On Some Predictive Ratio Type Estimators in Two Stage Sampling

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Some predictive ratio type estimators in two stage sampling are derived following the model free predictive criterion proposed by Basu (1971) and these estimators are compared with the classical multistage ratio estimator and also with Smith's (1969) multistage ratio estimator. A numerical illustration is provided to compare the efficiencies of different predictive estimators.

Keywords: Two stage sampling, Predictive estimator, Ratio Estimator.

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1. Introduction

Let a finite population U be partitioned into N clusters of first stage units (f.s.u.) denoted by $(U_1, U_2, \ldots, U_N)$ such that the number of second stage units (s.s.u.) in U_i be M_i $(i = 1, 2, \ldots, N)$. Further, let y_{ij} and x_{ij} be the values of the study variable (y) and the auxiliary variable (x) respectively corresponding to jth s.s.u. in ith f.s.u. $(i = 1, 2, \ldots, N;\ j = 1, 2, \ldots, M_i)$.

Define

$$\overline{Y}_i = \frac{1}{M_i}\sum_{j=1}^{M_i} y_{ij}, \quad \overline{X}_i = \frac{1}{M_i}\sum_{j=1}^{M_i} x_{ij}, \quad Y = \sum_{i=1}^{N}\sum_{j=1}^{M_i} y_{ij},$$

$$\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} u_i \overline{Y}_i, \quad X = \sum_{i=1}^{N}\sum_{j=1}^{M_i} x_{ij}, \quad \text{and} \quad \overline{X} = \frac{1}{N}\sum_{i=1}^{N} u_i \overline{X}_i,$$

where $u_i = \dfrac{M_i}{\overline{M}}$, $\overline{M} = \dfrac{\sum_{i=1}^{N} M_i}{N}$.

Also, $R = \dfrac{\overline{Y}}{\overline{X}}$ and $R_i = \dfrac{\overline{Y}_i}{\overline{X}_i}$ $(i = 1, 2, \ldots, N)$.

Assume that a simple random sample (without replacement) s of size n f.s.u. is selected from U and then at the second stage a sample s_i of size m_i s.s.u. from the ith selected f.s.u. U_i $(i = 1,2,\ldots,n)$ is selected according to simple random sampling without replacement.

Now define $\bar{y}_i = \frac{1}{m_i}\sum_{j=1}^{m_i} y_{ij}$, $\bar{x}_i = \frac{1}{m_i}\sum_{j=1}^{m_i} x_{ij}$.

Further define

$$S^2_{by} = \frac{1}{N-1}\sum_{i=1}^{N}(u_i\overline{Y}_i - \overline{Y})^2,$$

$$S^2_{bx} = \frac{1}{N-1}\sum_{i=1}^{N}(u_i\overline{X}_i - \overline{X})^2,$$

$$S_{byx} = \frac{1}{N-1}\sum_{i=1}^{N}(u_i\overline{Y}_i - \overline{Y})(u_i\overline{X}_i - \overline{X}),$$

$$\rho_{byx} = \frac{S_{byx}}{S_{by}S_{bx}}, \quad C_{by} = \frac{S_{by}}{\overline{Y}}, \quad C_{bx} = \frac{S_{bx}}{\overline{X}}.$$

For each U_i $(i = 1,2,\ldots,N)$, define

$$S^2_{iy} = \frac{1}{M_i - 1}\sum_{j=1}^{M_i}(y_{ij} - \overline{Y}_i)^2,$$

$$S^2_{ix} = \frac{1}{M_i - 1}\sum_{j=1}^{M_i}(x_{ij} - \overline{X}_i)^2,$$

$$S_{iyx} = \frac{1}{M_i - 1}\sum_{j=1}^{M_i}(y_{ij} - \overline{Y}_i)(x_{ij} - \overline{X}_i),$$

$$\rho_{iyx} = \frac{S_{iyx}}{S_{iy}S_{ix}}, \quad C_{iy} = \frac{S_{iy}}{\overline{Y}_i}, \quad C_{ix} = \frac{S_{ix}}{\overline{X}_i}, \quad (i = 1,2,\ldots,N).$$

For any given sample 's' we write the total of y as

$$\begin{aligned} Y &= \sum_{i=1}^{N}\sum_{j=1}^{M_i} y_{ij} \\ &= \sum_{i=1}^{n}\sum_{j=1}^{m_i} y_{ij} + \sum_{i=1}^{n}\sum_{j=m_i+1}^{M_i} y_{ij} + \sum_{i=n+1}^{N}\sum_{j=1}^{M_i} y_{ij} \\ &= \sum_{i\in s}\sum_{j\in s_i} y_{ij} + \sum_{i\in s}\sum_{j\in \bar{s}_i} y_{ij} + \sum_{i\in \bar{s}}\sum_{j\in U_i} y_{ij}, \end{aligned} \tag{1.1}$$

where $s \cup \bar{s} = U$ and $s_i \cup \bar{s}_i = U_i$.

As the first term in (1.1) is already known from the observed sample, we predict only second and third terms in (1.1) by model free prediction approach of Basu (1971) using auxiliary information on x, to arrive at an estimate of Y.

Thus, we may predict each y_{ij}, $j \in \bar{s}_i$ using the ratio method by $T_i x_{ij}$ and each y_{ij}, $i \in \bar{s}$ and $j \in U_i$, using also the ratio method by $T\ x_{ij}$. It may be mentioned here that T_i is an estimate of ratio $\frac{\overline{Y}_i}{\overline{X}_i}$ $(i = 1,2,\ldots,N)$ from the ith first stage sample and T is an estimate of $\frac{\overline{Y}}{\overline{X}}$ from the entire sample.

Different choices of T_i and T lead to different predictive estimators in two stage sampling.

In the following we examine some of these choices to derive predictive estimators and compare them with the non-predictive estimators used in practice.

2. Predictive Estimators

Case I

Let $T_i = \dfrac{\overline{y}_i}{\overline{x}_i}$ and $T = \dfrac{\sum_{i=1}^n u_i \overline{y}_i}{\sum_{i=1}^n u_i \overline{x}_i}$.

Then the Basu type model free predictive estimator of $\overline{Y}$ earlier suggested by Panda (1998) is given by

$$t_{p1} = \frac{\sum_{i=1}^n u_i \overline{y}_i}{\sum_{i=1}^n u_1 \overline{x}_i} \overline{X} + f \left[\frac{1}{n} \sum_{i=1}^n u_i \frac{\overline{y}_i}{\overline{x}_i} \overline{X}_i - \frac{\sum_{i=1}^n u_i \overline{y}_i}{\sum_{i=1}^n u_i \overline{x}_i} \frac{1}{n} \left(\sum_{i=1}^n u_1 \overline{X}_i \right) \right], \tag{2.1}$$

where $f = \dfrac{n}{N}$.

Bias of t_{p1} is $o\left(\frac{1}{n}\right)$. Neglecting the bias for large samples, the approximate mean square error (MSE) of t_{p1} is given by

$$\begin{aligned} MSE(t_{p1}) = &\left(\frac{1}{n} - \frac{1}{N}\right) \left(S_{by}^2 + R^2 S_{bx}^2 - 2R S_{byx}\right) \\ &+ \frac{1}{nN} \sum_{i=1}^N u_i^2 \left(\frac{1}{m_i} - \frac{1}{M_i}\right) \left(S_{iy}^2 + \Delta_{1i}^2 S_{ix}^2 - 2\Delta_{1i} S_{ixy}\right), \end{aligned} \tag{2.2}$$

where $\Delta_{1i} = R - f(R - R_i)$, $i = 1, 2, \ldots, N$.

Case II

Let $T_i = \dfrac{\overline{y}_i}{\overline{X}_i}$ and $T = \dfrac{\sum_{i=1}^n u_i \overline{y}_i}{\sum_{i=1}^n u_i \overline{x}_i}$.

The predictive estimator of $\overline{Y}$ is thus given by

$$\begin{aligned} t_{p2} &= \frac{1}{N\overline{M}} \left[\sum_{i=1}^n m_i \overline{y}_i + \sum_{i=1}^n \frac{\overline{y}_i}{\overline{X}_i} \left(M_i \overline{X}_i - m_i \overline{x}_i\right) + \frac{\sum_{i=1}^n u_i \overline{y}_i}{\sum_{i=1}^n u_i \overline{x}_i} \left(X - \sum_{i=1}^n M_i \overline{X}_i \right) \right] \\ &= \frac{\sum_{i=1}^n u_i \overline{y}_i}{\sum_{i=1}^n u_i \overline{x}_i} \overline{X} + f \left[\frac{1}{n} \sum_{i=1}^n u_i \overline{y}_i + \frac{1}{n} \sum_{i=1}^n f_i u_i \left(\overline{y}_i - \frac{\overline{y}_i \overline{x}_i}{\overline{X}_i} \right) - \frac{\sum_{i=1}^n u_i \overline{y}_i}{\sum_{i=1}^n u_i \overline{x}_i} \cdot \frac{1}{n} \sum_{i=1}^n u_i \overline{X}_i \right], \end{aligned} \tag{2.3}$$

where $f_i = \dfrac{m_i}{M_i}$ $(i = 1, 2, \ldots, n)$.

To the first order of approximation

$$MSE(t_{p2}) = \left(\frac{1}{n} - \frac{1}{N}\right)\left(S_{by}^2 + R^2 S_{bx}^2 - 2RS_{byx}\right) + \frac{1}{nN}\sum_{i=1}^{N} u_i^2 \left(\frac{1}{m_i} - \frac{1}{M_i}\right)\left(S_{iy}^2 + \Delta_{2i}^2 S_{ix}^2 - 2\Delta_{2i} S_{ixy}\right), \qquad (2.4)$$

where $\Delta_{2i} = R - f(R - f_i R_i)$, $i = 1,2,\ldots,N$.

Case III

Let $T_i = \dfrac{\overline{y}_i}{\overline{x}_i}$ and $T = \dfrac{\sum_{i=1}^n u_i \overline{y}_i}{\sum_{i=1}^n u_i \overline{X}_i}$.

In such a situation the predictive estimator of $\overline{Y}$ is given by

$$t_{p3} = \frac{\sum_{i=1}^{n} u_i \overline{y}_i}{\sum_{i=1}^{n} u_i \overline{X}_i}\overline{X} + f\left[\frac{1}{n}\sum_{i=1}^{n} u_i \frac{\overline{y}_i}{\overline{x}_i}\overline{X}_i - \frac{1}{n}\sum_{i=1}^{n} u_i \overline{y}_i\right]. \qquad (2.5)$$

The mean square error of t_{p3} to o$\left(\frac{1}{n}\right)$ is given by

$$MSE(t_{p3}) = \left(\frac{1}{n} - \frac{1}{N}\right)\left(S_{by}^2 + R^2 S_{bx}^2 - 2RS_{byx}\right) + \frac{1}{nN}\sum_{i=1}^{N} u_i^2 \left(\frac{1}{m_i} - \frac{1}{M_i}\right)\left(S_{iy}^2 + \Delta_{3i}^2 S_{ix}^2 - 2\Delta_{3i} S_{ixy}\right), \qquad (2.6)$$

where $\Delta_{3i} = fR_i$, $i = 1,2,\ldots,N$.

Case IV

Let $T_i = \dfrac{\overline{y}_i}{\overline{X}_i}$ and $T = \dfrac{\sum_{i=1}^n u_i \overline{y}_i}{\sum_{i=1}^n u_i \overline{X}_i}$.

The predictive estimator of $\overline{Y}$ in such a situation takes the form

$$t_{p4} = \frac{\sum_{i=1}^{n} u_i \overline{y}_i}{\sum_{i=1}^{n} u_i \overline{X}_i}\overline{X} + \frac{1}{n}\sum_{i=1}^{n} f f_i u_i \left(\overline{y}_i - \frac{\overline{y}_i \overline{x}_i}{\overline{X}_i}\right).$$

To the first order of approximation

$$MSE(t_{p4}) = \left(\frac{1}{n} - \frac{1}{N}\right)\left(S_{by}^2 + R^2 S_{bx}^2 - 2RS_{byx}\right) + \frac{1}{nN}\sum_{i=1}^{N} u_i^2 \left(\frac{1}{m_i} - \frac{1}{M_i}\right)\left(S_{iy}^2 + \Delta_{4i}^2 S_{ix}^2 - 2\Delta_{4i} S_{ixy}\right), \qquad (2.7)$$

where $\Delta_{4i} = f f_i R_i$, $i = 1,2,\ldots,N$.

Case V

Let $T_i = \dfrac{\overline{y}_i}{\overline{x}_i}$ and $T = \dfrac{\sum_{i=1}^{n} u_i \dfrac{\overline{y}_i}{\overline{x}_i}\overline{X}_i}{\sum_{i=1}^{n} u_i \overline{X}_i}$.

The predictive estimator of $\overline{Y}$ in this case is

$$t_{p5} = \frac{1}{N\overline{M}}\left[\sum_{i=1}^{n} m_i\overline{y}_i + \sum_{i=1}^{n}\frac{\overline{y}_i}{\overline{x}_i}\left(M_i\overline{X}_i - m_i\overline{x}_i\right) + \frac{\sum\limits_{i=1}^{n} u_i\frac{\overline{y}_i}{\overline{x}_i}\overline{X}_i}{\sum\limits_{i=1}^{n} u_i\overline{X}_i}\left(\overline{X} - \sum_{i=1}^{n} M_i\overline{X}_i\right)\right] = \frac{\sum\limits_{i=1}^{n} u_i\frac{\overline{y}_i}{\overline{x}_i}\overline{X}_i}{\sum\limits_{i=1}^{n} u_i\overline{X}_i}\overline{X}. \quad (2.8)$$

It may be seen that t_{p5} is internally congruent estimator in the words of Sampford (1978), as the form of t_{p5} is equivalent to T. Incidentally t_{p5} happens to be Murthy's (1967) chain ratio estimator.

To the first order of approximation

$$MSE(t_{p5}) = \left(\frac{1}{n} - \frac{1}{N}\right)\left(S_{by}^2 + R^2 S_{bx}^2 - 2RS_{byx}\right) + \frac{1}{nN}\sum_{i=1}^{N} u_i^2\left(\frac{1}{m_i} - \frac{1}{M_i}\right)\left(S_{iy}^2 + R_i^2 S_{ix}^2 - 2R_i S_{iyx}\right) \quad (2.9)$$

3. Comparison of predictive estimators

When comparing the predictive estimators among themselves it is worthwhile to compare them with an estimator without auxiliary information, classical two stage ratio estimator and Smith's (1969) two stage ratio estimator. Thus an unbiased estimator in two stage sampling without using auxiliary information is given by

$$\overline{y} = \frac{1}{n}\sum_{i=1}^{n} u_i\overline{y}_i \quad (3.1)$$

with

$$V(\overline{y}) = \left(\frac{1}{n} - \frac{1}{N}\right)S_{by}^2 + \frac{1}{nN}\sum_{i=1}^{N} u_i^2\left(\frac{1}{m_i} - \frac{1}{M_i}\right)S_{iy}^2. \quad (3.2)$$

Further the classical ratio estimator in two stage sampling using auxiliary information provided by x is given by

$$\overline{y}_R = \frac{\sum\limits_{i=1}^{n} u_i\overline{y}_i}{\sum\limits_{i=1}^{n} u_i\overline{x}_i}\overline{X}. \quad (3.3)$$

To the first order of approximation,

$$MSE(\overline{y}_R) = \left(\frac{1}{n} - \frac{1}{N}\right)\left(S_{by}^2 + R^2 S_{bx}^2 - 2RS_{byx}\right) + \frac{1}{nN}\sum_{i=1}^{N} u_i^2\left(\frac{1}{m_i} - \frac{1}{M_i}\right)\left(S_{iy}^2 + R^2 S_{ix}^2 - 2RS_{iyx}\right) \quad (3.4)$$

Smith's (1969) multistage ratio estimator is given by

$$\bar{y}'_R = \frac{\sum_{i=1}^{n} u_i \bar{y}_i}{\sum_{i=1}^{n} u_i \bar{X}_i} \bar{X}. \tag{3.5}$$

To the first order of approximation,

$$MSE\left(\bar{y}'_R\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\left(S_{by}^2 + R^2 S_{bx}^2 - 2RS_{byx}\right) + \frac{1}{nN}\sum_{i=1}^{N} u_i^2 \left(\frac{1}{m_i} - \frac{1}{M_i}\right) S_{iy}^2 \tag{3.6}$$

(i) The sufficient conditions when the predictive estimators t_{pi} $(i = 1, 2, \ldots, 5)$ would be more efficient than the unbiased estimator $\bar{y}$ are shown in Table 1.

Table 1. Comparison of predictive estimators with $\bar{y}$

Predictive Estimators	**Sufficient Conditions**
t_{p1}	(i) $\rho_{byx} > \frac{1}{2}\frac{C_{bx}}{C_{by}}$ (ii) $\beta_{iyx} > \frac{1}{2}[R - f(R - R_i)]$, for all $i \in U$
t_{p2}	(i) $\rho_{byx} > \frac{1}{2}\frac{C_{bx}}{C_{by}}$ (ii) $\beta_{iyx} > \frac{1}{2}[R - f(R - f_i R_i)]$, for all $i \in U$
t_{p3}	(i) $\rho_{byx} > \frac{1}{2}\frac{C_{bx}}{C_{by}}$ (ii) $\beta_{iyx} > \frac{1}{2} f R_i$, for all $i \in U$
t_{p4}	(i) $\rho_{byx} > \frac{1}{2}\frac{C_{bx}}{C_{by}}$ (ii) $\beta_{iyx} > \frac{1}{2} f f_i R_i$, for all $i \in U$
t_{p5}	(i) $\rho_{byx} > \frac{1}{2}\frac{C_{bx}}{C_{by}}$ (ii) $\beta_{iyx} > \frac{1}{2} R_i$, for all $i \in U$

(ii) The sufficient conditions when the predictive estimators t_{pi} $(i = 1, 2, \ldots, 5)$ would be more efficient than $\bar{y}_R$ are shown in Table 2.

Table 2. Comparison of efficiencies predictive estimators t_{pi} $(i = 1,2,\ldots,5)$ with $\overline{y}_R$

Predictive Estimators	**Sufficient Conditions**
t_{p1}	$\beta_{iyx} < \frac{1}{2}[2R - f(R - R_i)]$; $R > R_i$, for all $i \in U$ or $\beta_{iyx} > \frac{1}{2}[2R - f(R - R_i)]$; $R < R_i$, for all $i \in U$
t_{p2}	$\beta_{iyx} < \frac{1}{2}[2R - f(R - f_iR_i)]$; $R > f_iR_i$, for all $i \in U$ or $\beta_{iyx} > \frac{1}{2}[2R - f(R - f_iR_i)]$; $R < f_iR_i$, for all $i \in U$
t_{p3}	$\beta_{iyx} < \frac{1}{2}(R + fR_i)$; if $R > fR_i$, for all $i \in U$ or $\beta_{iyx} > \frac{1}{2}(R + fR_i)$; if $R < fR_i$, for all $i \in U$
t_{p4}	$\beta_{iyx} < \frac{1}{2}(R + ff_iR_i)$; $R > ff_iR_i$, for all $i \in U$ or $\beta_{iyx} > \frac{1}{2}(R + ff_iR_i)$; $R < ff_iR_i$, for all $i \in U$
t_{p5}	If $\beta_{iyx} < \frac{1}{2}R_i$, $R > R_i$, for all $i \in U$ or $\beta_{iyx} > \frac{1}{2}R_i$, $R_i > R$, for all $i \in U$

(iii) The sufficient conditions when the predictive estimators t_{pi} $(i = 1,2,\ldots,5)$ would be more efficient than $\overline{y}'_R$ are shown in Table 3.

Table 3. Comparison of predictive ratio type estimators with $\overline{y}'_R$

Predictive Estimators	**Sufficient Conditions**
t_{p1}	$\beta_{iyx} > \frac{1}{2}[R - f(R - R_i)]$, for all $i \in U$
t_{p2}	$\beta_{iyx} > \frac{1}{2}[R - f(R - f_iR_i)]$, for all $i \in U$
t_{p3}	$\beta_{iyx} > \frac{1}{2}fR_i$, for all $i \in U$
t_{p4}	$\beta_{iyx} > \frac{1}{2}ff_iR_i$, for all $i \in U$
t_{p5}	$\beta_{iyx} > \frac{1}{2}R_i$, for all $i \in U$

Note

(i) If $\beta_{iyx} > \frac{1}{2}[R - f(R - R_i)]$ for all $i \in U$, both t_{p1} and t_{p2} are more efficient than $\overline{y}'_R$.

(ii) If $\beta_{iyx} > \frac{1}{2}R_i$, for all $i \in U$, t_{p3}, t_{p4} and t_{p5} are more efficient than $\overline{y}'_R$.

Now, it would be worthwhile to compare the internally congruent predictive estimator t_{p5} with non-congruent predictive estimators – t_{p1}, t_{p2}, t_{p3} and t_{p4}. Thus, t_{p5} would be more efficient than

(i) t_{p1}, if for all $i \in U$,

$$\beta_{iyx} < \frac{1}{2}[(1-f)R + (1+f)R_i], \;\; R > R_i, \;\; \text{or}$$
$$\beta_{iyx} > \frac{1}{2}[(1-f)R + (1+f)R_i], \;\; R < R_i.$$

(ii) t_{p2}, if for all $i \in U$,

$$\beta_{iyx} < \frac{1}{2}[(1-f)R + (1+ff_i)R_i], \;\; R > \frac{1-ff_i}{1-f}R_i, \;\; \text{or}$$
$$\beta_{iyx} > \frac{1}{2}[(1-f)R + (1+ff_i)R_i], \;\; R < \frac{1-ff_i}{1-f}R_i.$$

(iii) t_{p3}, if for all $i \in U$,

$$\beta_{iyx} < \frac{1}{2}(1+f)R_i\,.$$

(iv) t_{p4}, if for all $i \in U$,

$$\beta_{iyx} < \frac{1}{2}(1+ff_i)R_i\,.$$

4. Numerical Illustration

To illustrate efficiencies of different predictive and non-predictive estimators we consider 1971 population census data of Odisha (India).

The population consists of 104 blocks (ssu) divided into $N = 15$ wards (fsu) of Berhampur City of Odisha (India). The number of blocks (M_i) in 15 wards are 6, 6, 12, 5, 6, 6,10, 5, 6, 6, 6, 6, 6, 12 and 6. The two variables i.e., number of educated females and female population are denoted by y and x respectively. For comparison of asymptotic mean square errors (MSE) of $\overline{y}_R$, $\overline{y}'_R$, t_{p1}, t_{p2}, t_{p3}, t_{p4} and t_{p5} we consider first stage sample of size $n = 5$. The sizes of the second stage samples m_i $(i = 1, 2, \ldots, 15)$ are fixed at 2, 2, 4, 2, 2, 2, 3, 2, 3, 3, 2, 2, 2, 2, 4, and 3 respectively.

The efficiencies of different estimators under comparison are compared in Table 4.

Table 4. Comparison of Efficiencies

Estimator	**MSE**	**Efficiency (%)**
$\overline{y}$	302.9529	100
$\overline{y}_R$	222.0110	136.4585
$\overline{y}'_R$	228.4768	132.5968
t_{p1}	223.1162	135.7826
t_{p2}	222.2914	136.2864
t_{p3}	224.6903	134.8313
t_{p4}	226.9665	133.4791
t_{p5}	221.9455	136.4988

5. Conclusion

The predictive ratio estimators including Murthy's chain ratio estimator (internally congruent predictive estimator) though logical in nature, are not uniformly more efficient than both the classical two stage ratio estimator $\overline{y}_R$ and Smith's two stage ratio estimator $\overline{y}'_R$ and their superiority depends on certain sufficient conditions.

It is observed from the given illustration that t_{p5} (internally congruent predictive estimator) is the most efficient among the estimators under comparison, although the increase in efficiency is only marginal.

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