau-Ginzburg theory, *Nucl. Phys. B*, 1994, V.413, 213.
- [2] Kachru S. and Witten E., Computing the complete massless spectrum of a Landau-Ginzburg Orbifold, *Nucl. Phys. B*, 1993, V.407, 637.
- [3] Kawai T. and Mohri K., Geometry of (0,2) Landau-Ginzburg orbifolds, *Nucl. Phys. B*, 1994, V.425, 191.
- [4] Yau S.T.(ed.), *Essays on Mirror Manifolds*, Int. Press Co., 1992.
- [5] Klemm A. and Schimmrigk R., Landau-Ginzburg string vacua, *Nucl. Phys. B*, 1993, V.411, 559.
- [6] Kachru S., Some three generation (0,2) Calabi-Yau models, Harvard preprint, HUTP-95/A003, hep-th/9501131.