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Invariance Analysis of the (2+1) Dimensional Long Dispersive Wave Equation

M. SENTHIL VELAN and *M. LAKSHMANAN*

Center for Nonlinear Dynamics, Department of Physics, Bharathidasan University Tiruchirapalli 620 024, India

Abstract

In this paper, we bring out the Lie symmetries and associated similarity reductions of the recently proposed (2+1) dimensional long dispersive wave equation. We point out that the integrable system admits an infinite-dimensional symmetry algebra along with Kac-Moody-Virasoro-type subalgebras. We also bring out certain physically interesting solutions.

1 Introduction

Professor Wilhelm Fushchych has been stressing for several decades the importance of symmetry analysis of nonlinear evolution equations to understand the basic properties of the underlying physical systems. It is a great pleasure to contribute the present article on his sixtieth birthday.

Soliton equations in (1+1) dimensions exhibit very interesting symmetry properties, both Lie and Lie-Bäcklund type [1-3]. These symmetries help one to understand the integrability properties of underlying nonlinear dynamical systems clearly. In (2+1) dimensions, solutions have much richer structures [4, 5]. As a consequence, the identification and study of symmetries will play a crucial role here also. Concentrating on Lie symmetries for the present, it has been realized that important classes of (2+1) dimensional extensions of soliton equations admit typically Lie symmetries involving infinite-dimensional symmetry algebras, often of the Kac-Moody-Virasoro type. Typical systems are the following: (i) Kadomtsev-Petviashvili equation [6], (ii) Davey-Stewartson equation [7], (iii) Nizhnik-Novikov-Veselov equation [8], (iv) nonlinear Schrödinger equation introduced by Fokas and the sine-Gordon equation [8]. However, there are certain integrable evolution equations which, though admit infinite-dimensional Lie algebras, do not seem to possess a Virasoro- type subalgebra [9]. The typical examples being (i) the breaking soliton equation and (ii) nonlinear Schrödinger type equation were studied by Strachan recently. So, the connection between integrability and Virasoro-type algebras deserves much study further.

In this contribution, we analyze the invariance properties of an important new evolution equation in (2+1) dimensions, namely, the (2+1) dimensional long dispersive wave equation and bring out the existence of an infinite-dimensional Lie algebra of symmetries along with Kac-Moody-Virasoro-type subalgebras. We also deduce the possible similarity reductions and some particular solutions.

The plan of the paper is as follows. In Sec.2 we discuss the Lie symmetries and Virasoro type subalgebras for the (2+1) dimensional long dispersive wave equation. In Sec.3 we construct the similarity variables and obtain the similarity reductions. In Sec.4 the non-trivial subcases are presented. In Sec.5 we present the invariance analysis of the reduced pde. In Sec.6 we present our conclusions.

2 Lie symmetries and Kac-Moody-Virasoro algebras of the (2+1) dimensional long dispersive wave (2LDW) equation

Recently Chakravarty, Kent and Newman[10] have introduced a new (2+1) dimensional long dispersive wave equation by symmetrically reducing the self-dual Yang-Mills equation. The resultant equation can be written in the form

$$\begin{aligned} &\lambda q_t + q_{xx} - 2q\int (qr)_x d\eta = 0, \\ &\lambda r_t - r_{xx} + 2r\int (qr)_x d\eta = 0, \end{aligned} \tag{1}$$

where $\partial_\eta = \partial_x - \lambda\partial_y$ and λ is a constant parameter. Eq.(1) is the (2+1) dimensional generalization of the one-dimensional long dispersive wave equation [11, 12]. It is interesting to note that eq.(1) reduces to the single nonlocal equation introduced recently by Fokas [13],

$$i\lambda q_t + q_{xx} - 2q\int |q|_x^2 d\eta = 0, \tag{2}$$

when $r = q^*$ and $t \to it$. Eq.(2) arises in plasma physics under appropriate circumstances [14] and it admits exponentially localized solutions and satisfies the Painlevé property [15]. Recently the integrability aspects of eq.(1) have been studied by Radha and Lakshmanan [16]. Using the bilinear approach, they have brought out the peculiar localization properties of solutions of eq.(1) by generating dromions for the physical quantity rq (composite field).

2.1 Lie symmetries

By introducing the transformation $(qr)_x = v_\eta$, where v is some arbitrary potential, we can rewrite eq.(1) as

$$\begin{aligned} &\lambda q_t + q_{xx} - 2qv = 0, \\ &\lambda r_t - r_{xx} + 2rv = 0, \\ &(qr)_x - v_\eta = 0. \end{aligned} \tag{3}$$

Now one can apply the Lie algorithm to eq.(3) and study the invariance properties. However, for our present study, we have considered the above equation in the form

$$\begin{aligned} &q_t + q_{xx} - 2qv = 0, \\ &r_t - r_{xx} + 2rv = 0, \\ &v_y - rq_x - qr_x = 0, \end{aligned} \tag{4}$$

wherein we have introduced the notational change $\eta \to y$ for convenience. The invariance of eq.(4) under the infinitesimal point transformations

$$\begin{aligned} &x \longrightarrow X = x + \varepsilon\xi_1(t,x,y,q,r,v), \quad q \longrightarrow Q = q + \varepsilon\phi_1(t,x,y,q,r,v), \\ &y \longrightarrow Y = y + \varepsilon\xi_2(t,x,y,q,r,v), \quad r \longrightarrow R = r + \varepsilon\phi_2(t,x,y,q,r,v), \\ &t \longrightarrow T = t + \varepsilon\xi_3(t,x,y,q,r,v), \quad v \longrightarrow V = v + \varepsilon\phi_3(t,x,y,q,r,v), \ \varepsilon \ll 1 \end{aligned}$$

leads to the following expressions for infinitesimals

$$\begin{aligned} &\xi_1 = \frac{x}{2}\dot{f}(t) + g(t), \qquad \xi_2 = m(y), \qquad \xi_3 = f(t), \\ &\phi_1 = \left[\frac{1}{2}x\dot{g}(t) + \frac{1}{8}\ddot{f}(t)x^2 - m'(y) - \frac{1}{2}\dot{f}(t) - N(y,t)\right]q, \\ &\phi_2 = \left[-\frac{1}{2}x\dot{g}(t) - \frac{1}{8}\ddot{f}(t)x^2 + N(y,t)\right]r, \\ &\phi_3 = -v\dot{f}(t) + \frac{1}{4}x\ddot{g}(t) + \frac{1}{16}\frac{d^3 f}{dt^3}x^2 + h(t), \end{aligned} \tag{5}$$

where $f(t)$, $g(t)$, $h(t)$ are arbitrary functions of t and $N(y,t)$ is an arbitrary function of (y,t) and dot and prime denote differentiations with respect ot t and y, respectively. In the above, the arbitrary functions $f(t)$, $h(t)$ and $N(y,t)$ are constrained by the following equation

$$\ddot{f}(t) + 4\dot{N}(y,t) + 8h(t) = 0. \tag{6}$$

The infinitesimals given in eqs.(5-6) are actually obtained using the symbolic program LIE [17].

2.2 Lie algebras

The presence of arbitrary functions $f(t)$, $g(t)$, $m(y)$ and $N(y,t)$ necessarily leads to an infinite-dimensional Lie algebra of symmetries. We can write a general element of this Lie algebra as

$$V = V_1(f) + V_2(g) + V_3(m) + V(N),$$

where

$$\begin{aligned} V_1(f) = &\frac{x}{2}\dot{f}(t)\frac{\partial}{\partial x} + f(t)\frac{\partial}{\partial t} + \left(\frac{1}{8}\ddot{f}(t)qx^2 - \frac{1}{2}q\dot{f}(t)\right)\frac{\partial}{\partial q} \\ &-\frac{1}{8}\ddot{f}(t)x^2 r\frac{\partial}{\partial r} + \left(\frac{1}{16}\frac{d^3 f}{dt^3}x^2 - v\dot{f}(t)\right)\frac{\partial}{\partial v}, \\ V_2(g) = &g(t)\frac{\partial}{\partial x} + \frac{1}{2}\dot{g}(t)xq\frac{\partial}{\partial q} - \frac{1}{2}\dot{g}(t)xr\frac{\partial}{\partial r} + \frac{1}{4}\ddot{g}(t)x\frac{\partial}{\partial v}, \\ V_3(m) = &m(y)\frac{\partial}{\partial y} - m'(y)q\frac{\partial}{\partial q}, \qquad V_4(N) = -qN(y,t)\frac{\partial}{\partial q} + rN(y,t)\frac{\partial}{\partial r}. \end{aligned}$$

The associated Lie algebra between these vector fields becomes

$$
\begin{aligned}
&[V_1(f_1), V_1(f_2)] = V_1(f_1\dot{f}_2 - f_2\dot{f}_1),\\
&[V_2(g_1), V_2(g_2)] = \frac{g_1\dot{g}_2 - g_2\dot{g}_1}{2}\left(q\frac{\partial}{\partial q} - r\frac{\partial}{\partial r}\right) + \frac{g_1\ddot{g}_2 - g_2\ddot{g}_1}{4}\frac{\partial}{\partial v},\\
&[V_3(m_1), V_3(m_2)] = V_3(m_1m_2' - m_2m_1'),\\
&[V_1(f), V_2(g)] = V_2\left(f\dot{g} - \frac{1}{2}g\dot{f}\right),\\
&[V_1(f), V_4(N)] = V_4(f\dot{N}),\\
&[V_3(m), V_4(N)] = V_4(mN'),
\end{aligned}
$$

which is obviously an infinite-dimensional Lie algebra of symmetries. A Virasoro-Kac-Moody-type subalgebra is immediately obtained by restricting the arbitrary functions f and m to Laurent polynomials so that we have the commutators

$$[V_1(t^n), V_1(t^m)] = (m-n)V_1(t^{n+m-1}), [V_3(y^n), V_3(y^m)] = (m-n)V_3(y^{n+m-1}).$$

It is interesting to note that a similar type of algebras also exist in other integrable systems mentioned in Introduction, namely, the Nizhnik-Novikov-Veselov equation, (2+1) dimensional nonlinear Schrödinger equation, and sine-Gordon equation [8].

3 Similarity variables and similarity reductions

The similarity variables associated with the infinitesimal symmetries (5) can be found by solving the characteristic equation

$$
\begin{aligned}
&\frac{dx}{\frac{x}{2}\dot{f}(t) + g(t)} = \frac{dy}{m(y)} = \frac{dt}{f(t)} = \frac{dq}{(\frac{1}{2}x\dot{g}(t) + \frac{1}{8}\ddot{f}(t)x^2 - m'(y) - \frac{1}{2}\dot{f}(t) - N(y,t))q}\\
&\quad = \frac{dr}{-(\frac{1}{2}x\dot{g}(t) + \frac{1}{8}\ddot{f}(t)x^2 - N(y,t))r} = \frac{dv}{-v\dot{f}(t) + \frac{1}{4}x\ddot{g}(t) + \frac{1}{16}\frac{d^3f}{dt^3}x^2 + h(t)}.
\end{aligned} \tag{7}
$$

Integrating eq.(7) with the condition that $f(t) \neq 0$, we get the following similarity variables:

$$
\begin{aligned}
&\tau_1 = \frac{x}{f^{\frac{1}{2}}(t)} - \int^t \frac{g(t')}{f^{\frac{3}{2}}(t')}dt', \qquad \tau_2 = \int^y \frac{dy'}{m(y')} - \int^t \frac{dt'}{f(t')},\\
&F = qe^{w_1}, \qquad G = re^{w_2}, \qquad H = vf(t) - w_3,
\end{aligned}
$$

where F, G and H are functions of τ_1 and τ_2 and

$$
\begin{aligned}
w_1 = {} & \frac{g(t)}{2f^{1/2}(t)}\int^t \frac{g(t')}{f^{3/2}(t')}dt' - \int^t \frac{g^2(t')}{2f^2(t')}dt' + \frac{g(t)\tau_1}{2f^{1/2}(t)} + \frac{\dot{f}(t)}{8}\left[\int^t \frac{g(t')dt'}{f^{3/2}(t')}\right]^2\\
&+\frac{\tau_1^2\dot{f}(t)}{8} + \tau_1\dot{f}(t)\int^t \frac{g(t')}{4f^{3/2}(t')}dt' - m'(y)\int^t \frac{1}{f(t')}dt' - \frac{1}{2}\log f(t) - \int^t \frac{N(y,t')}{f(t')}dt',\\
w_2 = {} & -\frac{g(t)}{2f^{1/2}(t)}\int^t \frac{g(t')}{f^{3/2}(t')}dt' + \int^t \frac{g^2(t')}{2f^2(t')}dt' - \frac{g(t)\tau_1}{2f^{1/2}(t)} - \frac{\dot{f}(t)}{8}\left[\int^t \frac{g(t')dt'}{f^{3/2}(t')}\right]^2\\
&-\frac{\tau_1^2\dot{f}(t)}{8} - \tau_1\dot{f}(t)\int^t \frac{g(t')}{4f^{3/2}(t')}dt' + \int^t \frac{N(y,t')}{f(t')}dt',
\end{aligned}
$$

$$w_3 = \dot{g} f^{1/2} \int^t \frac{g(t')}{4f^{3/2}(t')}dt' - \frac{\dot{f}g}{2f^{1/2}} \int^t \frac{g(t')}{4f^{3/2}(t')}dt' + \frac{\tau_1 \dot{g}}{4} f^{1/2} - \frac{\tau_1 \dot{f} g(t)}{8f^{1/2}(t)} - \frac{g^2}{8f} + \frac{\tau_1^2 f \ddot{f}}{16}$$
$$+\frac{\tau_1 f \ddot{f}}{2} \int^t \frac{g(t')}{4f^{3/2}} + f\ddot{f} \left[\int^t \frac{g(t')}{4f^{3/2}(t')}dt' \right]^2 - \frac{\dot{f}^2}{2} \left[\int^t \frac{g(t')}{4f^{3/2}(t')}dt' \right]^2$$
$$-\frac{\tau_1^2 \dot{f}^2}{32} - \tau_1 \dot{f}^2(t) \int^t \frac{g(t')}{16 f^{3/2}(t')}dt' + \int h(t')dt'.$$

Under the above similarity transformations, eq.(4) gets reduced to a system of pde in two independent variables τ_1 and τ_2:

$$\begin{aligned} &F_{\tau_2} - F_{\tau_1 \tau_1} + 2FH + kF = 0, \\ &G_{\tau_2} + G_{\tau_1 \tau_1} - 2GH - kG = 0, \\ &H_{\tau_2} - FG_{\tau_1} - GF_{\tau_1} = 0, \end{aligned} \tag{8}$$

where k is an arbitrary constant obtained by integrating eq.(6)

$$\frac{\partial}{\partial t} \left(\dot{f} + 4N(y,t) + 8 \int^t h(t')dt' \right) = 0.$$

Since the original (2+1) dimensional pde (4) satisfies the Painlevé property [17] for a general manifold, the (1+1) dimensional similarity reduced pde (8) is also expected to satisfy the P-property.

4 Subcases

In addition to the above general similarity reduction, one can also investigate particular cases by assuming one or more of vector fields to be zero. We list below some of important nontrivial cases.

Case:1 f(t)=0: The similarity variables are

$$\tau_1 = t, \quad \tau_2 = x - g(t) \int^y \frac{dy'}{m(y')}, \quad F = qe^{-w_1}, \quad G = re^{-w_2},$$
$$H = v - \frac{g\ddot{g}}{4} \int^y \frac{1}{m(y')} \left[\int^{y'} \frac{dy''}{m(y'')} \right] dy' - \frac{\tau_2 \ddot{g}}{4} \int^y \frac{dy'}{m(y')} - h(t) \int^y \frac{dy'}{m(y')},$$

where

$$w_1 = \frac{g\dot{g}}{2} \int^y \frac{1}{m(y')} \left[\int^{y'} \frac{dy''}{m(y'')} \right] dy' + \frac{\tau_2 \dot{g}}{2} \int^y \frac{dy'}{m(y')} - \log m(y) - \int^y \frac{N(y',t)dy'}{m(y')},$$
$$w_2 = \frac{-g\dot{g}}{2} \int^y \frac{1}{m(y')} \left[\int^{y'} \frac{dy''}{m(y'')} \right] dy' - \frac{\tau_2 \dot{g}}{2} \int^y \frac{dy'}{m(y')} + \int^y \frac{N(y',t)dy'}{m(y')}.$$

The reduced pde takes the form

$$\begin{aligned} &F_{\tau_1} + F_{\tau_2 \tau_2} - 2FH = 0, \\ &G_{\tau_1} - G_{\tau_2 \tau_2} + 2GH = 0, \\ &FG_{\tau_2} + GF_{\tau_2} - g(\tau_1) H_{\tau_2} - h(\tau_1) - (\tau_2/4) g''(\tau_1) = 0. \end{aligned}$$

In the above, prime denotes differentiation with respect to the variable τ_1.
Case:2 f(t)=g(t)=0: The similarity variables are

$$\tau_1 = x, \quad \tau_2 = t, \quad F = qe^{-w_1}, \quad G = re^{\int^y (-N/m)dy'}, \quad H = v - h(t)\int^y \frac{dy'}{m(y')},$$

where

$$w_1 = -\log m(y) - \int^y \frac{N(y',t)dy'}{m(y')}.$$

The reduced pde takes the form

$$F_{\tau_2} + F_{\tau_1\tau_1} - 2FH = 0,$$
$$G_{\tau_2} - G_{\tau_1\tau_1} + 2GH = 0,$$
$$(FG)_{\tau_1} = h(\tau_2).$$

Case:3 f(t)=m(y)=0: The similarity variables are

$$\tau_1 = t, \quad \tau_2 = y, \quad F = qe^{-w_1}, \quad G = re^{w_1}, \quad H = v - \frac{\ddot{g}}{8g}x^2 - \frac{h}{g}x,$$

where

$$w_1 = \frac{\dot{g}}{4g}x^2 - \frac{N}{g}x.$$

The reduced pde takes the form

$$F_{\tau_1} + \frac{g'(\tau_1)}{2g(\tau_1)}F + \frac{N^2(\tau_1,\tau_2)}{g^2(\tau_1)}F - 2FH = 0, \tag{9a}$$

$$G_{\tau_1} - \frac{g'(\tau_1)}{2g(\tau_1)}G - \frac{N^2(\tau_1,\tau_2)}{g^2(\tau_1)}G + 2GH = 0, \tag{9b}$$

$$H_{\tau_2} = 0. \tag{9c}$$

Eq.(9) can be integrated as follows. Eq.(9a) and (9b) admit an integral

$$G = \frac{P(\tau_2)}{F}, \tag{10}$$

where $P(\tau_2)$ is an arbitrary function of τ_2. Integrating eq.(9c), we get

$$H = H(\tau_1), \tag{11}$$

where $H(\tau_1)$ is an arbitrary function of τ_1. Substituting eq.(11) in eq.(9a) and integrating it, we get

$$F = f_1(\tau_2)\exp\left[\int\left(2H(\tau_1) - \frac{g'(\tau_1)}{2g(\tau_1)} - \frac{N^2(\tau_1,\tau_2)}{g^2(\tau_1)}\right)d\tau_1\right], \tag{12}$$

where $f_1(\tau_2)$ is arbitrary function of τ_2. Substituting the expression of F in eq.(10), we get

$$G = \frac{P(\tau_2)}{f_1(\tau_2)}\exp\left[-\int\left(2H(\tau_1) - \frac{g'(\tau_1)}{2g(\tau_1)} - \frac{N^2(\tau_1,\tau_2)}{g^2(\tau_1)}\right)d\tau_1\right]. \tag{13}$$

Eqs.(11)–(13) form the solution to pde (9). From this, one can also write down the solution to pde (4) as

$$
\begin{aligned}
q &= f_1(y)\exp\left[\frac{\dot g(t)}{4g(t)}x^2 - \frac{N(y,t)}{g(t)}x + \int\left(2H(t) - \frac{\dot g(t)}{2g(t)} - \frac{N^2(t,y)}{g^2(t)}\right)dt\right],\\
r &= \frac{P(y)}{f_1(y)}\exp\left[-\frac{\dot g(t)}{4g(t)}x^2 + \frac{N(y,t)}{g(t)}x - \int\left(2H(t) - \frac{\dot g(t)}{2g(t)} - \frac{N^2(t,y)}{g^2(t)}\right)dt\right],\\
v &= H(t) + \frac{\ddot g}{8g}x^2 + \frac{h}{g}x.
\end{aligned}
$$

5 Lie symmetries and similarity reduction of eqs.(8)

Now the reduced pde (8) in two independent variables can itself be further analyzed for its symmetry properties by looking at its own invariance property under the classical Lie algorithm again. In this case, we obtain the following five-parameter Lie symmetries,

$$
\begin{aligned}
&\xi_1 = \frac{c_1}{3}\tau_1 - c_2, \qquad \xi_2 = \frac{2c_1}{3}\tau_2 - c_3, \qquad \phi_1 = -\left(\frac{2c_1}{3}k\tau_2 + 2c_4\tau_2 + \frac{5}{6}c_1 + c_5\right)F,\\
&\phi_2 = \left(\frac{2c_1}{3}k\tau_2 + 2c_4\tau_2 - \frac{c_1}{6} + c_5\right)G, \qquad \phi_3 = -\left(\frac{2c_1}{3}H - c_4\right),
\end{aligned}
$$

where c_1, c_2, c_3, c_4 and c_5 are arbitrary constants. The associated vector fields are

$$
\begin{aligned}
V_1 &= \frac{\tau_1}{3}\frac{\partial}{\partial\tau_1} + \frac{2\tau_2}{3}\frac{\partial}{\partial\tau_2} - \left(\frac{2k}{3}\tau_2 F + \frac{5}{6}F\right)\frac{\partial}{\partial F} + \left(\frac{2k}{3}\tau_2 - \frac{1}{6}G\right)\frac{\partial}{\partial G} - \frac{2H}{3}\frac{\partial}{\partial H},\\
V_2 &= -2\tau_2 F\frac{\partial}{\partial F} + 2\tau_2 G\frac{\partial}{\partial G} + \frac{\partial}{\partial H}, \quad V_3 = -F\frac{\partial}{\partial F} + G\frac{\partial}{\partial G}, \quad V_4 = \frac{\partial}{\partial\tau_1}, \quad V_5 = \frac{\partial}{\partial\tau_2}.
\end{aligned}
$$

The nonzero commutation relations between the vector fields are

$$
[V_1, V_2] = \frac{2}{3}V_2, \quad [V_1, V_4] = \frac{-1}{3}V_4, \quad [V_1, V_5] = \frac{2}{3}(V_3 - V_5), \quad [V_2, V_5] = 2V_3.
$$

Solving the characteristic equation associated with the similarity variables, we obtain

$$
\begin{aligned}
&z = \left(\tau_1 - \frac{3c_2}{c_1}\right)^2\left(\tau_2 - \frac{3c_3}{2c_1}\right), \qquad w_1(z) = F\left(\tau_2 - \frac{3c_3}{2c_1}\right)^p \exp\left[\left(k + \frac{3c_4}{c_1}\right)\tau_2\right],\\
&w_2(z) = G\left(\tau_2 - \frac{3c_3}{2c_1}\right)^{-p-(3/2)} \exp\left[\left(k + \frac{3c_4}{c_1}\right)\tau_2\right], \quad w_3(z) = \left(H - \frac{3c_4}{2c_1}\right)\left(\tau_2 - \frac{3c_3}{2c_1}\right),
\end{aligned}
$$

where

$$
p = \frac{3c_3}{2c_1}\left(k + \frac{3c_4}{c_1}\right) + \frac{3c_5}{2c_1} + \frac{5}{4}.
$$

The associated similarity reduced ode turns out to be

$$
\begin{aligned}
&zw_1'' + \frac{z+2}{4}w_1' + \frac{p}{4}w_1 + \frac{w_1 w_3}{2} = 0,\\
&zw_2'' - \frac{z-2}{4}w_2' + \frac{p-6}{4}w_2 - \frac{w_2 w_3}{2} = 0,\\
&zw_3' + w_3 + 2z^{1/2}(w_1 w_2' + w_2 w_1') = 0.
\end{aligned}
$$

Even though it is very difficult to find a solution for the above equation, one can obtain interesting solutions by assuming one or more of the constants $c_i,\ i = 1,\ldots,5$ be zero. The following are some of nontrivial cases.
Case:1 $\underline{c_1 = 0}$: The similarity variables are

$$\begin{aligned} z &= \tau_1 - \frac{c_2}{c_3}\tau_2, & w_1 &= F\exp\left[-\left(\frac{c_4}{c_3}\tau_2^2 + \frac{c_5}{c_3}\tau_2\right)\right], \\ w_2 &= G\exp\left[\frac{c_4}{c_3}\tau_2^2 + \frac{c_5}{c_3}\tau_2\right], & w_3 &= H + \frac{c_4}{c_3}\tau_2. \end{aligned}$$

The reduced ode takes the form

$$\begin{aligned} & w_1'' + \frac{c_2}{c_3}w_1' - 2w_1w_3 - \left(k + \frac{c_5}{c_3}\right)w_1 = 0, \\ & w_2'' - \frac{c_2}{c_3}w_2' - 2w_2w_3 - \left(k + \frac{c_5}{c_3}\right)w_2 = 0, \\ & w_3' + \frac{c_3}{c_2}(w_1w_2)' + \frac{c_4}{c_2} = 0. \end{aligned}$$

This equation has not yet been fully analyzed.
Case:2 $\underline{c_1 = c_2 = 0}$: The similarity variables are

$$\begin{aligned} z &= \tau_1, & w_1 &= F\exp\left[-\left(\frac{c_4}{c_3}\tau_2^2 + \frac{c_5}{c_3}\tau_2\right)\right], \\ w_2 &= G\exp\left[\frac{c_4}{c_3}\tau_2^2 + \frac{c_5}{c_3}\tau_2\right], & w_3 &= H + \frac{c_4}{c_3}\tau_2. \end{aligned}$$

The reduced ode takes the form

$$\begin{aligned} & w_1'' - 2w_1w_3 - \left(k + \frac{c_5}{c_3}\right)w_1 = 0, \\ & w_2'' - 2w_2w_3 - \left(k + \frac{c_5}{c_3}\right)w_2 = 0, \\ & (w_1w_2)' + \frac{c_4}{c_3} = 0. \end{aligned} \tag{14}$$

Integrating eq.(14), we get the solution

$$\begin{aligned} w_1 &= I_3\left(z - \frac{I_1c_3}{c_4}\right)^{\left(\frac{1}{2} - \frac{I_2c_3}{2c_4}\right)}, \\ w_2 &= \frac{-c_4}{c_3I_3}\left(z - \frac{I_1c_3}{c_4}\right)^{-\left(\frac{1}{2} + \frac{I_2c_3}{2c_4}\right)}, \\ w_3 &= \frac{1}{2}\left(k + \frac{c_5}{c_3}\right)\left(\frac{1}{8} - \frac{I_2^2c_3^2}{8c_4^2}\right)\left(z - \frac{I_1c_3}{c_4}\right)^{-2}, \end{aligned}$$

where I_1, I_2 and I_3 are integration constants.
Case:3 $\underline{c_1 = c_3 = 0}$: The similarity variables are

$$\begin{aligned} z &= \tau_2, & w_1 &= F\exp\left[-\left(\frac{2c_4}{c_2}\tau_1\tau_2 + \frac{c_5}{c_2}\tau_1\right)\right], \\ w_2 &= G\exp\left[\frac{2c_4}{c_2}\tau_1\tau_2 + \frac{c_5}{c_2}\tau_1\right], & w_3 &= H + \frac{c_4}{c_2}\tau_1. \end{aligned}$$

The reduced ode takes the form

$$w_1' + 2w_1w_3 + kw_1 = 0, \qquad w_2' - 2w_2w_3 - kw_2 = 0, \qquad w_3' = 0. \tag{15}$$

Eq.(15) admits the following solution

$$w_1 = I_2 \exp[-(2I_1 + k)z], \qquad w_2 = I_3 \exp[(2I_1 + k)z], \qquad w_3 = I_1,$$

where I_1, I_2 and I_3 are integration constants.

6 Conclusions

In this paper, we have carried out an invariance analysis and similarity reductions of the (2+1) dimensional long dispersive wave (2LDW) equation and obtained particular solutions. We have pointed out the fact that the 2LDW equation admits an infinite-dimensional symmetry algebra and Kac-Moody-Virasoro type subalgebras, which typically exist in many other integrable (2+1) dimensional systems. It is yet to be clearly understood as to what is the significance of the existence or nonexistence of Kac-Moody-Virasoro-type subalgebras to nonlinear evolution equations as far as integrability is concerned. Such an understanding can throw some light on the classification of integrable systems. Currently we are investigating the possible similarity reductions of the above said equation through the nonclassical method and direct method of Clarkson and Kruskal.

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