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Algebra gl(λ) Inside the Algebra of Differential Operators on the Real Line

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Abstract

The Lie algebra gl(λ) with $\lambda \in \mathbb{C}$, introduced by B L Feigin, can be embedded into the Lie algebra of differential operators on the real line (see [7]). We give an explicit formula of the embedding of gl(λ) into the algebra $\mathcal{D}_\lambda$ of differential operators on the space of tensor densities of degree λ on $\mathbb{R}$. Our main tool is the notion of projectively equivariant symbol of a differential operator.

1 Introduction

The Lie algebra gl(λ) ($\lambda \in \mathbb{C}$) was introduced by B L Feigin in [7] for calculation the cohomology of the Lie algebra of differential operators on the real line. The algebra gl(λ) is defined as the quotient of the universal enveloping algebra $\mathrm{U}(\mathrm{sl}_2)$ of sl_2 with respect to the ideal generated by the element $\Delta - \lambda(\lambda - 1)$, where Δ is the Casimir element of $\mathrm{U}(\mathrm{sl}_2)$. gl($\lambda$) is turned into a Lie algebra by the standard method of setting $[a, b] = ab - ba$.

According to Feigin, gl(λ) can be considered as an analogue of gl(n) for $n = \lambda \in \mathbb{N}$; it is also called the algebra of matrices of complex size, see also [13, 16, 17, 12].

We consider the space $\mathcal{D}_\lambda$ of all linear differential operators acting on tensor densities of degree λ on $\mathbb{R}$. One of the main results of [7] is the construction of an embedding $\mathrm{gl}(\lambda) \to \mathcal{D}_\lambda$.

The purpose of this paper is to give an explicit formula of this embedding. We also show that this embedding realizes the isomorphism of Lie algebras $\mathrm{gl}(\lambda) \cong \mathcal{D}_\lambda^{\mathrm{pol}}$ constructed in [1, 2], where $\mathcal{D}_\lambda^{\mathrm{pol}} \subset \mathcal{D}_\lambda$ is the subalgebra of differential operators with polynomial coefficients.

The main idea of this paper is to use the *projectively equivariant symbol* of a differential operator, that is an sl_2-equivariant way to associate a polynomial function on $T^*\mathbb{R}$ to a differential operator. The notion of projectively equivariant symbol was defined in [4, 15] and used in [8, 9, 10] for study of modules of differential operators.

2 Basic definitions

2.1 The Lie algebra gl(λ). Let $\mathrm{Vect}(\mathbb{R})$ be the Lie algebra of smooth vector fields on $\mathbb{R}$ with complex coefficients: $X = X(x)\partial$, where $X(x)$ is a smooth complex function of one real variable; $X(x) \in C^\infty(\mathbb{R}, \mathbb{C})$, and where $\partial = \frac{d}{dx}$. Consider the Lie algebra $\mathrm{sl}_2 \subset \mathrm{Vect}(\mathbb{R})$ generated by the vector fields

$$\left\{\partial, x\partial, x^2\partial\right\}. \tag{2.1}$$

Denote $e_i := x^i\partial$, $i = 0, 1, 2$, the Casimir element

$$\Delta := e_1^2 - \frac{1}{2}(e_0e_2 + e_2e_0)$$

generates the center of $\mathrm{U}(\mathrm{sl}_2)$. The quotient

$$\mathrm{gl}(\lambda) := \mathrm{U}(\mathrm{sl}_2)/(\Delta - \lambda(\lambda - 1)), \qquad \lambda \in \mathbb{C}$$

is naturally a Lie algebra containing sl_2.

2.2 Modules of differential operators on $\mathbb{R}$. Denote $\mathcal{D}$ the Lie algebra of linear differential operators on $\mathbb{R}$ with complex coefficients:

$$A = a_n(x)\partial^n + a_{n-1}(x)\partial^{n-1} + \cdots + a_0(x), \tag{2.2}$$

with $a_i(x) \in C^\infty(\mathbb{R}, \mathbb{C})$.

For $\lambda \in \mathbb{C}$, $\mathrm{Vect}(\mathbb{R})$ is embedded into the Lie algebra $\mathcal{D}$ by:

$$X \mapsto L_X^\lambda := X(x)\partial + \lambda X'(x). \tag{2.3}$$

Denote $\mathcal{D}_\lambda$ the $\mathrm{Vect}(\mathbb{R})$-module structure with respect to the adjoint action of $\mathrm{Vect}(\mathbb{R})$ on $\mathcal{D}$. The module $\mathcal{D}_\lambda$ has a natural filtration: $\mathcal{D}_\lambda^0 \subset \mathcal{D}_\lambda^1 \subset \cdots \subset \mathcal{D}_\lambda^n \subset \cdots$, where $\mathcal{D}_\lambda^n$ is the module of n-th order differential operators (2.2).

Geometrically speaking, differential operators are acting on tensor densities, namely: $A : \mathcal{F}_\lambda \to \mathcal{F}_\lambda$, where $\mathcal{F}_\lambda$ is the space of tensor densities of degree λ on $\mathbb{R}$ (i.e., of sections of the line bundle $(T^*\mathbb{R})^{\otimes\lambda}, \lambda \in \mathbb{C}$), that is: $\phi = \phi(x)(dx)^\lambda$, where $\phi(x) \in C^\infty(\mathbb{R}, \mathbb{C})$.

It is evident that $\mathcal{F}_\lambda \cong C^\infty(\mathbb{R}, \mathbb{C})$ as linear spaces (but not as modules) for any λ. We use this identification throughout this paper. The Lie algebra structures of differential operators acting on the space of tensor densities and on the space of functions are also identified (see [8]).

The $\mathrm{Vect}(\mathbb{R})$-modules $\mathcal{D}_\lambda$ were considered by classics (see [3, 18]) and, recently, studied in a series of papers [5, 9, 8, 10, 14].

2.3 Principal symbol. Let $\mathrm{Pol}(T^*\mathbb{R})$ be the space of functions on $T^*\mathbb{R}$ polynomial in the fibers. This space is usually considered as the space of symbols associated to the space of differential operators on $\mathbb{R}$.

Recall that the *principal symbol* of a differential operator is the linear map $\sigma : \mathcal{D} \to \mathrm{Pol}(T^*\mathbb{R})$ defined by:

$$\sigma(A) = a_n(x)\xi^n,$$

where A is a differential operator (2.2) and ξ is the coordinate on the fiber.

One can also speak about the principal symbol of an element of $\mathrm{U}(\mathrm{sl}_2)$. Indeed, $\mathrm{U}(\mathrm{sl}_2)$ is canonically identified with the symmetric algebra $S(\mathrm{sl}_2)$ as sl_2-modules (see, e.g., [6, p.82]). Using the realization (2.1), the algebra $S(\mathrm{sl}_2)$ can be projected to $\mathrm{Pol}(T^*\mathbb{R})$. Therefore, one can define in a natural way the principal symbol on $S(\mathrm{sl}_2)$.

Our goal is to construct an sl_2-equivariant linear map $T_\lambda : \mathrm{U}(\mathrm{sl}_2) \to \mathcal{D}_\lambda$ which preserves the principal symbol, i.e., such that the following diagram commutes:

$$\begin{array}{ccc} \mathrm{U}(\mathrm{sl}_2) & \xrightarrow{T_\lambda} & \mathcal{D}_\lambda \\ \sigma\big\downarrow & & \big\downarrow\sigma \\ \mathrm{Pol}(T^*\mathbb{R}) & \xrightarrow{id} & \mathrm{Pol}(T^*\mathbb{R}) \end{array}$$

2.4 Projectively equivariant symbol. Viewed as a $\mathrm{Vect}(\mathbb{R})$-module, the space of symbols corresponding to $\mathcal{D}_\lambda$ has the form:

$$\mathrm{Pol}(T^*\mathbb{R}) \cong \mathcal{F}_0 \oplus \mathcal{F}_1 \oplus \cdots \oplus \mathcal{F}_n \oplus \cdots . \tag{2.4}$$

The space of polynomials of degree $\leq n$ is a submodule of $\mathrm{Pol}(T^*\mathbb{R})$ which we denote $\mathrm{Pol}_n(T^*\mathbb{R})$.

The following result of [8] allows one to identify, for arbitrary λ, $\mathcal{D}^n_\lambda$ with $\mathrm{Pol}_n(T^*\mathbb{R})$ as sl_2-modules:

(i) There exists a unique $\mathrm{sl}(2,\mathbb{R})$-isomorphism $\sigma_\lambda : \mathcal{D}^n_\lambda \to \mathrm{Pol}_n(T^*\mathbb{R})$ preserving the principal symbol.

(ii) σ_λ associates to each differential operator A the polynomial $\sigma_\lambda(A) = \sum\limits_{p=0}^{n} \bar{a}_p(x)\xi^p$, defined by:

$$\bar{a}_p(x) = \sum_{j=p}^{n} \alpha^j_p a_j^{(j-p)}, \tag{2.5}$$

where the constants α^j_p are given by:

$$\alpha^j_p = \frac{\binom{j}{p}\binom{2\lambda-p}{j-p}}{\binom{j+p+1}{2p+1}}$$

(the binomial coefficient $\binom{\lambda}{j} = \lambda(\lambda-1)\cdots(\lambda-j+1)/j!$ is a polynomial in λ).

The isomorphism σ_λ is called the *projectively equivariant symbol map*. Its explicit formula was first found in [4, 15] in the general case of pseudo-differential operators on a one-dimensional manifold (see also [15] for the multi-dimensional case).

3 Main result

In this section, we give the main result of this paper. We adopt the following notations:

$$[L^\lambda_{X_1} L^\lambda_{X_2} \cdots L^\lambda_{X_n}]_+ := \sum_{\tau\in S_n} L^\lambda_{X_{\tau(1)}} \circ L^\lambda_{X_{\tau(2)}} \circ \cdots \circ L^\lambda_{X_{\tau(n)}}$$

for a symmetric n-linear map from $\mathrm{Vect}(\mathbb{R})$ to $\mathcal{D}$ and

$$(X_1X_2\cdots X_n)_+ := \sum_{\tau\in S_n} X_{\tau(1)}X_{\tau(2)}\cdots X_{\tau(n)}$$

for a symmetric n-linear map from sl_2 to $\mathrm{U}(\mathrm{sl}_2)$, where S_n is the group of permutations of n elements and $X_i \in \mathrm{sl}_2$.

Theorem 1. *(i) For arbitrary $\lambda \in \mathbb{C}$, there exists a unique* sl_2*-equivariant linear map preserving the principal symbol:*

$$T_\lambda : \mathrm{U}(\mathrm{sl}_2) \to \mathcal{D}_\lambda$$

defined by

$$T_\lambda((X_1X_2\cdots X_n)_+) = [L^\lambda_{X_1}L^\lambda_{X_2}\cdots L^\lambda_{X_n}]_+, \tag{3.1}$$

where $X_i \in \{e_0, e_1, e_2\}$, $L^\lambda_{X_i}$ given by (2.3) and $n = 1, 2, \ldots$.

(ii) The operator T_λ is given in term of the sl_2*-equivariant symbol (2.5) by:*

$$\sigma_\lambda([L^\lambda_{X_1}L^\lambda_{X_2}\cdots L^\lambda_{X_n}]_+) = \sum_{\substack{0\le k\le n\\ k\ \mathit{even}}} P^n_k(\lambda)\mathcal{A}_k(X_1,\ldots,X_n)\xi^{n-k}, \tag{3.2}$$

where

$$\mathcal{A}_k(X_1,\ldots,X_n) = \sum_{2p+m=k} \binom{k/2}{p}(-2)^p(X_1''\ldots X_p''X_{p+1}'\cdots X_{p+m}'X_{p+m+1}\cdots X_n)_+ \tag{3.3}$$

and

$$P^n_k(\lambda) = \sum_{p=0}^{n}\sum_{l=n-k}^{n}(l-n+k)!\,\frac{\binom{l}{n-k}^2\binom{2\lambda-n+k}{l-n+k}}{\binom{n-k+l+1}{2n-2k+1}}\binom{n}{p}\left\{{p \atop l}\right\}\lambda^{n-p}, \tag{3.4}$$

where $\left\{{p \atop l}\right\}$ is the Stirling number of the second kind[1]*.*

It is worth noticing that the linear map T_λ does not depend on the choice of the PBW-base in $\mathrm{U}(\mathrm{sl}_2)$.

4 Proof of Theorem 1

By construction, the linear map T_λ is sl_2-equivariant.

4.1 sl_2-invariant symmetric differential operators. To prove part (ii) of Theorem 1 one needs the following

[1] We refer to [11] as a nice elementary introduction to the combinatorics of the Stirling numbers.

Proposition 1. *For arbitrary $\mu \in \mathbb{C}$ and $n = 1, 2, \ldots$, there exists at most one, up to proportionality, sl_2-equivariant symmetric operator $\otimes^n \mathrm{sl}_2 \to \mathcal{F}_\mu$ which is differential with respect to the vector fields $X_i \in \mathrm{sl}_2$. This operator exists if and only if $\mu = k - n$, where k is an even positive integer. It is denoted: $\mathcal{A}_k : \otimes^n \mathrm{sl}_2 \to \mathcal{F}_{k-n}$, and defined by the expression (3.3).*

Proof. Each k-th order differential operator $\mathcal{A} : \otimes^n \mathrm{sl}_2 \to \mathcal{F}_\mu$ is of the form:

$$\mathcal{A}(X_1, \ldots, X_n) = \sum_{2p+m=k} \beta_p(x) (X_1'' \cdots X_p'' X_{p+1}' \cdots X_{p+m}' X_{p+m+1} \cdots X_n)_+,$$

where $\beta_p(x)$ are some functions.

The condition of sl_2-equivariance for $\mathcal{A}$ reads as follows:

$$X[\mathcal{A}(X_1, \ldots, X_n)]' + \mu X' \mathcal{A}(X_1, \ldots, X_n) = \sum_{i=1}^n \mathcal{A}(X_1, \ldots, L_X^{-1}(X_i), \ldots, X_n),$$

where $X \in \mathrm{sl}_2$.

Substitute $X = \partial$ to check that the coefficients $\beta_p(x)$ do not depend on x. Substitute $X = x\partial$ to obtain the condition $\mu = k - n$. At last, substitute $X = x^2\partial$ and put $\beta_0 = 1$ to obtain, for even k, the coefficients from (3.3). If k is odd, one obtains $\beta_p = 0$ for all p.

Proposition 1 is proven. ■

The general form (3.2) is a consequence of Proposition 1 and decomposition (2.4).

4.2 Polynomials $P_k^n(\lambda)$. To compute the polynomials P_k^n, put $X_1 = \cdots = X_n = x\partial$. One readily gets, from (3.2),

$$\sigma_\lambda(T_\lambda(X_1, \ldots, X_n))|_{x=1} = n! \sum_{\substack{0 \le k \le n \\ k \text{ even}}} P_k^n(\lambda)\, \xi^{n-k}. \tag{4.1}$$

Furthermore, using the well-known expression $(x\partial)^n = \sum\limits_{l=0}^n \left\{ {n \atop l} \right\} x^l \partial^l$, one has:

$$\begin{aligned} T_\lambda(X_1, \ldots, X_n) &= n!\, (x\partial + \lambda)^n \\ &= n! \sum_{p=0}^n \binom{n}{p} (x\partial)^n \lambda^{n-p} = n! \sum_{p=0}^n \sum_{l=0}^n \binom{n}{p} \left\{ {n \atop l} \right\} x^l \partial^l \lambda^{n-p}. \end{aligned}$$

A straightforward computation gives the projectively equivariant symbol (2.5) of this differential operator:

$$\begin{aligned} &\sigma_\lambda(T_\lambda(X_1, \ldots, X_n))|_{x=1} \\ &\quad = n! \sum_{\substack{0 \le k \le n \\ k \text{ even}}} \sum_{p=0}^n \sum_{l=n-k}^n (l - n + k)! \frac{\binom{l}{n-k}^2 \binom{2\lambda - n + k}{l - n + k}}{\binom{n-k+l+1}{2n-2k+1}} \binom{n}{p} \left\{ {p \atop l} \right\} \lambda^{n-p} \xi^{n-k}. \end{aligned}$$

Compare with the equality (4.1) to obtain the formulae from (3.4).

Theorem 1 (ii) is proven.

4.3 Uniqueness. Let T be an sl_2-equivariant linear map $\mathrm{U}(\mathrm{sl}_2) \to \mathcal{D}_\lambda$ for a certain $\lambda \in \mathbb{C}$. In view of the decomposition (2.4), it follows from Proposition 1 that $\sigma_\lambda \circ T|_{\mathcal{F}_k} = c_k(\lambda)\mathcal{A}_k$, where $c_k(\lambda)$ is a constant depending on λ. Recall that $\mathrm{Pol}_n(T^*\mathbb{R})$ is a *rigid* sl_2-module, i.e., every sl_2-equivariant linear map on $\mathrm{Pol}_n(T^*\mathbb{R})$ is proportional to the identity (see, e.g., [15]). Assuming, now, that T preserves the principal symbol, the rigidity of $\mathrm{Pol}_n(T^*\mathbb{R})$ fixes the constants $c_k(\lambda)$ in a unique way. Hence the uniqueness of T_λ.

Theorem 1 is proven.

5 The embedding $\mathrm{gl}(\lambda) \to \mathcal{D}_\lambda$

A corollary of the uniqueness of the operator T_λ and results of [1, 2, 7, 17] is that the embedding $\mathrm{gl}(\lambda) \to \mathcal{D}_\lambda$ constructed in [7] coincides with T_λ.

More precisely, according to results of [1, 2, 17], there exists a homomorphism of Lie algebras $p_\lambda : \mathrm{U}(\mathrm{sl}_2) \to \mathcal{D}_\lambda$ preserving the principal symbol. The homomorphism p_λ is, in particular, sl_2-equivariant. By uniqueness of T_λ, one has $T_\lambda = p_\lambda$. It is also proven that the kernel of p_λ is a two-sided ideal of $\mathrm{U}(\mathrm{sl}_2)$ generated by $\Delta - \lambda(\lambda-1)$ (see [1, 2]). Taking the quotient, one then has an embedding $\tilde{T}_\lambda : \mathrm{gl}(\lambda) \to \mathcal{D}_\lambda$. Since the embedding from [7] preserves the principal symbol, it is equal to $\tilde{T}_\lambda$. Finally, it is obvious that the image of T_λ is the subalgebra $\mathcal{D}_\lambda^{\mathrm{pol}} \subset \mathcal{D}_\lambda$ of differential operators with polynomial coefficients. Therefore, $\tilde{T}_\lambda : \mathrm{gl}(\lambda) \to \mathcal{D}_\lambda^{\mathrm{pol}}$ is a Lie algebras isomorphism.

6 Examples

As an illustration of Theorem 1, let us give the expressions of the general formulae (3.1) and (3.2) for the order $n = 1, 2, 3, 4, 5$. Let X_1, X_2, X_3, X_4 and X_5 be arbitrary vector fields in sl_2.

1) The sl_2-equivariant symbol, defined by (2.5), of a first order operator of a Lie derivative $L^\lambda_{X_1}$ is

$$\sigma_\lambda(L^\lambda_{X_1}) = X_1(x)\xi.$$

2) The "anti-commutator" $[L^\lambda_{X_1} L^\lambda_{X_2}]_+$ has the following projectively equivariant symbol:

$$\sigma_\lambda([L^\lambda_{X_1} L^\lambda_{X_2}]_+) = (X_1X_2)_+\xi^2 + \frac{1}{3}\lambda(\lambda-1)((X_1'X_2')_+ - 2(X_1''X_2)_+)$$

which also following from (2.5).

3) The projectively equivariant symbol of a third order expression $[L^\lambda_{X_1} L^\lambda_{X_2} L^\lambda_{X_3}]_+$ can be also easily calculated from (2.5). The result is:

$$\sigma_\lambda([L^\lambda_{X_1} L^\lambda_{X_2} L^\lambda_{X_3}]_+) = (X_1X_2X_3)_+\xi^3$$
$$+ \frac{1}{5}(3\lambda^2 - 3\lambda - 1)((X_1'X_2'X_3)_+ - 2(X_1''X_2X_3)_+)\xi.$$

4) Direct calculation from (2.5) gives the projectively equivariant symbol of a fourth order expression $[L^\lambda_{X_1}L^\lambda_{X_2}L^\lambda_{X_3}L^\lambda_{X_4}]_+$, that is:

$$
\begin{aligned}
&\sigma_\lambda([L^\lambda_{X_1}L^\lambda_{X_2}L^\lambda_{X_3}L^\lambda_{X_4}]_+) = (X_1X_2X_3X_4)_+\xi^4 \\
&\quad + \frac{1}{7}(6\lambda^2-6\lambda-5)((X_1'X_2'X_3X_4)_+ - 2(X_1''X_2X_3X_4)_+)\xi^2 \\
&\quad + \frac{1}{15}\lambda(\lambda-1)(3\lambda^2-3\lambda-1)((X_1'X_2'X_3'X_4')_+ - 4(X_1''X_2'X_3'X_4)_+ \\
&\quad + 4(X_1''X_2''X_3X_4)_+).
\end{aligned}
$$

5) In the same manner, one can easily check that the sl_2-equivariant symbol of a fifth order expression $[L^\lambda_{X_1}L^\lambda_{X_2}L^\lambda_{X_3}L^\lambda_{X_4}L^\lambda_{X_4}]_+$ is:

$$
\begin{aligned}
&\sigma_\lambda([L^\lambda_{X_1}L^\lambda_{X_2}L^\lambda_{X_3}L^\lambda_{X_4}L^\lambda_{X_5}]_+) = (X_1X_2X_3X_4X_5)_+\xi^5 \\
&\quad + \frac{5}{9}(2\lambda^2-2\lambda-3)((X_1'X_2'X_3X_4X_5)_+ - 2(X_1''X_2X_3X_4X_5)_+)\xi^3 \\
&\quad + \frac{1}{7}(3\lambda^4-6\lambda^3+3\lambda+1)((X_1'X_2'X_3'X_4'X_5)_+ - 4(X_1''X_2'X_3'X_4X_5)_+ \\
&\quad + 4(X_1''X_2''X_3X_4X_5)_+)\xi.
\end{aligned}
$$

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