A New Scheme to Improve the BER Performance Based on Optimizing Energy Allocation

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Abstract - How to improve the channel coding capacity and reduce the bit error rate have been the main research directions of channel coding. In order to improve the bit error rate (BER) performance based on a formula derived from the Union Bound which can estimate the bit error rate for every position in the codeword sequence, this paper presented a new method to optimize the bit energy. By the new energy allocation, the energy of each bit in the codeword is optimized. The simulation shows that BER performance can be improved noticeable not only at high signal noise ratio (SNR), but also at low and moderate SNR.

Index Terms - Channel, coding, BER, Union Bound, Energy allocation.

I . Introduction

For modern communication system, error correcting code is a means of reliability guarantee, and the BER performance is the measure of an error correcting code technology quality. Therefore, how to improve the BER performance of error correcting code is the most important task. There are many methods toward this destination. One of these is to change the code structure and algorithms, Such as in [1] and [2]. Another method is to reallocate the bit energy of the codeword sequence. The advantage of such method is easily to be implemented nearly without any change on the encoding and decoding schemes. There are some papers researching on this aspect. Such as in [3] and [4], the authors presented asymmetric energy allocation strategies to design different energies to the systemic bits and parity check bits. In [5], based on the low weight distributions of turbo codes, the authors allocated the energies among the codeword that have different weights instead of between the systematic and parity bits. In this scheme more energy is assigned to the lowest codeword to improve the “error floor” of Turbo codes. In [7], another method to reallocate the bit’s energy is presented. This method considers the all lowest weight code words connecting to every bit and better BER performance is achieved. However, when the codeword length is small, it is not difficult to get these parameters. But if the codeword length is longer, it would be hard to calculate the required parameters for this method. In this paper, a simple way to solve this problem is proposed. This new scheme only requires the BER distribution curves of simulations under two different SNRs without knowing any construction information of the code. Therefore this new scheme is very easily realized and the simulations show that BER performance can be improved not only at high SNR, but also at low and moderate SNR.

II. Union Bound And The Formulas Of The Energy Allocation

For an AWGN channel, the BER is bounded by the Union Bound as

$$P_b \le \sum_{i=1}^{2^k} \frac{w_i}{2k} erfc\left(\sqrt{d_i \frac{R_c E_b}{N_0}}\right) \tag{1}$$

Where w_i and d_i are the information weight and total Hamming weight, respectively, of the *i*-th codeword, k is the input length, R_c is the code rate. E_b is the bit energy of the codeword and the N_0 is the noise power spectrum density.

From (1), a formula to estimate every position’s bit error rate can be derived as[6]

$$p_b(j) \approx \frac{1}{2} n_{\min}(j) e^{-d_{\min}(j) R_c E_b / N_0} \tag{2}$$

Where $d_{min}(j)$ is the lowest weight code words connecting to j-th position and $n_{min}(j)$ is its number. j=0,1,2,…,N-1. N is the length of codeword.

Generally, the $p_b(j)$ for all j are not constant, especially at high SNR, The average BER is mainly decided by the bit error rate at the such positions which have the higher BER. So in the paper [7] more energy is assigned to the positions which have the higher BER in order to reduce the BER. If the total energy remains unchanged, consequently, the BER of some positions will be likely to be increased due to less energy to be assigned to them. If we assign the energy in such a way that the $p_b(j)$ is constant for all j, the average BER will be reduced. To realize this, by replacing the constant parameter E_b with $E_b(j)$, which is the optimized energy for the bit in position j and replacing $p_b(j)$ with $\overline{p_b}$, which is the new bit error rate for position *j* relating to $E_b(j)$ in (2), we can get

$$E_b(j) = \frac{N_0}{R_c d_{\min}(j)} \ln \frac{n_{\min}(j)}{2\overline{p_b}} \tag{3}$$

With the binding condition of energy conservation

$$\sum_{j=0}^{N-1} E_b(j) = NE_b \tag{4}$$

We get[7]

$$\overline{p_b} = \frac{1}{2}\exp \frac{\sum_{j=0}^{N-1}\frac{\ln n_{\min}(j)}{d_{\min}(j)} - NR_c \frac{E_b}{N_0}}{\sum_{j=0}^{N-1}\frac{1}{d_{\min}(j)}} \quad (5)$$

Therefore, if we get the required parameters, $d_{min}(j)$ and $n_{min}(j)$, we can calculate the $\overline{p_b}$ by (5) for every position. Then the optimized energy allocation can be calculated by (3). In this energy allocation scheme, there have two problems need to be solved. The first one is that the formula (2) gives a good estimation of the BER distribution at high SNR, but not for low SNR, at the low SNR, the bit error rate is not only determined by the codeword of lowest weights, we need more information about the code structure to get an accurate result. So this scheme has no significant effect in this case.The second problem is that when the codeword length is small, it is not difficult to get $d_{min}(j)$ and $n_{min}(j)$, but when the codeword length is longer, it would be hard to calculate them. In order to solve these problems, we will propose a simple method in the next section.

III. Optimizing The Energy Allocation

For the sake of simulation, let SNR be in dB and generally E_b=1, so the N_0 in the above formulas can be substituted by

$$N_0 = 10^{\frac{-dB}{10}} \quad (6)$$

Set $d_{min}(j)$ and $n_{min}(j)$ be unknown parameters. Now if we have two simulation curves, noted by $p_{b1}(j)$ and $p_{b2}(j)$, under two different SNRs, noted by dB1 and dB2, separately, then from the following equation group

$$\begin{cases} p_{b1}(j) \approx \frac{1}{2} n_{\min}(j) e^{-d_{\min}(j) R_c 10^{\frac{\mathrm{dB}_1}{10}}} \\ p_{b2}(j) \approx \frac{1}{2} n_{\min}(j) e^{-d_{\min}(j) R_c 10^{\frac{\mathrm{dB}_2}{10}}} \end{cases} \quad (7)$$

The parameters $d_{min}(j)$ and $n_{min}(j)$ are determined by

$$d_{\min}(j) = \frac{\ln \frac{p_{b2}(j)}{p_{b1}(j)}}{R_c (10^{\frac{dB_1}{10}} - 10^{\frac{dB_2}{10}})} \quad (8)$$

And

$$n_{\min}(j) = 2 p_{b1}(j) \exp\left(\frac{10^{\frac{dB_1}{10}} \ln(p_{b1}(j) / p_{b2}(j))}{10^{\frac{dB_2}{10}} - 10^{\frac{dB_1}{10}}}\right) \quad (9)$$

It should be noted that, In (3), the optimized $E_b(j)$ is determined by the $d_{min}(j)$ and $n_{min}(j)$, which are exactly the lowest codewords' weight and their numbers connecting to the j position. But in this method, the BER distributions are involved. Because the BER distribution gives our more information about the code structure, instead of only the lowest code words and their numbers, therefore the d_{min} (j) in (8) and n_{min} (j) in (9) no long reserve the original meanings as in the definitions. Therefore, when using this method, regardless of the codeword length, we can obtained $d_{min}(j)$ and $n_{min}(j)$ by (8) and (9), then calculate the $\overline{p_b}$ by (5) for every position, finally the optimized energy allocation can be calculated by (3).

IV. Simulation And Analytical Results

Firstly we give a simple example to show the method's efficiency. Here a (2, 1) convolutional code without tail bits is used, its generator matrix is (171,133). The length of input sequences is 32.

Figure 1 shows the simulation curve of (2,1) convolutional code. In the convolutional code, from 0 to 5 dB, the optimized curve noted as (p_b0, p_b2.5) is produced by simulation curves under 0dB and 2.5dB.

There have three curves in Figure 1, the curve "-o-"is produced under the normal case, that is, Eb=1. The curve "-*-"is produced by (5) and the solid curve is produced by (7).

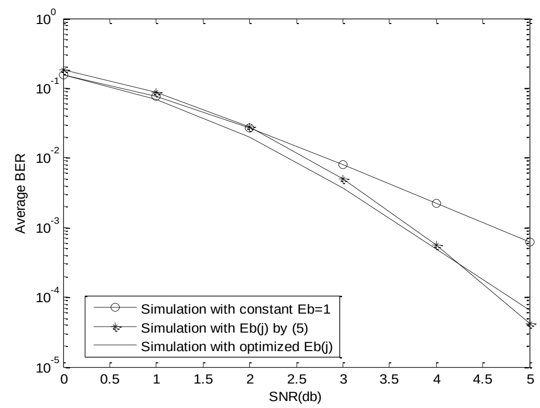

Fig. 1 The simulation BER curves for the (2, 1) convolutional code without tail bits

From figure 1 we can see that the simulation curves, which are achieved by using the formula (8) and (9), are lower than other two curves at the low SNRs. This is because $d_{min}(j)$ and $n_{min}(j)$ are calculated by the two simulation curves at low SNRs at this time, so the optimized energy allocation formula about this case contains more information about the code structure and the condition of channel under the low SNRs , therefore the BER performance can be improved in these SNRs. But at the high SNRs, the optimized energy allocation formula can't play a good improvement effect, the simulation curve of optimized is no long lower than the simulation curve produced by the original energy allocation scheme presented in [7]. This is because that the two BER distributions to calculate the $d_{min}(j)$ and $n_{min}(j)$ are produced under the low SNR(0dB) and moderate SNR(2.5dB). So the parameters $d_{min}(j)$ and $n_{min}(j)$ reflect more information about the code for the low and moderate SNR, and less information for the high

SNR. Therefore, we can use the two simulation curves at high SNRs to calculate $d_{min}(j)$ and $n_{min}(j)$, then apply them to the energy allocation formula, the improvement for the high SNRs will be achieved. Next we will use this approach to improve the code performance at high SNRs.

Figure 1 and 2 use the same code except that in figure 2, from 0 to 3 dB, the optimized curve noted as (p_b0, $p_b2.5$) is produced by simulation curves under 0dB and 2.5dB. From 4 to 5 dB, the optimized curve noted as (p_b2, p_b4) is produced by simulation curves under 2dB and 4dB.

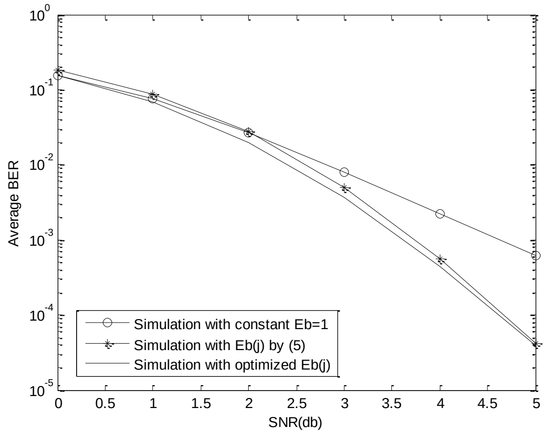

Fig. 2 The simulation BER curves for the (2, 1) convolutional code without tail bits

Figure 2 shows that the simulation curves, which are achieved by using the formula (8) and (9), are lower than other two curves under different SNRs. So we can come to a preliminary conclusion, that is through choose the proper $p_{b1}(j)$ and $p_{b2}(j)$ according to the different SNRs to optimize the energy allocation scheme, the energy allocation formula can contain more information about the code structure, and the BER performance can be improved not only at high SNR, but also at low and moderate SNR. Other examples are given in the following figures to prove our conclusion.

Figure 3 shows the simulation curve of (15, 4) linear code. The code's generator matrix is

$$g = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

In Figure 3, from 0 to 3 dB, the optimized curve noted as (p_b0, p_b4) is produced by simulation curves under 0dB and 4dB. From 4 to 7 dB, the optimized curve noted as (p_b4, p_b6) is produced by simulation curves under 4dB and 6dB.

Figure 4 shows the simulation curve of Turbo code. The code's generator matrix is g = (1, 1101/1011). An 8X8 block interleaver with size 64 is used. The puncture pattern is p = [1 0; 0 1] which produces a 1/2 code rate. Iteration time is 5 and the decoding algorithm is BCJR. In this figure, from 0 to 5 dB, the optimized curve noted as (p_b-10, p_b5) is produced by simulation curves under -10dB and 5dB.

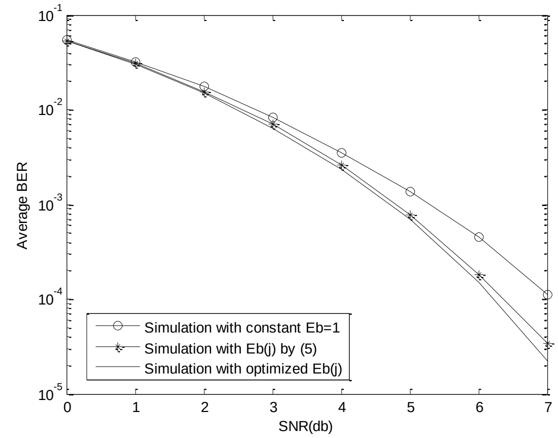

Fig. 3 The simulation BER curves for the (15, 4) linear code

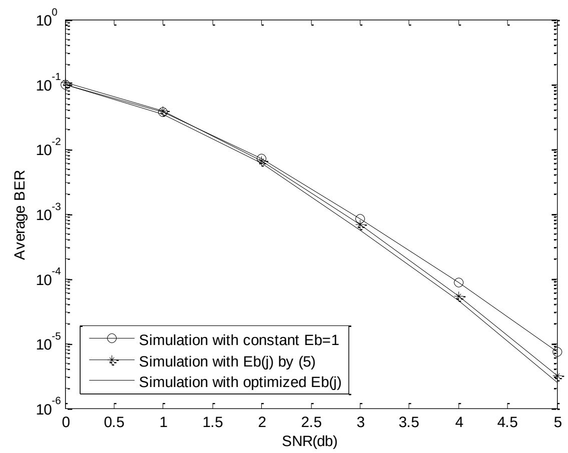

Fig. 4 The simulation BER curves for the Turbo code

From figure 3 and 4 we can see the similar result as Figure 2 does. The optimized curve, which is achieved by using the formula (8) and (9), are lower than other two curves under different SNRs. The BER performance can be improved not only at high SNR, but also at low and moderate SNR. This confirms the method proposed in this article is valid; it can solve the existing problems in the original energy allocation scheme[7]. But in this case, different selections of the $p_b(j)$may produce different results, so how to choose the proper $p_b(j)$to make the BER performance to be optimization should be studied further.

V. Conclusion

A new method to optimize the bit energy is presented in this paper. Under the optimized energy allocation, each bit in the codeword can easily obtain quantitative energy, by this method the average BER is minimized not only at high SNR, but also at low and moderate SNR. This scheme is applied to a variety of error correcting codes. When the codeword length is long, this optimized energy allocation scheme also has the advantage of simple and easy to operate. By applying this scheme to the different type of error-correcting codes, the simulation results show that its improvements for the BER performance are noticeable.

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