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On the Analytical Approach to the *N*-Fold Bäcklund Transformation of Davey-Stewartson Equation

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Abstract

N-fold Bäcklund transformation for the Davey-Stewartson equation is constructed by using the analytic structure of the Lax eigenfunction in the complex eigenvalue plane. Explicit formulae can be obtained for a specified value of N. Lastly it is shown how generalized soliton solutions are generated from the trivial ones.

Introduction: Inverse scattering transform holds a central place in the analysis of nonlinear integrable system in either (1+1)- or (2+1)-dimensions [1]. On the other hand it has been found that explicit soliton solutions can also be obtained by the use of Bäcklund transformations in a much easier way [2]. These Bäcklund transformations are also useful in proving the superposition formulae for these solutions. There have been many attempts to construct explicit N-soliton solutions for nonlinear integrable system either by Bäcklund transformations or the inverse scattering method. A separate and elegant approach was developed by Zakharov *et al* [3] which relied on the pole structure of the Lax eigenfunction and use of projection operators. In this letter we have used an approach similar to that of Zakharov *et al* but have generated a formulae for the N-fold BT of the nonlinear field variables occurring in the (2+1)-dimensional Davey-Stewartson equation [4]. Our approach is very similar to that of gauge transformation repeatedly applied to any particular seed solution. Lastly we demonstrate how non-trivial solutions are generated by starting with known trivial ones.

Formulation: The Davey-Stewartson equation under consideration can be written as

$$
\begin{aligned}
&ir_t + r_{xx} - r_{yy} + r(A_2 - A_1) = 0,\\
&iq_t + q_{yy} - q_{xx} + q(A_1 - A_2) = 0,\\
&A_{1x} = -\frac{1}{2}(q_y r + r_y q), \qquad A_{2y} = -\frac{1}{2}(q_x r + r_x q).
\end{aligned}
\tag{1}
$$

Equation (1) is known to be a result of the consistency of the operators T_1 and T_2 written as $[T_1, T_2]\Psi = 0$, where

$$T_1\Psi = \left\{2\begin{pmatrix}\partial_x & 0\\ 0 & \partial_y\end{pmatrix} + \begin{pmatrix}0 & q\\ r & 0\end{pmatrix}\right\}\Psi = 0,$$

$$T_2\Psi = \frac{1}{2}\left\{\left(i\partial_t + \partial_x^2 + \partial_y^2\right) + \begin{pmatrix}0 & q_x\\ r_y & 0\end{pmatrix} + A\right\}\Psi = -\frac{K^2}{2}\Psi,$$

and $A = \begin{pmatrix}A_1 & 0\\ 0 & A_2\end{pmatrix}$, $K = \text{const.}$

To proceed further we set

$$A_1 = f_{1y}, \qquad A_2 = f_{2x}$$

and

$$\Psi = \Phi \exp\left\{i\left(\alpha + \lambda^{-2}\right)x - i\left(\beta - \lambda^{-2}\right)y\right\}.$$

Whence the Lax pair becomes,

$$M\Phi = U\Phi, \qquad \Phi_t = V\Phi \tag{2}$$

with

$$M = \begin{pmatrix}\partial_x & 0\\ 0 & \partial_y\end{pmatrix}, \qquad U = \begin{pmatrix}-i\left(\alpha + \lambda^{-2}\right) & -q/2\\ -r/2 & i\left(\beta - \lambda^{-2}\right)\end{pmatrix} \tag{3}$$

and

$$V = \begin{pmatrix}i\Lambda + Q(\partial_x, \partial_y) + if_{1y} & iq_x\\ ir_y & i\Lambda + Q(\partial_x, \partial_y) + if_{2y}\end{pmatrix}. \tag{4}$$

With

$$\Lambda = K^2 - \left(\alpha + \lambda^{-2}\right)^2 - \left(\beta - \lambda^{-2}\right)^2,$$
$$Q(\partial_x, \partial_y) = i\left(\partial_x^2 + \partial_y^2\right) - 2\left\{\left(\alpha + \lambda^{-2}\right)\partial_x - \left(\beta - \lambda^{-2}\right)\partial_y\right\},$$

we can construct particular Jost solutions, corresponding to $q = q_0 = \text{const}$, $r = r_0 = \text{const}$, $A_1 = A_{10} = f_{1y}^0$ and $A_2 = A_{20} = f_{2x}^0$ with $f_{1y}^0 = f_{2x}^0 = \text{const}$. This particular eigenvector Φ_0 turns out to be

$$\hat{\Phi}_0 = \begin{pmatrix}\exp(\theta_1 x + \chi_1 y + \xi_1 t) & \exp(\theta_2 x + \chi_2 y + \xi_2 t)\\ m_0\exp(\theta_1 x + \chi_1 y + \xi_1 t) & n_0\exp(\theta_2 x + \chi_2 y + \xi_2 t)\end{pmatrix}$$

with

$$\theta_1 = -i\left(\alpha + \lambda^{-2}\right) + am_0, \qquad \theta_2 = -i\left(\alpha + \lambda^{-2}\right) + an_0,$$
$$\chi_1 = b/m_0 + i\left(\beta - \lambda^{-2}\right), \qquad \chi_2 = b/n_0 + i\left(\beta - \lambda^{-2}\right),$$

ξ_1, ξ_2 are arbitrary complex constants, m_0, n_0 are arbitrary constants and $m_0 \neq n_0$.

Note that $\det\Phi_0 \neq 0$ so that Φ_0^{-1} exists.

Now suppose that Φ_{n-1} denotes the Lax eigenfunction, corresponding to the $(n-1)$ soliton solution, and $B_n(x,y,t)$ be the transformation which yields the Φ_n (solution corresponding to the n soliton case when applied to Φ_{n-1}), that is

$$\Phi_n(x,y,t,\lambda) = B_n(x,y,t,\lambda)\Phi_{n-1}(x,y,t,\lambda).$$

Using the above Lax equations (2) and (4), we can at once deduce the equations satisfied by B_n

$$MB_n = U_n B_n - B_n U_{n-1}, \qquad \partial_t B_n = V_n B_n - B_n V_{n-1}, \tag{5}$$

where U_n, V_n denote the Lax matrices.

Corresponding to the n-soliton solution: Note that U, V are even functions of λ, so that we can assume that

$$B_n(-\lambda) = B_n(\lambda).$$

We now assume B_n to have simple pole structure in the complex λ-plane, so that

$$B_n(x,y,t,\lambda) = Q_n + \frac{2\lambda_n}{\lambda^2-\lambda_n^2}P_n. \tag{6}$$

We also assume that

$$B_n^{-1}(x,y,t,\lambda) = Q'_n + \frac{2\lambda'_n}{\lambda^2-\lambda_n^2}P'_n, \tag{7}$$

where P_n, Q_n, P'_n, Q'_n are matrix functions of (x,y,t). The condition $B_nB_n^{-1} = B_n^{-1}B_n = I$ leads to

$$B_n(\lambda'_n)P'_n = 0, \qquad P_nB_n^{-1}(\lambda_n) = 0,$$
$$B_n^{-1}(\lambda_n)P_n = 0, \qquad P'_nB_n(\lambda'_n) = 0.$$

Calculation of the matrices Q_n, P_n, Q'_n, P'_n: Let us now go back to equation (5) and use the expression (6) and (7). Rewriting U_n as

$$U_n = -i\lambda^{-2}I + U'_n$$

and using the form of B_n given in (6) we get:

$$\begin{aligned} MQ_n + \frac{2\lambda_n}{\lambda^2-\lambda_n^2}(MP_n) &= \left(-i\lambda^{-2}I + U'_n\right)\left(Q_n + \frac{2\lambda_n}{\lambda^2-\lambda_n^2}P_n\right) \\ &\quad - \left(Q_n + \frac{2\lambda_n}{\lambda^2-\lambda_n^2}P_n\right)\left(-i\lambda^{-2}I + U'_{n-1}\right) \end{aligned}$$

which yields equations satisfied by P_n and Q_n:

$$M(P_n\Phi_{n-1}(\lambda_n)) = U_n(\lambda_n)(P_n\Phi_{n-1}(\lambda_n)),$$
$$M(Q_n\Phi_{n-1}(\lambda_n)) = U_n(\lambda_n)(Q_n\Phi_{n-1}(\lambda_n)).$$

Using the same form of V_n as give in equation (3) in the time part, we get the following equations:

$$\partial_x\left(Q_n - \frac{2}{\lambda_n}P_n\right) = -\partial_y\left(Q_n - \frac{2}{\lambda_n}P_n\right),$$
$$\partial_t(P_n\Phi_{n-1}(\lambda_n)) = V_n(\lambda_n)(P_n\Phi_{n-1}(\lambda_n)),$$
$$Q_{nt} = OP_2Q_n + 2i(Q_{nx}\partial_x + Q_{ny}\partial_y) + D_nQ_n - Q_nD_{n-1},$$

where operator $OP_2 \equiv i\left(\partial_x^2+\partial_y^2\right)-2(\alpha\partial_x-\beta\partial_y)$, $\alpha,\beta=$ const., and $D_n=\begin{pmatrix} if_{1y}^n & iq_{nx} \\ ir_{ny} & if_{2x}^n \end{pmatrix}$.

As per the ansatz of Zakharov we search for P_n in the form,

$$P_n=\begin{pmatrix}\gamma_{n1}\\ \gamma_{n2}\end{pmatrix}(\delta_{n1},\delta_{n2}).$$

It is interesting to observe that

$$(\delta_{n1},\delta_{n2})=(a_{n1},a_{a2})\Phi_{n-1}^{-1}(n),$$

where a_{n1}, a_{n2} are practically two constants.

Similarly for P'_n and Q'_n, we set

$$P'_n=\begin{pmatrix}\gamma'_{n1}\\ \gamma'_{n2}\end{pmatrix}(\delta'_{n1},\delta'_{n2}) \quad \text{and} \quad Q'_n=\begin{pmatrix}\alpha'_n & \alpha''_n\\ \beta'_n & \beta''_n\end{pmatrix}.$$

Whence we get

$$B_l(\lambda'_l)=\begin{pmatrix}F_l^{11}+\sigma_l\gamma_{l1}\delta_{l1} & F_l^{12}+\sigma_l\gamma_{l2}\delta_{l2}\\ F_l^{21}+\sigma_l\gamma_{l2}\delta_{l1} & F_l^{22}+\sigma_l\gamma_{l2}\delta_{l2}\end{pmatrix},$$

where

$$F_l^{11}=f_l^{11}(t)\exp\{im_{1l}(x-y)\}, \qquad F_l^{12}=f_l^{12}(t)\exp\{im'_{1l}(x-y)\},$$
$$F_l^{21}=f_l^{21}(t)\exp\{im_{2l}(x-y)\}, \qquad F_l^{22}=f_l^{22}(t)\exp\{im'_{2l}(x-y)\},$$

and

$$\sigma_l=\frac{2}{\lambda_l}-\frac{2\lambda_l}{\lambda_l^2-\lambda_l'^2},$$
$$\delta_{l1}=-\frac{\delta'_{l1}F_l^{11}+\delta'_{l2}F_l^{21}}{\sigma_l(\delta'_{l1}\gamma_{l1}+\delta'_{l2}\gamma_{l2})}, \qquad \delta_{l2}=-\frac{\delta'_{l1}F_l^{12}+\delta'_{l2}F_l^{22}}{\sigma_l(\delta'_{l1}\gamma_{l1}+\delta'_{l2}\gamma_{l2})},$$
$$\gamma_{l1}=-\frac{\gamma'_{l1}F_l^{11}+\gamma'_{l2}F_l^{12}}{\sigma_l(\gamma'_{l1}\delta_{l1}+\gamma'_{l2}\delta_{l2})}, \qquad \gamma_{l2}=-\frac{\gamma'_{l1}F_l^{21}+\gamma'_{l2}F_l^{22}}{\sigma_l(\gamma'_{l1}\delta_{l1}+\gamma'_{l2}\delta_{l2})}.$$

$$B_l^{-1}(\lambda_l)=\begin{bmatrix}\alpha'_l+\varepsilon'_l\gamma'_{l1}\delta'_{l1} & \alpha''_l+\varepsilon'_l\gamma'_{l1}\delta'_{l2}\\ \beta'_l+\varepsilon'_l\gamma'_{l2}\delta'_{l1} & \beta''_l+\varepsilon'_l\gamma'_{l2}\delta'_{l2}\end{bmatrix}.$$

Here$\varepsilon'_l=\dfrac{2\lambda'_l}{\lambda_l^2-\lambda_l'^2}$, along with

$$\delta'_{l1}=-\frac{\alpha'_l\delta_{l1}+\beta'_l\delta_{l2}}{\varepsilon'_l(\delta_{l1}\gamma'_{l1}+\delta_{l2}\gamma'_{l2})}, \qquad \delta'_{l2}=-\frac{\alpha''_l\delta_{l1}+\beta''_l\delta_{l2}}{\varepsilon'_l(\delta_{l1}\gamma'_{l1}+\delta_{l2}\gamma'_{l2})},$$
$$\gamma'_{l1}=-\frac{\gamma_{l1}\alpha'_l+\gamma_{l2}\alpha''_l}{\varepsilon'_l(\gamma_{l1}\delta'_{l1}+\gamma_{l2}\delta'_{l2})}, \qquad \gamma'_{l2}=-\frac{\gamma_{l1}\beta'_l+\gamma_{l2}\beta''_l}{\varepsilon'_l(\gamma_{l1}\delta'_{l1}+\gamma_{l2}\delta'_{l2})}.$$

Finally the matrix Q_l is given as

$$Q_l=\begin{pmatrix} f_l^{11}(t)\exp\{im_{1l}(x-y)\}+\dfrac{2}{\lambda_l}\gamma_{l1}\delta_{l1} & f_l^{12}(t)\exp\{im'_{1l}(x-y)\}+\dfrac{2}{\lambda_l}\gamma_{l1}\delta_{l2}\\ f_l^{21}(t)\exp\{im_{2l}(x-y)\}+\dfrac{2}{\lambda_l}\gamma_{l2}\delta_{l2} & f_l^{22}(t)\exp\{im'_{2l}(x-y)\}+\dfrac{2}{\lambda_l}\gamma_{l2}\delta_{l2}\end{pmatrix}.$$

It is also very convenient to rewrite the matrix elements of Q and P in terms of Lax eigenfunctions Φ. We collect these results below without giving the detailed derivation,

$$Q_l^{11} = \frac{R_l\Phi_{l-1}^{21}(\lambda_l) + R_l'\Phi_{l-1}^{22}(\lambda_l)}{a_l\Phi_{l-1}^{22}(\lambda_l) - b_l\Phi_{l-1}^{21}(\lambda_l)}, \qquad Q_l^{12} = \frac{M_l\Phi_{l-1}^{21}(\lambda_l) + M_l'\Phi_{l-1}^{22}(\lambda_l)}{a_l\Phi_{l-1}^{22}(\lambda_l) - b_l\Phi_{l-1}^{21}(\lambda_l)},$$
$$Q_l^{21} = \frac{L_l\Phi_{l-1}^{21}(\lambda_l) + L_l'\Phi_{l-1}^{22}(\lambda_l)}{a_l\Phi_{l-1}^{22}(\lambda_l) - b_l\Phi_{l-1}^{21}(\lambda_l)}, \qquad Q_l^{22} = \frac{N_l\Phi_{l-1}^{21}(\lambda_l) + N_l'\Phi_{l-1}^{22}(\lambda_l)}{a_l\Phi_{l-1}^{22}(\lambda_l) - b_l\Phi_{l-1}^{21}(\lambda_l)},$$

where

$$R_l = -\frac{\lambda_l'^2 b_l f_l^{11}(t)\exp\{im_{1l}(x-y)\}}{\lambda_l'^2},$$
$$R_l' = \frac{1}{\lambda_l'^2}\left\{\lambda_l^2 a_l f_l^{11}(t)\exp\{im_{1l}(x-y)\} + \left(\lambda_l^2 - \lambda_l'^2\right) b_l f_l^{12}(t)\exp\{im_{1l}'(x-y)\}\right\},$$
$$M_l = \frac{1}{\lambda_l'^2}\left\{-\left(\lambda_l^2 - \lambda_l'^2\right) a_l f_l^{11}(t)\exp\{im_{1l}(x-y)\} - \lambda_l^2 b_l f_l^{12}(t)\exp\{im_{1l}'(x-y)\}\right\},$$
$$M_l' = \frac{\lambda_l'^2 a_l f_l^{12}(t)\exp\{im_{1l}'(x-y)\}}{\lambda_l'^2}, \qquad L_l = -\frac{\lambda_l'^2 b_l f_l^{21}(t)\exp\{im_{2l}(x-y)\}}{\lambda_l'^2},$$
$$L_l' = \frac{1}{\lambda_l'^2}\left\{\lambda_l^2 a_l f_l^{21}(t)\exp\{im_{2l}(x-y)\} + \left(\lambda_l^2 - \lambda_l'^2\right) b_l f_l^{22}(t)\exp\{im_{2l}'(x-y)\}\right\},$$
$$N_l = \frac{1}{\lambda_l'^2}\left\{-\left(\lambda_l^2 - \lambda_l'^2\right) a_l f_l^{21}(t)\exp\{im_{2l}(x-y)\} - \lambda_l^2 b_l f_l^{22}(t)\exp\{im_{2l}'(x-y)\}\right\},$$
$$N_l' = \frac{\lambda_l'^2 a_l f_l^{22}(t)\exp\{im_{2l}(x-y)\}}{\lambda_l'^2}.$$

The elements of the P_l matrix are:

$$P_l^{11} = -\frac{F_l}{\sigma_l}\frac{\Phi_{l-1}^{22}(\lambda_l)}{a_l\Phi_{l-1}^{22}(\lambda_l) - b_l\Phi_{l-1}^{21}(\lambda_l)}, \qquad P_l^{12} = \frac{F_l}{\sigma_l}\frac{\Phi_{l-1}^{21}(\lambda_l)}{a_l\Phi_{l-1}^{22}(\lambda_l) - b_l\Phi_{l-1}^{21}(\lambda_l)},$$
$$P_l^{21} = -\frac{F_l'}{\sigma_l}\frac{\Phi_{l-1}^{22}(\lambda_l)}{a_l\Phi_{l-1}^{22}(\lambda_l) - b_l\Phi_{l-1}^{21}(\lambda_l)}, \qquad P_l^{22} = \frac{F_l'}{\sigma_l}\frac{\Phi_{l-1}^{21}(\lambda_l)}{a_l\Phi_{l-1}^{22}(\lambda_l) - b_l\Phi_{l-1}^{21}(\lambda_l)},$$

with

$$F_l = a_l f_l^{11}(t)\exp\{im_{1l}(x-y)\} + b_l f_l^{12}(t)\exp\{im_{1l}'(x-y)\},$$
$$F_l' = a_l f_l^{21}(t)\exp\{im_{2l}(x-y)\} + b_l f_l^{22}(t)\exp\{im_{2l}'(x-y)\},$$
$$\sigma_l = \frac{2\lambda_l'^2}{\lambda_l\left(\lambda_l'^2 - \lambda_l^2\right)}.$$

Construction of the nonlinear fields: Once the form of the matrices P_l and Q_l are determined we can construct the matrix B_l, so that the Lax eigenfunction $\Phi_l(\lambda)$ for the next stage can be determined from that of the previous one via,

$$\Phi_l(\lambda) = B_l(\lambda)\Phi_{l-1}(\lambda).$$

These expressions are very complicated, so we just quote one of them to display their structure. For example,

$$
\begin{aligned}
&\Phi_l^{11}(\lambda) = N_l/D_l,\\
&D_l = a_l\Phi_{l-1}^{21}(\lambda_l) - b_l\Phi_{l-1}^{22}(\lambda_l),\\
&N_l = R_l\Phi_{l-1}^{21}(\lambda_l)\Phi_{l-1}^{11}(\lambda) + \{R_l' - f_l(\lambda)F_l\}\Phi_{l-1}^{22}(\lambda_l)\Phi_{l-1}^{11}(\lambda)\\
&\qquad +\{M_l + f_l(\lambda)F_l\}\Phi_{l-1}^{21}(\lambda_l)\Phi_{l-1}^{21}(\lambda) + M_l'\Phi_{l-1}^{22}(\lambda_l)\Phi_{l-1}^{21}(\lambda),
\end{aligned}
\tag{8}
$$

with similar expression for other elements Φ^{12}, Φ^{21} and Φ^{22}.

Now, for the determination of nonlinear fields, consider

$$U_n'Q_n = M_nQ_n + Q_nU_{n-1}',$$

where

$$M = \begin{pmatrix} \partial_x & 0 \\ 0 & \partial_y \end{pmatrix}, \qquad U_n' = \begin{pmatrix} -i\alpha & -\dfrac{q_n}{2} \\ -\dfrac{r_n}{2} & i\beta \end{pmatrix}, \qquad U_{n-1}' = \begin{pmatrix} -i\alpha & -\dfrac{q_{n-1}}{2} \\ -\dfrac{r_{n-1}}{2} & i\beta \end{pmatrix}.$$

In the expression for U_n' and U_{n-1}' we take $\alpha = \beta = 0$, which at once yields

$$q_n = -\frac{2Q_{nx}^{12}}{Q_n^{22}} + \frac{Q_n^{11}}{Q_n^{22}}q_{n-1}. \tag{9}$$

This is nothing but a simple recursion relation. Similarly,

$$r_n = -\frac{2Q_{ny}^{21}}{Q_n^{11}} + \frac{Q_n^{22}}{Q_n^{11}}r_{n-1}. \tag{10}$$

Explicitly are can write,

$$
\begin{aligned}
n=1,\quad & q_1 = -\frac{2Q_{1x}^{12}}{Q_1^{22}} + \frac{Q_1^{11}}{Q_1^{22}}q_0, \quad r_1 = -\frac{2Q_{1y}^{21}}{Q_1^{11}} + \frac{Q_1^{22}}{Q_1^{11}}r_0,\\
n=2,\quad & q_2 = -\frac{2Q_{2x}^{12}}{Q_2^{22}} - \frac{2Q_2^{11}Q_{1x}^{12}}{Q_2^{22}Q_1^{22}} + \frac{Q_2^{11}Q_1^{11}}{Q_2^{22}Q_1^{22}}q_0,\\
& r_2 = -\frac{2Q_{2y}^{21}}{Q_2^{11}} - \frac{2Q_2^{22}Q_{1y}^{21}}{Q_2^{11}Q_1^{11}} + \frac{Q_2^{22}Q_1^{22}}{Q_2^{11}Q_1^{11}}r_0.
\end{aligned}
$$

So far we have considered $f_l^{11}(t)$, $f_l^{12}(t)$, $f_l^{21}(t)$, $f_l^{22}(t)$ to be functions of time or constants; m_{1l}, m_{1l}', m_{2l}, m_{2l}' to be arbitrary constants; a_l, b_l to be arbitrary constants for all l. Now assume that $f_l^{12}(t) = f_l^{21}(t) = 0$ and $b_l = 0$ for all l values. So that $R_l = M_l' = L_l = L_l' = N_l = F_l' = 0$ for all l. In this case the form of $B_l(\lambda)$ turns out to be:

$$B_l(\lambda) = \begin{pmatrix} \frac{\lambda_l^2}{\lambda_l'^2}\left(1 - \frac{\lambda_l'^2-\lambda_l^2}{\lambda^2-\lambda_l^2}\right) f_l^{11}(t)e^{im_{1l}(x-y)} & \left(1 + \frac{\lambda_l^2}{\lambda^2-\lambda_l^2}\right)\left(1 - \frac{\lambda_l^2}{\lambda_l'^2}\right) f_l^{11}(t)e^{im_{1l}(x-y)}\theta_{l-1} \\ 0 & f_l^{22}(t)e^{im_{2l}(x-y)} \end{pmatrix},$$

where $\theta_{l-1} = \dfrac{\Phi_{l-1}^{21}(\lambda_l)}{\Phi_{l-1}^{22}(\lambda_l)}$.

To write the formulae for the n-soliton solution in a compact form, we note that

$$Q_n^{11}Q_{n-1}^{11}Q_{n-2}^{11}\dots Q_R^{11} = A_{n-R+1}^n F_{n-R+1}^n(t)\exp\left\{ i\sum_{j=1}^{n-R+1} m_{1,n-j+1}(x-y)\right\},$$

where A_{n-R+1}^n, $F_{n-R+1}^n(t)$ stands for

$$A_{n-R+1} = \prod_{l=1}^{n-R+1}\frac{\lambda_{n-l+1}^2}{\lambda_{n-l+1}'^2}, \qquad F_{n-R+1}^n(t) = \prod_{l=1}^{n-R+1} f_{n-l+1}^{11}(t).$$

Similar expressions can be written for Q_n^{ij} and its products. Using these, we at once obtain:

$$q_n = \frac{-2\overline{m}_n Q_n^{12}}{G_1^n(t)\exp\{im_{2,n}'(x-y)\}} + \frac{A_n^n F_n^n(t)\exp\left\{i\sum\limits_{j=1}^{n} m_{1,n-j+1}(x-y)\right\}}{G_n^n(t)\exp\left\{i\sum\limits_{j=1}^{n} m_{2,n-j+1}(x-y)\right\}}$$
$$-2\sum_{K=1}^{n-1}\frac{A_K^n F_K^n(t)\exp\left\{i\sum\limits_{j=1}^{K} m_{1(n-j+1)}(x-y)\right\}}{G_{K+1}^n(t)\exp\left\{i\sum\limits_{j=1}^{K+1} m_{2(n-j+1)}(x-y)\right\}}\overline{m}_{n-K}Q_{n-K}^{12},$$

where we have set

$$\overline{m}_0 = a(m_0-n_0), \quad G_K^n(t) = \prod_{l=1}^{K} f_{n-l+1}^{22}(t), \quad \overline{m}_n' = -im_{1n} + \overline{m}_0',$$

$$\overline{m}_0' = b(1/m_0 - 1/n_0), \quad F_k^n = \prod_{l=1}^{k} f_{n-l+1}^{11}(t), \quad T_n = \frac{m_0}{n_0}\left(1-\frac{\lambda_n^2}{\lambda_n'^2}\right),$$

$$Q_n^{12} = T_n f_n^{11}(t)\exp(im_{1n}x)\exp(\overline{m}_{0x} + \overline{m}_n' y + \delta t).$$

Here δ is a complex constant.

Discussions: In the above analysis we have demonstrated how the pole type ansatz of Zakharov *et al* [3] can be used to generate a compact formulae for the N-fold Bäcklund transformation in the case of the Davey-Stewartson equation. The study yields two main results exhibited in equations (8) and (9). While the equation gives a recursive procedure for the determination of the Lax eigenfunction (Φ_l corresponds to the l-soliton state) equation (9) and (10) gives the corresponding recursion relation for the nonlinear fields. We have actually checked that for $n = 1$ one obtains the one soliton solution well known in the literature.

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