On Chase-Like Bound-Distance Decoding Algorithms*

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Abstract - For the decoding of a binary linear block code of Hamming distance of d over AWGN channels, a soft-decision decoder is said to be bounded-distance (BD) decoding if its squared error-correction radius is equal to d. A Chase-like algorithm outputs the best (most likely) codeword in a list of candidates generated by a conventional algebraic binary decoder whose input vectors are determined by the reliability order of the hard-decisions. Let $\Delta(d)$ denote the smallest size of input vector sets of Chase-like algorithms which achieve BD decoding. When d approaches to infinity, the best known upper bound on $\Delta(d)$ is $\Delta(d) \le (\lambda + o(1))d^{1/2}$, where $\lambda \approx 2.414$. In this paper, we show $\Delta(d) \le (\psi + o(1))d^{1/2}$, where $\psi \approx 2.218$.

Index Terms - Chase-like algorithm, algebraic binary decoder, bounded-distance decoding

I. Introduction

In this paper, we consider the decoding of binary linear block codes over additive white Gaussian noise (AWGN) channels. As the algorithms proposed by Chase in [1], a Chase-like algorithm outputs the best (most likely) codeword in a list of candidates generated by a conventional algebraic binary decoder whose input vectors are determined by the reliability order of the hard-decisions. A decoding algorithm is called a bounded-distance (BD) decoding if its error-correction radius reaches the maximum. It is well-known that any BD decoding is asymptotically optimal. When applied to a binary linear block code of length n and minimal Hamming distance d, the original Chase algorithms [1] achieve BD decoding while the numbers of input vectors are $C_N^{d/2}$, $2^{\lfloor d/2 \rfloor}$ and $\lfloor d/2 \rfloor + 1$, respectively. Since the decoding complexity of a Chase-like algorithm is by and large proportional to the number of the input vectors, it is of interest to design Chase-like BD decoding algorithms with as least input vectors as possible. Let $\Delta(d)$ denote the smallest size of input vector sets of Chase-like BD decoding algorithms. In 2003, $\Delta(d) \le \lceil (d+2)/4 \rceil$ and $\Delta(d) \le \lceil d/6 \rceil + 1$ were proved in [2] and [3], respectively. When the minimal Hamming distance d approaches to infinity, $\Delta(d) \le \mathrm{O}(d^{2/3})$, $\Delta(d) \le \mathrm{O}(d^{1/2+\varepsilon})$, $\Delta(d) \le \mathrm{O}(\sqrt{d \ln d})$ were shown in [4], [5], [6], respectively. The best known asymptotic upper bound on $\Delta(d)$ is shown in [7]: $\Delta(d) \le (\lambda + o(1))d^{1/2}$, where $\lambda \approx 2.414$. In this paper, we will improve this upper bound further.

II. Preliminaries

Let V^N denote the set of binary vectors of length N. For $\vec{u} = (u_1, u_2, \cdots u_N) \in V^N$, let $s(\vec{u}) \triangleq ((-1)^{u_1}, (-1)^{u_2}, \cdots, (-1)^{u_N})$ be the bipolar vector corresponding to $\vec{u}$. For two real vectors $\vec{x}$, $\vec{y} \in R^N$, their squared Euclidean distance is defined as $d_E(\vec{x}, \vec{y}) \triangleq (x_1 - y_1)^2 + \cdots + (x_N - y_N)^2$, where x_i and y_i are the i-th entry of $\vec{x}$ and $\vec{y}$, respectively. Suppose that a linear binary block code $C \subset V^N$ of Hamming distance d is used for error control over the additive white Gaussian noise (AWGN) channel with BPSK signaling. When the transmitted codeword is $\vec{c} = (c_1, c_2, \cdots c_N) \in C$, the conditional density function of the received vector $\vec{r} \in R^N$ is

$$p(\vec{r} \mid \vec{c}) = \frac{1}{(\pi N_0)^{N/2}} e^{-d_E(\vec{r}, s(\vec{c}))/N_0}.$$

For given received vector $\vec{r}$, a vector $\vec{u} \in V^N$ is said to be better (or more likely) than another vector $\vec{v} \in V^N$ if $d_E(\vec{r}, s(\vec{u})) < d_E(\vec{r}, s(\vec{v}))$. Hence, a maximum-likelihood (ML) decoder always outputs the best codeword.

Suppose that $\vec{r} = (r_1, r_2, \cdots r_N) \in R^N$ is a received vector. Let $\vec{z} = (z_1, z_2, \cdots z_N) \in V^N$ denote the hard-decision vector defined by: $z_i = 0$ for $r_i > 0$ and $z_i = 1$ for $r_i \le 0$. For simplicity, without loss of generality, we assume further that the entries have been permuted according to the reliability order of the hard-decisions such that

$$|r_1| \le |r_2| \le \cdots |r_N|.$$

Like in [4] to [9], we assume further that the Hamming distance d of the code is odd for simplicity. Let $\tau \triangleq (d-1)/2$. Assume that a conventional bounded-distance- τ algebraic binary decoder, which outputs a codeword within Hamming distance τ of the sum of the hard-decision vector $\vec{z}$ and the input vector, if any, is available. For any set $U \subset V^N$, let $C(U)$ denote the Chase-like algorithm which outputs the best codewords in a list of candidates generated by the algebraic decoder with U as the input vector set. For a decoding algorithm A of a binary block code, its squared error-correction radius (SECR) is defined as the largest number, denoted $\rho(A)$, such that A decodes correctly whenever the received vector is within squared Euclidean distance $\rho(A)$ of the bipolar vector corresponding to the transmitted codeword.

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Then, a decoding algorithm A achieves BD decoding if and only if $\rho(A)=d$.

For $U\subset V^N$ and positive integer l, let $\Omega_l(U)$ denote the set of vectors in V^N which are beyond Hamming distance l of each vector in U. $\Omega_l(U)$ is called the unchecked region of the Chase-like algorithm $C(U)$. The SECR of $C(U)$ can be computed [8] by

$$\rho(C(U))=\min\{d,\min_{v\in\Omega_\tau(U)}\sigma(\vec{v})\} \qquad (1)$$

where $\sigma(\vec{v})$ is the minimal squared Euclidean distance (MSED) between the vector $s(\vec{v})$ and the vectors in

$$H_N \triangleq \{(x_1,\cdots,x_N)\in R^N : 0\le x_1\le\cdots\le x_N\le 1\}.$$

Since the size of the unchecked region $\Omega_\tau(U)$ is very large, it is not easy to estimate the minimum of $\sigma(\vec{v})$ over $\Omega_\tau(U)$ for a general input set U.

For $0\le j\le j'\le m$ and a vector $\vec{u}=(u_1,u_2,\cdots u_m)$, let $\gamma_{j,j'}(\vec{u})$ denote the sub-vector $(u_{j+1},u_{j+2},\cdots,u_{j'})$ of $\vec{u}$. Let $w_{j,j'}(\vec{u})$ denote the Hamming weight of $\gamma_{j,j'}(\vec{u})$. By convention, $w_{0,m}(\vec{u})$ is also abbreviated as $w(\vec{u})$. For two different vectors $\vec{u},\vec{v}\in V^m$, $\vec{u}$ is said to be smaller than $\vec{v}$ if $w_{i,m}(\vec{u})\le w_{i,m}(\vec{v})$ for all $0\le i<m$. For $\vec{u},\vec{v}\in V^m$, it is proved in [8,10,11] that the MSED of $\vec{u}$ is not larger than that of $\vec{v}$ if $\vec{u}$ is smaller than $\vec{v}$. When the nonzero entries of the input vectors are confined in the leftmost positions (the most unreliable positions), it is shown in [4,5,6,7] that there is a unique minimal vector in the unchecked region $\Omega_\tau(U)$.

For any binary vector $\vec{u}$, let $\vec{u}^{\,j}$ denote the concatenation of j $\vec{u}$'s. For $0\le i\le N$, let $\vec{t}_i$ denotes the vector 1^i0^{N-i}. To improve the upper bound on $\Delta(d)$, we will investigate the Chase-like algorithm whose input vector set U is of form

$$U_J=\{\vec{t}_0,\vec{t}_d,\vec{t}_{d+1},0^d10^{N-d-1}\}\cup\{\vec{t}_j : j\in J\} \qquad (2)$$

where J is a set of odd integers between 1 and $d-2$.

III . The Minimal Vector in $\Omega_\tau(U_J)$

If U_J is a set of form (2), the following theorem shows that there is a unique minimal vector in $\Omega_\tau(U_J)$.

Lemma 1 Let $J=(a_1,a_2,\cdots a_{k-1})$ be a set of odd integers with $1\le a_1<a_2<\cdots<a_{k-1}<a_k=d$. The set $\Omega_\tau(U_J)$ has a unique minimal sequence

$$\vec{f}_J=1^{c_0-1}0^{c_0}1^{c_1}0^{c_1}\cdots1^{c_{k-1}}0^{c_{k-1}+1}10^{N-d-2} \qquad (3)$$

where $c_0=(a_1+1)/2$ and $c_j=(a_{j+1}-a_j)/2$ for $j=1,\cdots,k-1$.

Proof: From (3), we see

$$\gamma_{0,a_1}(\vec{f}_J)=1^{c_0-1}0^{c_0} \qquad (4)$$

$$\gamma_{a_j,a_{j+1}}(\vec{f}_J)=1^{c_j}0^{c_j}, 0<j<k \qquad (5)$$

$$\gamma_{d,N}(\vec{f}_J)=010^{N-d-2} \qquad (6)$$

Then, we have

$$d_H(\vec{f}_J,\vec{t}_0)=w(\vec{f}_J)=\sum_{j=0}^{k-1}c_j,$$

$$d_H(\vec{f}_J,\vec{t}_{d+1})=(d-w_{0,d}(\vec{f}_J))+2=\sum_{j=0}^{k-1}c_j+2,$$

$$d_H(\vec{f}_J,0^d10^{N-d-1})=w_{0,d}(\vec{f}_J)+2=\sum_{j=0}^{k-1}c_j+1,$$

and, for $1\le j\le k$,

$$d_H(\vec{f}_J,\vec{t}_{a_j})=(a_j-w_{0,a_j}(\vec{f}_J))+w_{a_j,N}(\vec{f}_J)=\sum_{j=0}^{k-1}c_j+1.$$

Therefore, from $\sum_{j=0}^{k-1}c_j=(a_k+1)/2=\tau+1$, we see $\vec{f}_J\in\Omega_\tau(U_J)$.

Now we assume that $\vec{u}$ is an arbitrary vector in $\Omega_\tau(U_J)$. For $1\le j\le k$, from $d_H(\vec{u},\vec{t}_0)=w_{0,a_j}(\vec{u})+w_{a_j,N}(\vec{u})\ge\tau+1$ and $d_H(\vec{u},\vec{t}_{a_j})=(a_j-w_{0,a_j}(\vec{u}))+w_{a_j,N}(\vec{u})\ge\tau+1$, we see that

$$w_{a_j,N}(\vec{u})\ge\tau+1-(a_j-1)/2=w_{a_j,N}(\vec{f}_J). \qquad (7)$$

Furthermore, we can conclude that

$$w_{d+1,N}(\vec{u})\ge 1 \qquad (8)$$

Assume in contrary that $w_{d+1,N}(\vec{u})=0$. Then, from $w_{d,N}(\vec{u})=w_{a_k,N}(\vec{u})\ge 1$, we see that the $(d+1)$-th entry of $\vec{u}$ is equal to 1, from $d_H(\vec{u},\vec{t}_0)\ge\tau+1$ and $d_H(\vec{u},\vec{t}_{d+1})\ge\tau+1$, we see that $w_{0,d+1}(\vec{u})=\tau+1$. Therefore, $d_H(\vec{u},0^d10^{N-d-1})=w_{0,d+1}(\vec{u})-1=\tau$, contradicts to $\vec{u}\in\Omega_\tau(U_J)$.

Let i be an arbitrary integer with $1\le i<N$. If $1\le i<a_1$, from (4) and (7), we have

$$\begin{aligned}w_{i,N}(\vec{u})&\ge\max\{w_{a_1,N}(\vec{u}),w_{0,N}(\vec{u})-i\}\\&\ge\max\{w_{a_1,N}(\vec{f}_J),w_{0,N}(\vec{f}_J)-i\}=w_{i,N}(\vec{f}_J)\end{aligned}.$$

If $a_j\le i<a_{j+1}$ for some j with $1\le j<k$, from (5) and (7), we have

$$\begin{aligned}w_{i,N}(\vec{u})&\ge\max\{w_{a_{j+1},N}(\vec{u}),w_{a_j,N}(\vec{u})-(i-a_j)\}\\&\ge\max\{w_{a_{j+1},N}(\vec{f}_J),w_{a_j,N}(\vec{f}_J)-(i-a_j)\}=w_{i,N}(\vec{f}_J).\end{aligned}$$

If $i\ge a_k$, from (6) and (8), we also have $w_{i,N}(\vec{u})\ge w_{i,N}(\vec{f}_J)$. Hence, $\vec{f}_J$ is smaller than $\vec{u}$ if $\vec{u}\ne\vec{f}_J$.

According to (1) and Lemma 1, the Chase-like algorithm $C(U_J)$ achieves BD decoding if and only if $\sigma(\vec{f}_J)\geq d$.

IV. Conditions for $C(U_J)$ Achieving BD Decoding

To give conditions for $\sigma(\vec{f}_J)\geq d$, we show some properties of the MSEDs at first.

Lemma 2 For any vectors $\vec{u}\in V^m$ and $\vec{v}\in V^m$ and integers a,b with $a>b>0$,

$$\sigma(\vec{u}1^{a-b}0^a1^b\vec{v})\geq\sigma(\vec{u}0^b1^a0^{a-b}\vec{v}) \tag{9}$$

Proof: Let $s=(a-b)/(a+b)$. Suppose that $\vec{x}=(x_1,x_2,...,x_{m+n+2a})\in H_{m+n+2a}$ is a vector such that $d_E(\vec{s}(\vec{u}1^{a-b}0^a1^b\vec{v}),\vec{x})=\sigma(\vec{u}1^{a-b}0^a1^b\vec{v})$. Then, we have $x_m=x_{m+1}=...=x_{m+a-b}$ and $x_{m+a-b+1}=x_{m+a-b+2}=...=x_{m+2a}=t$, where t is the number defined by

$$t=\begin{cases}x_m, & if\ s<x_m,\\ x_{m+2a+1}, & if\ s>x_{m+2a+1},\\ s, & if\ x_m\leq s\leq x_{m+2a+1}.\end{cases}$$

Let $\vec{y}=(y_1,y_2,...,y_{m+n+2a})$ be the vector defined by

$$y_i=\begin{cases}x_i, & if\ i\leq m\ or>m+2a,\\ x_m, & if\ m<i\leq m+a+b,\\ x_{m+2a+1}, & if\ m+a+b<i\leq m+2a.\end{cases}$$

Then,

$$\begin{aligned}&\sigma(\vec{u}0^b1^a0^{a-b}\vec{v})-\sigma(\vec{u}1^{a-b}0^a1^b\vec{v})\\ &\leq d_E(\vec{s}(\vec{u}0^b1^a0^{a-b}\vec{v}),\vec{y})-d_E(\vec{s}(\vec{u}1^{a-b}0^a1^b\vec{v}),\vec{x})\\ &=b(1-x_m)^2+b(1+x_m)^2+(a-b)(1-x_{m+2a+1})^2\\ &\quad -a(1-t)^2-b(1+t)^2.\end{aligned} \tag{10}$$

Let ϕ be the right part of the equality (10). Then, if $s<x_m$, we have

$$\phi=(a-b)((1-x_{m+2a+1})^2-(1-x_m)^2)\leq 0.$$

If $s>x_{m+2a+1}$, we also have

$$\begin{aligned}\phi&=b((1-x_m)^2+(1+x_m)^2-(1-x_{m+2a+1})^2-(1+x_{m+2a+1})^2)\\ &=2b(x_m^2-x_{m+2a+1}^2)\leq 0\end{aligned}$$

If $x_m\leq s\leq x_{m+2a+1}$, we still have

$$\begin{aligned}\phi&=2b(1+x_m^2)+(a-b)(1-x_{m+2a+1})^2-a(1-s)^2-b(1+s)^2\\ &\leq 2b(1+s^2)+(a-b)(1-s)^2-a(1-s)^2-b(1+s)^2=0.\end{aligned}$$

Hence, (9) is valid.

The following lemma can be found in [11].

Lemma 3 Let $\vec{u}$ be an arbitrary vector in V^m. If $w_{i,m}(\vec{u})/(m-i)\geq\min\{1/2,w(\vec{u})/m\}$ holds for all i with $0\leq i<m$, the MSED $\sigma(\vec{u})$ is given by

$$\sigma(\vec{u})=\begin{cases}m-(m-2w(\vec{u}))^2/m, & if\ w(\vec{u})/m<1/2,\\ m, & otherwise.\end{cases}$$

To design Chase-like algorithms achieving BD decoding with a small set of input vectors, according to Lemma 2, we can only consider the Chase-like algorithms $C(U_J)$ such that the numbers defined in Lemma 1 satisfy $c_0\geq c_1\geq...\geq c_{k-1}$. Furthermore, we assume that the set J contains $d-2$. Then, $c_{k-1}=1$. Assume that

$$h_0=1<h_1<h_2<...<h_p \tag{11}$$

are the distinct integers in the list $c_0,c_1,...,c_{k-1}$ and, for each $0\leq j\leq p$, h_j repeats g_j times in such list. Then, we have

$$\sum_{i=0}^{p}g_ih_i=\tau+1. \tag{12}$$

and (3) can be rewritten further as

$$\vec{f}_J=\zeta\xi_{p-1}\cdots\xi_0 0010^{N-d-2} \tag{13}$$

where $\zeta=1^{h_p-1}(0^{h_p}1^{h_p})^{g_p-1}$ and

$$\xi_i=0^{h_{i+1}}1^{h_i}(0^{h_i}1^{h_i})^{g_i-1},0\leq i<p.$$

Now we give a sufficient condition for the Chase-like algorithm $C(U_J)$ achieves BD decoding.

Theorem 1 The Chase-like algorithm $C(U_J)$ achieves BD decoding if

$$\sum_{i=0}^{p-1}\frac{(h_{i+1}-h_i)^2}{2g_ih_i+h_{i+1}-h_i}\leq 5/3 \tag{14}$$

Proof: For any vector $\vec{u}\in V^m$ and integer i with $0\leq i<m$, from the definition of MSED, one can show easily that

$$\sigma(\vec{u})\geq\sigma(\gamma_{0,i}(\vec{u}))+\sigma(\gamma_{i,m}(\vec{u})) \tag{15}$$

Since the vectors 001, ζ and ξ_i, $0\leq i<p$ satisfy the condition of Lemma 3, respectively, according to (13), (15) and Lemma 3, we have

$$\begin{aligned}\sigma(\vec{f}_J)&\geq\sigma(\zeta)+\sigma(001)+\sum_{i=0}^{p-1}\sigma(\xi_i)\\ &=(d+2)-\frac{1}{3}-\sum_{i=0}^{p-1}\frac{(h_{i+1}-h_i)^2}{2g_ih_i+h_{i+1}-h_i}\end{aligned} \tag{16}$$

Hence, from (14), (16) and Lemma 1, the Chase-like algorithm $C(U_J)$ achieves BD decoding.

Let a,b be numbers with $0<a<1/2$ and $b>0$. For $i\geq 0$, let $h_i=i+1$ and $f_i=\lceil b(\tau+1)^a(i+1)^{-2a}\rceil$. Let p be the integer such that

$$\sum_{i=0}^{p-1} f_i(i+1) \le \tau+1 < \sum_{i=0}^{p} f_i(i+1). \tag{17}$$

Let $g_p = \lfloor (\tau+1-\sum_{i=0}^{p-1} f_i(i+1))/(p+1) \rfloor$ and $i_0 = \tau+1-\sum_{i=0}^{p-1} f_i(i+1) - g_p(p+1)$. For $0 \le i < p$, let

$$g_i = \begin{cases} f_i+1, & \text{if } i = i_0 - 1, \\ f_i, & \text{otherwise}. \end{cases} \tag{18}$$

Then $h_0, g_0, h_1, g_1, \ldots, h_p, g_p$ are positive integers satisfying (11) and (12). From the left inequality of (17), we see

$$b^{-1}(\tau+1)^{1-a} \ge \sum_{i=0}^{p-1} (i+1)^{1-2a} \ge \int_0^p x^{1-2a} dx = p^{2-2a}/(2-2a),$$

and thus

$$p \le (\tau+1)^{1/2}((2-2a)/b)^{1/(2-2a)}. \tag{19}$$

Hence,

$$\begin{aligned}
&\sum_{i=0}^{p-1} \frac{(h_{i+1}-h_i)^2}{2g_i h_i + h_{i+1} - h_i} = \sum_{i=0}^{p-1} \frac{1}{2g_i(i+1)+1} \\
&\le \frac{1}{2b(\tau+1)^a} \sum_{i=0}^{p-1} (i+1)^{2a-1} \le \frac{1}{2b(\tau+1)^a}\left(1+\int_1^p x^{2a-1}dx\right) \\
&= \frac{1}{2b(\tau+1)^a}\left(1+\frac{p^{2a-1}}{2a}\right) \le \frac{p^{2a}}{4ab(\tau+1)^a} \le \frac{((2-2a)b^{-1})^{a/1-a}}{4ab}.
\end{aligned}$$

If we choose b further as

$$b = 3^{1-a}(20a)^{a-1}(2-2a)^a, \tag{20}$$

Then we have

$$\sum_{i=0}^{p-1} \frac{(h_{i+1}-h_i)^2}{2g_i h_i + h_{i+1} - h_i} \le 5/3,$$

and thus according to Theorem 1, the Chase-like algorithm $C(U_J)$ achieves BD decoding.

From $g_p \le f_p - 1$, (19) and (20), we have

$$\begin{aligned}
&\sum_{i=0}^{p} g_i \le p+1+\sum_{i=0}^{p} b(\tau+1)^a (i+1)^{-2a} \\
&\le p+1+b(\tau+1)^a\left(1+(p+1)^{-2a}+\int_1^p x^{-2a}dx\right) \\
&= p+1+b(\tau+1)^a\left(1+(p+1)^{-2a}+\frac{p^{1-2a}-1}{1-2a}\right) \\
&\le 1+2b(\tau+1)^a \\
&\quad +\left((2-a)^{\frac{1}{2-2a}} b^{\frac{-1}{2-2a}} + \frac{1}{1-2a}(2-2a)^{\frac{1-2a}{2-2a}} b^{\frac{1}{2-2a}}\right)(\tau+1)^{1/2} \\
&= 1+2a(\tau+1)^a + \sqrt{\frac{40a(1-a)}{3}}\left(1+\frac{3}{20a(1-2a)}\right)(\tau+1)^{1/2}.
\end{aligned}$$

Hence, we have proved the following theorem.

Theorem 2 When the Hamming distance d of the code approaches infinity, the Chase-like algorithms can achieve BD decoding with $(\psi + o(1))d^{1/2}$ input vectors, where

$$\psi = \min_{0<a<1/2}\left(1+\frac{3}{20a(1-2a)}\right)\sqrt{\frac{20a(1-a)}{3}} \approx 2.218.$$

V . Conclusions

In literature, there are many works to estimate the smallest size, denoted by $\Delta(d)$ for binary block code of Hamming distance d , of input vector sets of Chase-like algorithms which achieve BD decoding. Unlike most of these works, we deal with in this paper some Chase-like algorithms with an additional input vector whose nonzero entries are not confined in the most unreliable positions. With a similar method used in [7], we show that such a Chase-like algorithm has also a unique minimal vector in its unchecked region and then improve the best known upper bound on $\Delta(d)$ to: $\Delta(d) \le (\psi + o(1))d^{1/2}$, where $\psi \approx 2.218$.

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