Intuitionistic Fuzzy Neural Networks based on Extended Kalman Filter Training algorithm

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Abstract—In this paper, an intuitionistic fuzzy neural network model is proposed. The network structure has five layers, and adopts Mandani’s fuzzy reasoning. A new fuzzy inference system is applied in the model, which contains hesitation margin as a part. A training algorithm based on Extended Kalman Filter(EKF) is development. The EKF procedure to update parameter is introduced. The derivation of EKF based on adaptation algorithm for intuitionistic fuzzy adaptive equalizer is given. An example is given to demonstrate the intuitionistic fuzzy neural network based on EKF training algorithm has a good function approximate performance.

Keywords—Intuitionistic fuzzy neural networks;training algorithm;Extended Kalman Filter

I. INTRODUCTION

Intuitionistic fuzzy sets(IFSs) are introduced by Atanassov[1][2], which are generalization of the concept of fuzzy sets by adding an additional attribute parameter called non-membership[3]. IFSs are successful applied in many areas, multi-criteria fuzzy decision-making[4][5], pattern recognition[6], air quality modelling[7], time series prediction[8], etc.

During the past decade, a fuzzy neural network has been found to solve many problems which cannot be solved before[9]. For instance, the fuzzy neural network has been successfully applied in system identification[10], intelligent control[11], etc. Since IFSs have proved to be more powerful to deal with vagueness and uncertainty than fuzzy sets, combination of IFSs and artificial neural networks is investigated by many experts. In [12], a max-min intuitionistic fuzzy Hopfield neural network (IFHNN) is proposed. In [13], an intuitionistic fuzzy neural model is present based on an simple intuitionistic inference system. In [14], an intuitionistic fuzzy inference system based on game theory is proposed. In[15], an intuitionistic fuzzy neural network based on two steps gradient descent algorithm is present. In this paper, an intuitionistic fuzzy neural network based on EKF training algorithm is present. The EKF procedure to update intuitionistic fuzzy parameters is introduced. An example is given to show the EKF training yield more improved performance than using gradient descent algorithm.

This paper is organized as follows. In Section II, the new intuitionistic fuzzy inference system will be introduced. In Section III, a intuitionistic fuzzy neural network model with five layers is builded. In Section IV, the EKF training algorithm will be given. In Section V, An example is given to show the EKF training yield more
The Kalman gain matrix; is the target vector and (4) is error covariance matrix; The second equation, rules can be calculated as is measurement and process noise covariance, and the recurrent node activations, the network is the coefficient, which need to be designed. The first equation, known as the process equation, where the state of the system is given by the network’s membership and non-membership function parameter \( w_k = [c_k, \sigma_k, k] \). The process noise \( \omega_k \) is characterized as aero-mean, white noise with covariance given by \( E[\omega_k \omega_k^T] = \delta_{k,j} R_k \). The second equation, known as the observation or measurement equation, represents the network’s desired response vector \( y_k \) as a nonlinear function of the input vector \( u_k \), the network’s membership and non-membership function parameter \( w_k \), and the recurrent node activations \( v_k \). The measurement noise \( v_k \) is typically characterized as aero-mean, white noise with covariance given by \( E[v_k v_k^T] = \delta_{k,j} R_k \).

The training problem using Extended Kalman filter theory can now be described as finding the minimum mean-squared error estimate of the state \( w \) using all observed data so far. The Extended Kalman filter solution to the training problem is given by the following recursion[17]:

\[
A_k = \left[ R_k + H_k^T P_k H_k \right]^{-1}
\]

\[
K_k = P_k H_k^T A_k
\]

\[
\hat{w}_{k+1} = \hat{w}_k + K_k \xi_k
\]

\[
P_{k+1} = P_k - K_k H_k^T P_k + Q_k
\]

Where \( k \) is the discrete time index; the vector \( \hat{w}_k \) is the estimate of the state of the system at update step \( k \); \( K_k \) is the Kalman gain matrix; \( \xi_k = y_k - \hat{y}_k \) is error vector, where \( y_k \) is the target vector and \( \hat{y}_k \) is the network’s output vector; \( P_k \) is error covariance matrix; \( R_k \) and \( Q_k \) is measurement and process noise covariance matrices.

The extended Kalman filter training is carried out in a sequential fashion, one step of training involves the following steps:

1. The polynomial parameters \( \{c, \sigma, k\} \).
2. The membership and non-membership function parameter \( [c, \sigma, k] \).

The polynomial parameters \( \{c, \sigma, k\} \) can be solved by least square regression techniques[15].

So in this part, only a training algorithm based on extended Kalman filter how to optimize the membership and non-membership function parameter is introduced.

A intuitionistic fuzzy neural network’s behaviour can be described by the following nonlinear discrete-time system[16]:

\[
w_{k+1} = w_k + \omega_k
\]

\[
y_k = h_k(w_k, u_k, v_{k-1}) + v_k
\]

Layer 3: It is fuzzy inference layer. Each node represents a fuzzy rule. The degree of fulfillment and non-fulfillment of the \( i \) th rule is represented by the following equations:

\[
\bar{\mu}_j = \mu_{1j} \mu_{2j} \cdots \mu_{nj} = \prod_{i=1}^{n} \mu_{ij}
\]

\[
\bar{\gamma}_j = \gamma_{1j} \gamma_{2j} \cdots \gamma_{nj} = \prod_{i=1}^{n} \gamma_{ij}
\]

Layer 4: It normalized the degree of fulfillment and non-fulfillment of the fuzzy and calculated the hesitation margin index.

\[
\bar{\varphi}_j = \frac{\bar{\mu}_j}{\sum_{i=1}^{m} \bar{\mu}_j}
\]

\[
\bar{\phi}_j = \frac{\bar{\gamma}_j}{\sum_{i=1}^{m} \bar{\gamma}_j}
\]

\[
\pi_j = 1 - \bar{\varphi}_j - \bar{\phi}_j
\]

Layer 5: The output of the intuitionistic neural network with \( n \) rules can be calculated as:

\[
y = \sum_{j=1}^{m} \left( (1 - \pi_j) \omega_j \bar{\varphi}_j + \pi_j \varphi_j \bar{\phi}_j \right) = \sum_{j=1}^{m} y_j
\]

IV. IFNN TRAINING ALGORITHM BASED ON EXTENDED KALMAN FILTER

A. An training algorithm based on EKF

The IFNN architecture consists of two trainable parameter sets:
Initialize:
The number of iteration, \( n \)
\[ c_i = [c_{i1}, c_{i2}, \cdots, c_{im}] \] in the range of \([c_{\text{min}}, c_{\text{max}}] \)
\[ \sigma_i = [\sigma_{i1}, \sigma_{i2}, \cdots, \sigma_{im}] \] in the range of \([\sigma_{\text{min}}, \sigma_{\text{max}}] \)
\[ k_i = [k_{i1}, k_{i2}, \cdots, k_{im}] \] in the range of \([k_{\text{min}}, k_{\text{max}}] \)
\[ P_{\phi_i}(0), P_{\gamma_i}(0), P_{k_i}(0) \] each equals to an identify matrix of \( m \times m \)

Recursion:
For \( k = 1, 2, \cdots, n \) do
1. Calculate the membership \( \mu_i(x) \) and non-membership \( \gamma_i(x) \);
2. Calculate the Jacobians \( H_{\phi_i}(k) \) \( H_{\sigma_i}(k) \) and \( H_{k_i}(k) \) respectively.
3. Calculate the Kalman gain matrices \( K_{\phi_i}(k) \) \( K_{\sigma_i}(k) \) and \( K_{k_i}(k) \);
4. Update the error covariance matrix \( P_{\phi_i}(k) \), \( P_{\sigma_i}(k) \) and \( P_{k_i}(k) \);
5. Update the parameter \( c_i, \sigma_i \) and \( k_i \).

B. Derivation of Extended Kalman Filter based on adaptation algorithm

The Jacobians \( H_{\phi_i}(k) \) \( H_{\sigma_i}(k) \) and \( H_{k_i}(k) \) are calculated as folowed:
\[
H_{\phi_i}(k) = \left( \frac{\partial y_j}{\partial y_j} \right)\left( \frac{\partial \phi_j}{\partial \phi_j} \frac{\partial \phi_j}{\partial \mu_j} \frac{\partial \mu_j}{\partial y_j} + \frac{\partial \phi_j}{\partial \phi_j} \frac{\partial \phi_j}{\partial \gamma_j} \frac{\partial \gamma_j}{\partial y_j} \right) + \frac{\partial y_j}{\partial y_j} \left( \frac{\partial \mu_j}{\partial \phi_j} \frac{\partial \phi_j}{\partial y_j} + \frac{\partial \phi_j}{\partial \mu_j} \frac{\partial \mu_j}{\partial y_j} + \frac{\partial \phi_j}{\partial \gamma_j} \frac{\partial \gamma_j}{\partial y_j} + \frac{\partial \gamma_j}{\partial \phi_j} \frac{\partial \phi_j}{\partial y_j} + \frac{\partial \gamma_j}{\partial \mu_j} \frac{\partial \mu_j}{\partial y_j} + \frac{\partial \gamma_j}{\partial \gamma_j} \frac{\partial \gamma_j}{\partial y_j} \right) + \frac{\partial y_j}{\partial y_j} \left( \frac{\partial \phi_j}{\partial \phi_j} \frac{\partial \phi_j}{\partial y_j} \right)
\]

Because \( y = \sum_{j=1}^{m} y_j \) so \( \frac{\partial y_j}{\partial y_j} = 1 \)
\[
\gamma(x) = \left( 1 - \exp \left( \frac{-(x-c_{ij})^2}{2\sigma_{ij}^2} \right) \right)^{k_{ij}}, k_{ij} \geq 1 \quad \text{so}
\]
\[
\frac{\partial \gamma_{ij}}{\partial k_{ij}} = k_{ij} \left( 1 - \exp \left( \frac{-(x-c_{ij})^2}{2\sigma_{ij}^2} \right) \right)^{k_{ij}-1}
\]
Substituting these into the update equation we get

\[
H_{\phi_i}(k) = \left( \frac{1 - \phi_j}{\sum_{j=1}^{m} \mu_j} \right) \left( \frac{\gamma_j}{\sum_{j=1}^{m} \gamma_j} \right) \left( \frac{\phi_j}{\mu_j} \frac{x-c_{ij}}{\sigma_{ij}^2} \right)
\]

\[
H_{\sigma_i}(k) = \left( \frac{1 - \phi_j}{\sum_{j=1}^{m} \mu_j} \right) \left( \frac{\gamma_j}{\sum_{j=1}^{m} \gamma_j} \right) \left( \frac{\sigma_j}{\mu_j} \frac{x-c_{ij}}{\sigma_{ij}^2} \right)
\]

V. Example

In this section, the effectiveness of the intuitionistic fuzzy neural based on EKF training algorithm is demonstrated. The simulation is carried out in the unified running environment of Matlab2011a.
In the experiment, the function $\sin(x)$ is needed to be approximated:
\[ f(x) = \sin(x) \]

The intuitionistic fuzzy neural network is constructed with four fuzzy rules. A random sampling of the interval $[-4, 4]$ is used in obtaining 200 input-output data pairs for the training set. $c_i$ in the range of $[-5, 5]$, $\sigma_i$ in the range of $[1,4]$, $k_i$ in the range of $[1,5]$, $\omega_i, \sigma_i$ in the range of $[-1,1]$. Fig 2 shows the training results and training results. We can see that the resulting intuitionistic fuzzy neural network can approximate well to the original function. Comparisons of the gradient descent algorithm, Fig 4 plot root mean squared error during the training process of EKF and gradient descent algorithm. The results illustrate the good approximate performance.

In this paper, a training algorithm based on EKF for intuitionistic fuzzy neural network has been developed. An intuitionistic fuzzy neural network with five layers has been constructed. The EKF has been applied to parameter identification of intuitionistic fuzzy neural network. The EKF procedure to update parameter is introduced. The derivation of EKF based on adaptation algorithm for intuitionistic fuzzy adaptive equalizer is given. Simulation results show that the intuitionistic fuzzy neural network based on EKF training algorithm has a good function approximate performance than gradient descent training algorithm.

**REFERENCES**