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Minimizing the Number of Tardy Jobs with Gammadistributed Processing Times

Haohao Li
Department of Mathematics, State Key Lab of CAD \& CG
Zhejiang University,
Hangzhou, 310027, China
e-mail: hhlzju@126.com

***Abstract*—This paper investigates two scheduling problems i.e., single machine scheduling problem with minimizing the number of tardy jobs and two machine flow shop scheduling problem with a common due date and minimizing the number of tardy jobs, in a stochastic setting in the class of non-preemptive static list policies. It is assumed that the processing times of jobs are Gamma distributed stochastic variables. In previous results on these stochastic models, the strict condition– that all scale parameters are the same – is required. However, this condition may be difficult to be specified in real world problems. This paper presents new stochastic models for these two problems. Approximate deterministic problems are generated for these stochastic problems based on a new probability inequality, under mild conditions. The former stochastic models are special cases of our results.**

Keywords-stochastic scheduling; single machine scheduling; flow shop scheduling; tardy jobs

I. INTRODUCTION

Consider two problems of scheduling *n* jobs for the purpose of minimizing the number of tardy jobs in the class of non-preemptive static list policies. The first involves a single machine problem. The other involves a two machine flow shop problem with a common due date. Following the three-field notation proposed by Lawler et al. [1], the problems are denoted by $1 \| \sum U_j$ and $F_2 \mid d_j = d \mid \sum U_j$, respectively.

Recently, Elyasi and Salmasi [2] develop two interesting stochastic versions of problems $1 \| \sum U_j$ and $F_2 \mid d_j = d \mid \sum U_j$. In their research, the job processing times are assumed to be independent random variables with Gamma distributions. To obtain the effective models for these problems, Elyasi and Salmasi assume that the second parameter are all the same. That is, assume that $A_j \sim \Gamma(k_j, \theta),\ j = 1,2,\ldots n$,where A_j is the processing times of the job *j*. However, these conditions may be difficult to be satisfied in real world problems.

In this paper, we develop approximate stochastic versions of the problems $1 \| \sum U_j$ and $F_2 \mid d_j = d \mid \sum U_j$ in which the processing times of the job follow general Gamma distributions. The scale parameter can be different in our models. Also, Elyasi and Salmasi's models are special cases of our models.

The rest of this paper is organized as follows. In Section 2, we provide necessary background and briefly present stochastic versions of problems of $1 \| \sum U_j$ and $F_2 \mid d_j = d \mid \sum U_j$ developed in [2] for the readers convenience. An new probability inequality is proved in the Section 3, which is used to establish new stochastic models for the problems of $1 \| \sum U_j$ and $F_2 \mid d_j = d \mid \sum U_j$ in Section 4. Finally, conclusions and suggestions for future research areas are provided in Section 5.

II. STOCHASTIC VERSIONS OF THE PROBLEMS $1 \| \sum U_j$ AND $F_2 \mid d_j = d \mid \sum U_j$

In the classic version of the problem $1 \| \sum U_j$, a set $N = \{1,2,\ldots,n\}$ of independent and non-preemptive jobs is available at the beginning of the planning horizon to be processed on the machine. All jobs have equal weights. The machine is always available, and it can process at most one job at a time. Let p_j and d_j denote the processing time and the due date of job *j*, respectively. The job due dates are assumed to be given exogenously. If the process of job *j* is completed after its due date, the job is considered as tardy. The objective is to minimize the number of tardy jobs.

Let the jobs be re-indexed according to the earliest due date(EDD) first rule, i.e., $d_1 \le d_2 \le \cdots \le d_n$. Then, the following integer programming model maximizes the number of on-time jobs (or equivalently minimizes the number of tardy jobs) on a single machine [3]:

$$\max\ z = \sum_{j=1}^{n} x_j$$

$$s.t. \sum_{j=1}^{i} p_j x_j \le d_j, i = 1,2,\ldots n, \qquad (1)$$

$$x_j \in \{0,1\},\ j = 1,2,\ldots,n,$$

where $x_j = 1$ if job *j* is on-time, and 0 otherwise.

The classic version of the problem $F_2 \mid d_j = d \mid \sum U_j$ is a flow shop with two machines, M_1 and M_2. A set $N = \{1,2,\ldots,n\}$ of independent and

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non-preemptive jobs is available at the beginning of the planning horizon to be processed sequentially on machine M_1 and then on M_2. Each machine cannot process more than one job and each job cannot be processed by more than one machine at a time. The jobs have a common and deterministic due date *d*, which is given exogenously. The objective is to minimize the number of tardy jobs.

Let a_i and b_j be the processing times of job *j* on machines M_1 and M_2, respectively. Without loss of generality, assume that jobs are re-indexed according to the Johnson algorithm [4], i.e., first set the jobs with $a_i \le b_j$ in a non-decreasing order of a_j, then set the remaining jobs in a non-increasing order of b_j. Then, Della Croce et al. [5] present the following integer programming model for the problem $F_2 \mid d_j = d \mid \sum U_j$:

$$\begin{aligned} &\max\ z = \sum_{j=1}^{n} x_j \\ &s.t. \sum_{j=1}^{k} a_j x_j + \sum_{j=k}^{n} b_j x_j \le d, k = 1,\ldots,n, \quad (2) \\ &x_j \in \{0,1\}, j = 1,\ldots,n, \end{aligned}$$

where $x_j = 1$ if job *j* is on-time, and 0 otherwise.

Stochastic versions of the $1 \| \sum U_j$ problem have been studied by some authors, see e.g., [6, 7, 8, 9]. The problem can be modeled as a chance constrained program as follows:

$$\begin{aligned} &\max\ z = \sum_{j=1}^{n} x_j \\ &s.t. P\left[\sum_{j=1}^{i} A_j x_j \le d_i\right] \ge \rho, i = 1,\ldots,n, \quad (3) \\ &x_j \in \{0,1\}, j = 1,\ldots,n, \end{aligned}$$

where $A_j, 1 \le j \le n$ denote the stochastic processing time of job *j* on the machine, $P(\bullet)$ is the probability of $\bullet$, $0 \le \rho \le 1$ is the desired confidence level for satisfaction of the constraint set. Fortz and Poss [10] prove the following theorem to transform the chance constrained formulation of a combinatorial optimization problem to a deterministic one.

Theorem 1 ([10]). Consider *n* independent random variables

$$A_j \sim \Gamma(k_j, \theta), 1 \le j \le n,,$$

with $\theta \ge 0$ and $k_j \ge 0$ and assume that $d > 0$. Then, if $x_j \in \{0,1\}$ for each $1 \le j \le n$, the following constraints are equivalent:

$$\begin{aligned} &P\left[\sum_{j=1}^{n} A_j x_j \le d\right] \ge \rho \\ &\Leftrightarrow \sum_{j=1}^{n} k_j x_j \le k^* \end{aligned}, \quad (4)$$

where k^* is the unique root of the equation

$$\frac{\int_0^d z^{k-1} e^{-\frac{z}{\theta}} dz}{\Gamma(k)\theta^k} = \rho$$

and the gamma function is defined by

$$\Gamma(k) = \frac{\int_0^\infty z^{k-1} e^{-\frac{z}{\theta}} dz}{\theta^k}.$$

Let the jobs be re-indexed so that

$$k_1^* \le k_2^* \le \cdots \le k_n^*$$

Using Theorem 1, Elyasi and Salmasi [2] obtain the equivalent deterministic model of the problem $1 \| \sum U_j$ with Gamma distributed processing times as follows:

$$\begin{aligned} &\max\ z = \sum_{j=1}^{n} x_j \\ &s.t. \sum_{j=1}^{i} k_j x_j \le k_i^*, i = 1,2,\ldots,n \quad (5) \\ &x_j \in \{0,1\}, j = 1,2,\ldots,n, \end{aligned}$$

where k_i^* is the unique root of the equation $\frac{\int_0^{d_i} z^{k_i-1} e^{-\frac{z}{\theta}} dz}{\Gamma(k_i)\theta^{k_i}} = \rho$, for each $1 \le i \le n$.

Also, Elyasi and Salmasi [2] consider stochastic version of the problem $F_2 \mid d_j = d \mid \sum U_j$, where the job processing times on the machines are gamma distributed with a common scale parameter, i.e.,

$$A_j \sim \Gamma(k_{A_j}, \theta)$$

and

$$B_j \sim \Gamma(k_{B_j}, \theta).$$

Recall that if $A_j \sim \Gamma(k_{A_j}, \theta)$ and $B_j \sim \Gamma(k_{B_j}, \theta)$, then

$$A_j + B_j \sim \Gamma(k_{A_j} + k_{B_j}, \theta).$$

Using Theorem 1, each chance constraint can be replaced by a deterministic linear constraint if the processing times of job *j* on machines M_1 and M_2 follow $\Gamma(k_{A_j}, \theta)$ and $\Gamma(k_{B_j}, \theta)$, respectively. Let the jobs be re-indexed according to the Johnson algorithm [4] assuming that the processing times of job j on machines M_1 and M_2 are k_{A_j} and k_{B_j}, respectively. In other words, first set the jobs with $k_{A_j} \le k_{B_j}$ in a non-decreasing order of k_{A_j}, then set the remaining jobs in non-increasing order of k_{B_j}. Therefore, the chance constrained formulation of the stochastic problem with gamma distributions is turned into an equivalent deterministic model as follows:

$$\max\ z = \sum_{j=1}^{n} x_j$$
$$s.t. \sum_{j=1}^{k} k_{A_j} x_j + \sum_{j=k}^{n} k_{B_j} x_j \le k^*, k = 1,2,\ldots,n, \quad (6)$$
$$x_j \in \{0,1\}, j = 1,\ldots,n,$$

where k^* is the unique root of the equation $\dfrac{\int_0^d z^{k-1} e^{-\frac{z}{\theta}} dz}{\Gamma(k)\theta^k} = \rho$.

In models (5) and (6), the job processing times on the machines are gamma distributed with a common scale parameter θ. In this paper, we construct new stochastic versions of the problems $1 \| \sum U_j$ and $F_2 \mid d_j = d \mid \sum U_j$, where different scale parameters are allowed in Gamma distributions.

III. A PROBABILITY INEQUALITY

In this section, we establish a probability inequality which will be used to formulate new stochastic versions of the problems $1 \| \sum U_j$ and $F_2 \mid d_j = d \mid \sum U_j$.

Theorem 2 Consider *n* independent random variables

$$A_j \sim \Gamma(k_j, \theta_j), k_j > 0, j = 1,2,\ldots,n.$$

Assume that $\theta_1 \le \theta_2 \le \cdots \le \theta_n$ and $b > 0$. Then, if $x_j \in \{0,1\}$ for each $1 \le j \le n$, the following inequality holds:

$$p_{A_1 x_1 + \cdots + A_n x_n}(x) \ge \left(\frac{\theta_1}{\theta_n}\right)^{k_1} \left(\frac{\theta_2}{\theta_n}\right)^{k_2} \cdots \left(\frac{\theta_{n-1}}{\theta_n}\right)^{k_{n-1}} p_{\eta_n}(x) \quad (7)$$

Proof We prove (7) by induction on *n*. Denote by $p_\eta(x)$ the density function of random variable η. For $n = 2$, note that $\theta_1 \le \theta_2$, and $u \le x$ we have

$$p_{A_1 + A_2}(x)$$
$$= \int_{-\infty}^{+\infty} p_{A_1}(u) p_{A_2}(x-u) du$$
$$= \frac{\theta_1^{k_1} \theta_2^{k_2}}{\Gamma(k_1)\Gamma(k_2)} \int_0^x u^{k_1 - 1}(x-u)^{k_2 - 1} e^{-\theta_1 u - \theta_2 x + \theta_2 u} du$$
$$= \frac{\theta_1^{k_1} \theta_2^{k_2}}{\Gamma(k_1)\Gamma(k_2)} e^{-\theta_2 x} \int_0^x u^{k_1 - 1}(x-u)^{k_2 - 1} e^{(\theta_2 - \theta_1)u} du$$
$$\ge \frac{\theta_1^{k_1} \theta_2^{k_2}}{\Gamma(k_1)\Gamma(k_2)} e^{-\theta_2 x} \int_0^x u^{k_1 - 1}(x-u)^{k_2 - 1} du$$
$$= \frac{\theta_1^{k_1} \theta_2^{k_2}}{\Gamma(k_1)\Gamma(k_2)} e^{-\theta_2 x} \frac{\Gamma(k_1)\Gamma(k_2)}{\Gamma(k_1 + k_2)} x^{k_1 + k_2 - 1} \quad (8)$$
$$= \frac{\theta_1^{k_1} \theta_2^{k_2}}{\Gamma(k_1 + k_2)} x^{k_1 + k_2 - 1} e^{-\theta_2 x}$$
$$= \left(\frac{\theta_1}{\theta_2}\right)^{k_1} \frac{\theta_2^{k_1 + k_2}}{\Gamma(k_1 + k_2)} x^{k_1 + k_2 - 1} e^{-\theta_2 x}$$
$$= \left(\frac{\theta_1}{\theta_2}\right)^{k_1} \Gamma(k_1 + k_2, \theta_2)$$

Now let

$$\theta_1 \le \theta_2 \le \cdots \le \theta_{n+1}, A_{n+1} \sim \Gamma(k_{n+1}, \theta_{n+1}),$$

And

$$\eta \sim \Gamma(k_1 + \cdots + k_n, \theta_n)$$

Assume that

$$p_{A_1 + \cdots A_n}(x) \ge \left(\frac{\theta_1}{\theta_n}\right)^{k_1} \left(\frac{\theta_2}{\theta_n}\right)^{k_2} \cdots \left(\frac{\theta_{n-1}}{\theta_n}\right)^{k_{n-1}} \Gamma(k_1 + \cdots + k_n, \theta_n) \quad (9)$$

Then from (8), (9) we have

$$p_{A_1 + \cdots + A_{n+1}}(x)$$
$$= p_{(A_1 + \cdots + A_n) + A_{n+1}}(x)$$
$$= \int_{-\infty}^{+\infty} p_{A_1 + \cdots + A_n}(u) p_{A_{n+1}}(x-u) du$$
$$\ge \left(\frac{\theta_1}{\theta_n}\right)^{k_1} \left(\frac{\theta_2}{\theta_n}\right)^{k_2} \cdots \left(\frac{\theta_{n-1}}{\theta_n}\right)^{k_{n-1}} \int_{-\infty}^{+\infty} p_\eta(u) p_{A_{n+1}}(x-u) du$$
$$\ge \left(\frac{\theta_1}{\theta_n}\right)^{k_1} \left(\frac{\theta_2}{\theta_n}\right)^{k_2} \cdots \left(\frac{\theta_{n-1}}{\theta_n}\right)^{k_{n-1}} \left(\frac{\theta_n}{\theta_{n+1}}\right)^{k_1 + \cdots + k_n} \cdot \Gamma(k_1 + \cdots + k_{n+1}, \theta_{n+1})$$
$$= \left(\frac{\theta_1}{\theta_{n+1}}\right)^{k_1} \left(\frac{\theta_2}{\theta_{n+1}}\right)^{k_2} \cdots \left(\frac{\theta_n}{\theta_{n+1}}\right)^{k_n} \Gamma(k_1 + \cdots + k_{n+1}, \theta_{n+1})$$

By induction we have proved that

$$p_{A_1 + \cdots A_n}(x) \ge \left(\frac{\theta_1}{\theta_n}\right)^{k_1} \left(\frac{\theta_2}{\theta_n}\right)^{k_2} \cdots \left(\frac{\theta_{n-1}}{\theta_n}\right)^{k_{n-1}} \Gamma(k_1 + \cdots + k_n, \theta_n)$$

Thus, if x_j are binary numbers, we have also that

$$p_{A_1 + \cdots A_n}(x) \ge \left(\frac{\theta_1}{\theta_n}\right)^{k_1} \left(\frac{\theta_2}{\theta_n}\right)^{k_2} \cdots \left(\frac{\theta_{n-1}}{\theta_n}\right)^{k_{n-1}} p_{\eta_n}(x),$$

where $\eta_n \sim \Gamma(k_1 x_1 + \cdots + k_n x_n, \theta_n)$.

This completes the proof of the Theorem 2.

For a given desired confidence level ρ, denoted by $\bar{\rho} = \left(\frac{\theta_n}{\theta_1}\right)^{k_1} \left(\frac{\theta_n}{\theta_2}\right)^{k_2} \cdots \left(\frac{\theta_n}{\theta_{n-1}}\right)^{k_{n-1}} \rho$

Corollary If

$$P\left[p_{\eta_n}(x)\le d\right]\ge \overline{\rho} \qquad (10)$$

Then

$$P\left[p_{A_1+\cdots+A_n}(x)\le d\right]\ge \rho \qquad (11)$$

Proof The proof follows easily from (7) and thus omitted here.

IV. NEW STOCHASTIC VERSIONS OF THE PROBLEMS $1\|\sum U_j$ AND $F_2\,|\,d_j=d\,|\sum U_j$

In this section, we establish new stochastic versions of the problems $1\|\sum U_j$ and $F_2\,|\,d_j=d\,|\sum U_j$, under a relaxed condition. Strict conditions–all Gamma distributions have a common scale parameter– will be dropped from the modeling process.

4.1 New stochastic version of the problem $1\|\sum U_j$

Using Theorem 2, each chance constraint (11) can be approximately replaced by a deterministic linear constraint(10) if the conditions in Theorem 2 are satisfied. Therefore, for a given confidence level ρ ,the chance constrained formulationof the stochastic problem $1\|\sum U_j$ with gamma distributions is turned into an approximately equivalent deterministic model as follows:

$$\begin{aligned}&\max\ z=\sum_{j=1}^{n}x_j\\ &s.t.\sum_{j=1}^{i}k_jx_j\le k_i^*,\ i=1,\ldots,n \qquad (12)\\ &x_j\in\{0,1\},\ j=1,\ldots,n\ ,\end{aligned}$$

where k_i^* is the unique root of the equation $\frac{\int_0^{d_i} z^{k_i-1}e^{-\frac{z}{\theta}}dz}{\Gamma(k_i)\theta^{k_i}}=\overline{\rho_i}$,and $\overline{\rho_i}=\frac{\theta_i^{k_1+\cdots k_{i-1}}}{\theta_1^{k_1}\theta_2^{k_2}\cdots\theta_{i-1}^{k_{i-1}}}\rho$ .The gamma function is defined by $\Gamma(k)=\frac{\int_0^{\infty} z^{k-1}e^{-\frac{z}{\theta}}dz}{\theta^k}$.

4.2 New stochastic versions of the problems $F_2\,|\,d_j=d\,|\sum U_j$

Consider the stochastic problem $F_2\,|\,d_j=d\,|\sum U_j$ where the job processing times on the machines are gamma distributed, $A_j\sim\Gamma\left(k_{A_j},\theta_{A_j}\right)$ and $B_j\sim\Gamma\left(k_{B_j},\theta_{B_j}\right)$. Assume that $\theta_{A_1}\le\theta_{A_2}\le\cdots\le\theta_{A_n}$, $\theta_{B_1}\le\theta_{B_2}\le\cdots\le\theta_{B_n}$ and $\theta_{A_j}\le\theta_{B_j},\ j=1,\cdots,n$.

Let the jobs be re-indexed according to the Johnson algorithm[4] assuming that the processing times of job j on machines M_1 and M_2 are k_{A_j} and k_{B_j} , respectively. In other words, first set the jobs with $k_{A_j}\le k_{B_j}$ in a non-decreasing order of k_{A_j}, then set the remaining jobs in non-increasing order of k_{B_j}. Therefore, the chance constrained formulation of the stochastic problem $F_2\,|\,d_j=d\,|\sum U_j$ with gamma distributions is turned into an approximately deterministic model as follows:

$$\begin{aligned}&\max\ z=\sum_{j=1}^{n}x_j\\ &s.t.\sum_{j=1}^{h}k_{A_j}x_j+\sum_{j=h}^{n}k_{B_j}x_j\le k^*,\ h=1,\ldots,n, \qquad (13)\\ &x_j\in\{0,1\},\ j=1,2,\ldots,n,\end{aligned}$$

where k^* is the unique root of the equation $\frac{\int_0^{d} z^{k-1}e^{-\frac{z}{\theta}}dz}{\Gamma(k)\theta^{k}}=\overline{\rho}$, and $\overline{\rho}=\frac{\theta_{B_n}^{k_{A_1}+\cdots+k_{A_h}+k_{B_h}+\cdots+k_{B_{n-1}}}}{\theta_{A_1}^{k_{A_1}}\cdots\theta_{A_h}^{k_{A_h}}\theta_{B_h}^{k_{B_h}}\cdots\theta_{B_{n-1}}^{k_{B_{n-1}}}}\rho$.

V. CONCLUSIONS

In this paper we investigate two stochastic models of scheduling problems $1\|\sum U_j$ and $F_2\,|\,d_j=d\,|\sum U_j$. Compared to their stochastic counterparts proposed in [2], the new models have some advantages. The strict condition that all gamma distributions have same scale parameter, required by other models of problems $1\|\sum U_j$ and $F_2\,|\,d_j=d\,|\sum U_j$, is dropped here. Thus, the new models apply to a wider class of problems under a mild condition.

Theoretically speaking, we obtain approximate optimal solutions by models (12) and (13), rather than the exact optimal solutions obtained by models (5) and (6). Our models perform well in the situation that the gaps between μs are not too big. However, it is worth mentioning that the corresponding models proposed in [2] are special cases of our results. In fact, let $\theta_1=\theta_2=\cdots=\theta_n$ in model (12) and $\theta_{A_1}=\cdots\theta_{A_n}=\cdots=\theta_{B_1}=\cdots=\theta_{B_n}$ in model (13), respectively. It is easy to see that we obtain the same models proposed in [2] in this situation.

There are several directions for future research. The first one is to establish new stochastic versions of models of the problems $1\|\sum U_j$ and $F_2\,|\,d_j=d\,|\sum U_j$ with the processing times following other distribution functions. Also, develop stochastic versions of models $F_2\,|\,d_j=d\,|\sum U_j$ is interesting.

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