Communicating with Lunar Orbiter by Relay Satellites at Earth-Moon Lagrange Points

Hui Li
School of Aeronautics and Astronautics, Zhejiang University, Hangzhou, 310027, China
zzlihui@zju.edu.cn

Abstract - In order to continuous communication with lunar orbiter and even far side station at the back of Moon, Lagrange points L_1, L_2, $L_4$and L_5 is considered to be candidates for relay satellites or orbiters in the Earth-Moon restricted three body system. Positions of Lagrange points in solar system including Earth-Moon system were calculated and missions before and future around these points, especially Sun-Earth L_1 and L_2 were listed. Lagrange points satellites, orbiters and local networks on the ground of a celestial body will constitute a planetary networks connected by a interplanetary backbone in the whole architecture of Inter Pla Netary Internet.

Index Terms - Deep Space Communication, Three Body System, Lagrange Points, Relay Satellites.

1. Introduction

Some of the new deep space missions do not have direct link between Earth and final destination, therefore data must be relayed between a series of spacecraft each providing a store & forward capability until the final destination is reached. For distance increasing in deep space exploration and Earth rotation and other planets' motions, the communication link between the spacecraft and the ground mission control center may not be permanent, even via several data relay satellites and several ground antenna.

In 1772, French mathematician Joseph L. Lagrange analyzed restricted three-body problem in space during the gravity research: how a third, small body would orbit around two orbiting large ones. His solution was astronomically confirmed in 1906 with the discovery of the Trojan asteroids orbiting at the Sun-Jupiter L_4 and L_5 points. The Voyager probes found tiny moonlets at the Saturn-Dione L_4 point and at the Saturn-Tethys L_4 and L_5 points[1,2].

In his conclusions, there are 5 balancing points in Earth-Moon system and also in Sun-Earth system, named Lagrange points as define in table 1 and shown in figure 1. At these points, an entity is in a balancing state due to gravitation and tracking movement. Of the five Lagrange points, three are unstable and two are stable. The unstable Lagrange points labeled L_1, L_2 and L_3 lie along the line connecting the two large masses: Sun and Earth or Earth and Moon. The stable Lagrange points, labeled L_4 and L_5, form the apex of two equilateral triangles that have the large masses at their vertices. They are analogous to geosynchronous orbits in that they allow an object to be in a "fixed" position in space rather than an orbit in which its relative position changes continuously.

In Sun-Earth system, from 1978, when the first Lagrange point-1 satellite ISEE-3 was launched successful, these ideal balancing points are high concerned in deep space missions. Now the ESA/NASA's SOHO solar watchdog is positioned there. And Sun-Earth L_2 is supposed to be home for ESA missions such as Herschel, Planck and Darwin, etc [2].

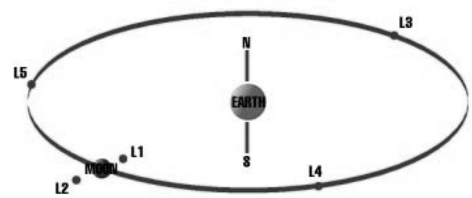


Fig.1. Lagrange points in Earth-Moon three body system

2. Lagrange Points in Earth-Moon Three Body System

In Earth-Moon system, data and images can be transmitted from lunar orbit to Earth timely, without store and forward on board save a little longer delay. The lander and rover are able to explore back side of Moon with adequate energy. Due to the direct link existing between the near side lunar station and Earth, Lagrange point L_1 is not considered in my study and also the point L_3 on the back of Earth. The distance from L_1 to the centroid of Moon is about 5.776×10^4 km, and 6.5348×10^4 km for L_2 and centroid of Moon. An object at L_1, L_2, or L_3 is meta-stable, like a ball sitting on top of a hill.

A little push or bump starts its moving away. A spacecraft at one of these points has to use frequent, small rocket firings or other means to remain in the area[3,4].

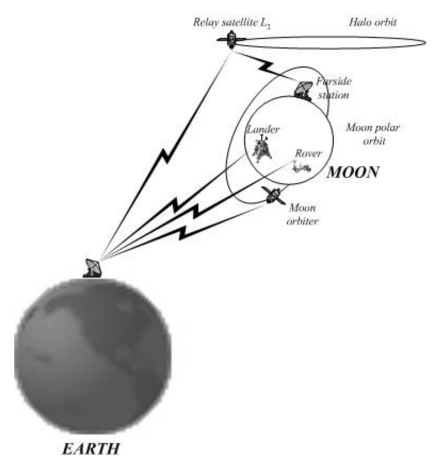
Researches on gravity field of Earth-Moon system improve that an "aisle" naming zone of metastability of weak stability is along the line of the Earth and Moon, including three Lagrange points L_1, L_2 and L_3. A spacecraft positioning in this aisle would be neither disengaged from the system nor captured by Earth or Moon. A tiny push may force the spacecraft orbit around a metastable Lagrange point, which is called halo orbit, [5] as shown in figure 2-(a).

An object at L_4 or L_5 is truly stable, like a ball in a bowl: when gently pushed away, it orbits the Lagrange point without drifting farther and farther, and without the need of frequent rocket firings.

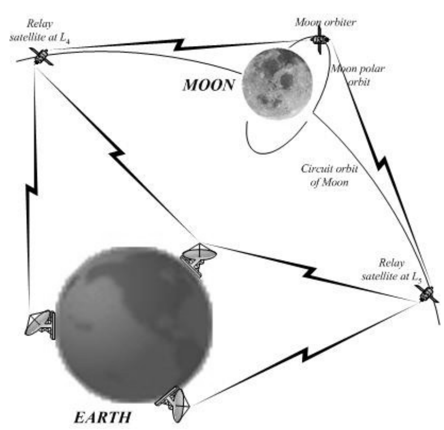
In Earth-Moon system, utilization of Lagrange points is being regarded with the re-entry of Moon. Continuous communication is a task in lunar exploration and beyond. When lunar orbiter rotates around the Moon in polar orbit, almost in half of the orbit-period, the orbiter could not communicate with Earth in the shadow of Moon. Metastable point L_2 and stable points L_4 and L_5 can be used as location of relay satellite for lunar orbiters as shown in figure 2 below.

Table.1. Definition & Position of Lagrange Points and Their Utilization

Name	Definition & position	Function, projects and plans	
		Sun-Earth system	Earth-Moon system
Lagrange point L_1	on the line defined by the two large masses m_1 and m_2, and between them	observations of the Sun: Solar and Heliospheric Observatory (SOHO), Advanced Composition Explorer (ACE)	half-way manned space station intended to help transport cargo and personnel to the Moon and back
Lagrange point L_2	on the line defined by the two large masses, beyond the smaller of the two	space-based observatories: Wilkinson Microwave Anisotropy Probe, future Herschel Space Observatory , Gaia probe, and James Webb Space Telescope	communications satellite covering the Moon's far side
Lagrange point L_3	on the line defined by the two large masses, beyond the larger of the two	Not yet	Not yet
Lagrange point L_4 &L_5	at the third point of an equilateral triangle whose base is the line between the two masses, such that the point is ahead of (L_4), or behind (L_5), the smaller mass in its orbit around the larger mass	Space habitats of future colonization	communications and relay satellites



(a) Relay satellite at L_2



(b) Relay satellite at L_4 and L_5

Fig.2. Relay satellite utilizing Lagrange points in Earth-Moon system

3. Calculation of Lagrange Points

Suppose mass of two big celestial bodies P1 and P2 are m1 and m2 in a circular system. The movement of a small celestial body P in the system constituted by P1 and P2 is a circular restricted three-body problem (CR3BP). In the centroidal inertial coordinates system O-XYZ, the initial point is located on the center of mass—barycenter, and XY plane of coordinates is the relative movement plane of two bodies P1 and P2. At the initial time t=t0, P1 and P2 are on the axis of coordinates OX as shown in figure 3. In this coordinates, vectors of coordinates of P, P1 and P2 are , and , and

$$\vec{R}_1 = \vec{R} - \vec{R}'_1, \vec{R}_2 = \vec{R} - \vec{R}'_2 \qquad (1)$$

Table.2. Lagrange points L_1, L_2, and L_3 in solar system

System	μ	x_1	x_2	x_3
Sun-Mercury	0.00000017	-0.99618898	-1.00382039	1.00000007
Sun-Venus	0.00000245	-0.99067832	-1.00937503	1.00000102
Sun-Earth	0.00000304	-0.98999093	-1.01007019	1.00000126
Sun-Mars	0.00000032	-0.99524867	-1.00476578	1.00000013
Sun-Jupiter	0.00095388	-0.93236559	-1.06883052	1.00039745
Sun-Saturn	0.00028550	-0.95476098	-1.04605727	1.00011896
Sun-Uranus	0.00004373	-0.97572949	-1.02458081	1.00001822
Sun-Neptune	0.00005177	-0.97433032	-1.02601130	1.00002157
Sun-Pluto	0.00000278	-0.99028227	-1.00977551	1.00000116
Earth-Moon	0.01215057	-0.83691521	-1.15568210	1.00506264

The two big celestial bodies are both in circular orbit around barycenter O, and

$$\vec{R}_1' = \begin{bmatrix} \mu\cos t \\ \mu\sin t \\ 0 \end{bmatrix}, \vec{R}_2' = \begin{bmatrix} -(1-\mu)\sin t \\ -(1-\mu)\cos t \\ 0 \end{bmatrix} \tag{2}$$

$$\theta(t) = \omega(t - t_0) \tag{3}$$

in which $\overline{OP_1} = |R_1'| = \mu$, $\overline{OP_2} = |R_2'| = 1-\mu$, and

$$\mu = \frac{m_2}{m_1 + m_2}, 1-\mu = \frac{m_1}{m_1 + m_2} \tag{4}$$

So the equation of small body's movement in O-XYZ is:

$$\ddot{\vec{R}} = \left(\frac{\partial U}{\partial \vec{R}}\right)^T = -(1-\mu)\frac{\vec{R}_1}{{R_1}^3} - \mu\frac{\vec{R}_2}{{R_2}^3} \tag{5}$$

in which

$$U = U(R_1, R_2) = \frac{1-\mu}{R_1} + \frac{\mu}{R_2} \tag{6}$$

in which

$$\begin{cases} R_1 = |\vec{R} - \vec{R}_1'| = [(X - \mu\cos t)^2 + (Y - \mu\sin t)^2 + Z^2]^{1/2} \\ R_2 = |\vec{R} - \vec{R}_2'| = [(X + (1-\mu)\sin t)^2 + (Y + (1-\mu)\cos t)^2 + Z^2]^{1/2} \end{cases} \tag{7}$$

in which, vector of the small body in O-XYZ system is [X,Y, Z]. And in the centroidal revolution coordinates system O-xyz, vectors of three celestial bodies are $\vec{r}$, $\vec{r}_1'$ and $\vec{r}_2'$, and

$$\vec{r}_1 = \vec{r} - \vec{r}_1', \vec{r}_2 = \vec{r} - \vec{r}_2' \tag{8}$$

in which

$$\vec{r}_1' = \begin{bmatrix} \mu \\ 0 \\ 0 \end{bmatrix}, \vec{r}_2' = \begin{bmatrix} -(1-\mu) \\ 0 \\ 0 \end{bmatrix} \tag{9}$$

So

$$\begin{cases} r_1 = [(x-\mu)^2 + y^2 + z^2]^{1/2} = R_1 \\ r_2 = [(x+1-\mu)^2 + y^2 + z^2]^{1/2} = R_2 \end{cases} \tag{10}$$

The relationship of $\vec{r}$ and $\vec{R}$ are

$$\vec{r} = R_T(t)\vec{R} = \begin{bmatrix} X\cos t + Y\sin t \\ -X\sin t + Y\cos t \\ Z \end{bmatrix} \tag{11}$$

$$\vec{R} = R_T(-t)\vec{r} = \begin{bmatrix} x\cos t - y\sin t \\ x\sin t + y\cos t \\ z \end{bmatrix} \tag{12}$$

in which $R_T(t)$ is the transforming matrix:

$$R_T(t) = \begin{bmatrix} \cos t & \sin t & 0 \\ -\sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{13}$$

And $R_T^{-1}(t) = (R_T(t))^T = R_T(-t)$, we get

$$\dot{\vec{R}} = \dot{R}_T(-t)\vec{r} + R_T(-t)\dot{\vec{r}} \tag{14}$$

$$\ddot{\vec{R}} = \ddot{R}_T(-t)\vec{r} + 2\dot{R}_T(-t)\dot{\vec{r}} + R_T(-t)\ddot{\vec{r}} \tag{15}$$

in which

$$R_T(-t) = \begin{bmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix}, \dot{R}_T(-t) = \begin{bmatrix} -\sin t & -\cos t & 0 \\ \cos t & -\sin t & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{16}$$
$$\ddot{R}_T(-t) = \begin{bmatrix} -\cos t & \sin t & 0 \\ -\sin t & -\cos t & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Utilizing equations above we can obtain the equation of small body's movement in O-xyz is

$$\ddot{\vec{r}} + 2\begin{bmatrix} -\dot{y} \\ \dot{x} \\ 0 \end{bmatrix} = \left(\frac{\partial \Omega}{\partial \vec{r}}\right)^T \tag{17}$$

in which

$$\Omega = (x^2 + y^2)/2 + U(r_1, r_2) \tag{18}$$

in which

$$U(r_1, r_2) = \frac{1-\mu}{r_1} + \frac{\mu}{r_2} \tag{19}$$

From equation (17) we get

$$\dot{x}\ddot{x} + \dot{y}\ddot{y} + \dot{z}\ddot{z} = \frac{\partial \Omega}{\partial x}\dot{x} + \frac{\partial \Omega}{\partial y}\dot{y} + \frac{\partial \Omega}{\partial z}\dot{z} \tag{20}$$

which can also be written as

$$\begin{cases} \frac{1}{2}\frac{d}{dt}(v^2) = \frac{d\Omega}{dt} \\ v^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2 \end{cases} \tag{21}$$

In CR3BP the only one Jacobi integration in O-xyz is

$$2\Omega - v^2 = C \tag{22}$$

in which C is a Jacobi constant. And the Jacobi integration in O-XYZ is

$$\begin{cases} 2U - [V^2 + 2(\dot{X}Y - X\dot{Y})] = C - \mu(1-\mu) \\ U = \frac{1-\mu}{R_1} + \frac{\mu}{R_2} \end{cases} \tag{23}$$

The equilibrium solution of equation (17) should fulfill the following restrictive qualification:

$$x(t) \equiv x_0, y(t) \equiv y_0, z(t) \equiv z_0 \tag{24}$$

x_0, y_0, z_0 are initial state, and correspondingly

$$\begin{aligned} \dot{x} = 0, \dot{y} = 0, \dot{z} = 0 \\ \ddot{x} = 0, \ddot{y} = 0, \ddot{z} = 0 \end{aligned} \tag{25}$$

So the equilibrium points in space should fulfill

$$\Omega_x = 0, \Omega_y = 0, \Omega_z = 0 \tag{26}$$

which is also written as

$$\begin{cases} x - \dfrac{(1-\mu)(x-\mu)}{r_1^3} - \dfrac{\mu(x+1-\mu)}{r_2^3} = 0 \\ y(1 - \dfrac{1-\mu}{r_1^3} - \dfrac{\mu}{r_2^3}) = 0 \\ z(\dfrac{1-\mu}{r_1^3} + \dfrac{\mu}{r_2^3}) = 0 \end{cases} \tag{27}$$

Because $\frac{1-\mu}{r_1^3} + \frac{\mu}{r_2^3} \neq 0$, so $z = z_0 = 0$, which means the equilibrium points are all in xy plane. From equations (26), we get two situations:

$$\begin{aligned} & y = 0 \\ & x - \frac{1-\mu}{(x-\mu)^2} - \frac{\mu}{(x+1-\mu)^2} = 0 \end{aligned} \tag{28}$$

and

$$\begin{aligned} & y \neq 0, \quad 1 - \frac{1-\mu}{r_1^3} - \frac{\mu}{r_2^3} = 0 \\ & x - \frac{(1-\mu)(x-\mu)}{r_1^3} - \frac{\mu(x+1-\mu)}{r_2^3} = 0 \end{aligned} \tag{29}$$

From equation (28), we obtain three equilibrium points alone the Ox axis as shown in figure 4, which are $x_1(\mu)=-(1-\mu)+\xi^{(1)}$, $x_2(\mu)=-(1-\mu)-\xi^{(2)}$ and $x_3(\mu)=\mu+\xi^{(3)}$, in which

$$\xi^{(1)} = (\frac{\mu}{3})^{1/3}[1 - \frac{1}{3}(\frac{\mu}{3})^{1/3} - \frac{1}{9}(\frac{\mu}{3})^{2/3} - \cdots] \tag{30}$$

$$\xi^{(2)} = (\frac{\mu}{3})^{1/3}[1 + \frac{1}{3}(\frac{\mu}{3})^{1/3} - \frac{1}{9}(\frac{\mu}{3})^{2/3} + \cdots] \tag{31}$$

$$\begin{cases} \xi^{(3)} = 1 - \nu[1 + \frac{23}{84}\nu^2 + \frac{23}{84}\nu^3 + \frac{761}{2352}\nu^4 + \frac{3163}{7056}\nu^5 + \frac{30703}{49392}\nu^6] + O(\nu^8) \\ \nu = \frac{7}{12}\mu \end{cases} \tag{32}$$

And from equation (29), we obtain two equilibrium points at the vertexes of equilateral triangles.

$$\begin{cases} x_4 = x_5 = -1/2 + \mu \\ y_4 = +\sqrt{3}/2, y_5 = -\sqrt{3}/2 \end{cases} \tag{33}$$

Then the three metastable equilibrium points L_1, L_2, and L_3 in solar system are listed in table 2, and Jacobi constant in equation (22) is in table 3. More careful consideration should be in a state of elliptical restricted three bodies rather than the circular one.

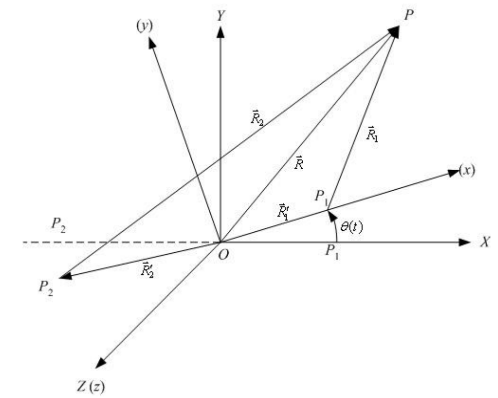


Fig.3. Centroidal inertial coordinates system O-XYZ and centroidal revolution coordinates system O-xyz

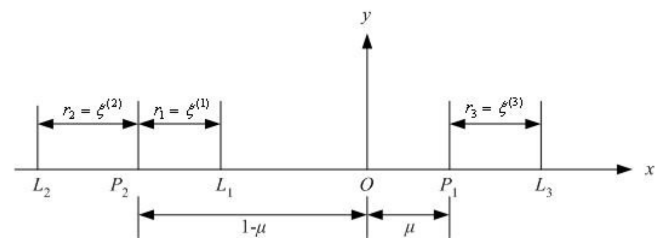


Fig.4. Relative position of Lagrange points L_1, L_2, L_3 and two celestial bodies P_1 and P_2

Table.3. Jacobi constant of Lagrange points L_1, L_2, and L_3

System	C_1	C_2	C_3
Sun-Mercury	3.00013043	3.00013065	3.00000033
Sun-Venus	3.00077756	3.00078083	3.00000490
Sun-Earth	3.00089604	3.00090009	3.00000607
Sun-Mars	3.00020261	3.00020304	3.00000065
Sun-Jupiter	3.03844172	3.03971380	3.00190682
Sun-Saturn	3.01771636	3.01809709	3.00057092
Sun-Uranus	3.00521010	3.00536840	3.00008745
Sun-Neptune	3.00582087	3.00588991	3.00010354
Sun-Pluto	3.00084481	3.00084851	3.00000556
Earth-Moon	3.18416325	3.20034388	3.02415006

4. Missions and Projects Around Sun-Earth Lagrange Points

Agency like ESA has some space missions and projects under consideration and studying around Lagrange points especially Sun-Earth L_2 point as listed in table 4[6]. Formation flying spacecrafts locating Lagrange point is a big challenge not only for orbit-control[7,8] and formation-maintenance, but also for cooperative interferometry and communication with earth [6].

NASA's missions are mainly concerned with Sun-Earth Lagrange points 1 and 2. Their missions include: International Cometary Explorer (1982)[9], SOHO (1995)[10], Advanced Composition Explorer (1997)[11,12], Genesis (2001)[13] and Wilkinson Microwave Anisotropy Probe (2001)[14] with the last one on L_2 and other four on L_1.

5. Conclusions

Utilization of Lagrange points for continuous communication with lunar orbiter and far side stations is a bold and challenging image in Moon exploration and research. Missions before around Sun-Earth L_1 and L_2 provide human being a wider field of view of exploring universe, and Earth-Moon L_1, L_2, $L_4$and L_5 will play an important role in future projects concerning with Moon.

Satellites around a celestial body, its local network and the Lagrange points in a certain 3–body system are to construct a planetary network which is an ingredient in a supposed InterPlaNetary Internet. Moreover Lagrange points will play more importance role in future deep space exploration for continuous communication and navigation. These points will home future formation flying spacecrafts as Darwin project supposed to be and even served as habitats for space colonization.

Table.4. ESA future mission at Sun-Earth Lagrange point 2

Missions	Date	Missions and goal	Instrumentation onboard
Herschel	2007	exploring formation of stars and galaxies	3.5-metre diameter infrared telescope and three scientific instruments: Photodetector Array Camera and Spectrometer (PACS); Spectral and Photometric Imaging REceiver (SPIRE); Heterodyne Instrument for the Far Infrared (HIFI)
Planck	2007	study the cosmic microwave background radiation and the fabric of the Universe's birth and evolution.	1.5-metre telescope; two highly sensitive detectors called the Low Frequency Instrument and the High Frequency Instrument
James Webb Space Telescope	2010	study the very distant Universe, looking for the first stars and galaxies that ever emerged	Visible/Near Infrared Camera; Near-Infrared Multi-Object Dispersive Spectrograph; Mid-Infrared Camera-Spectrograph
Gaia	2011	make the largest, most precise map of our Galaxy by surveying an unprecedented number of stars - more than a thousand million	three optical telescopes, etc.
Eddington	–	mapping stellar evolution, determine the size and precise chemical composition of the stars, and search for other Earth-sized worlds that harbour extraterrestrial life	wide-field, high-accuracy optical photometer, etc.
Darwin	–	Finding Earth-like planets, survey 1000 of the closest stars, looking for small, rocky planets	four (or possibly five) separate spacecraft. Three of the spacecraft will carry 3-4 metre 'space telescopes', or more accurately light collectors, based on the Herschel design. These will redirect light to the central hub spacecraft.

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