An Backbone Guided Extremal Optimization Method for Solving the Hard Maximum Satisfiability Problem

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Abstract-**The original Extremal Optimization (EO) algorithm and its modified versions have been successfully applied to a variety of NP-hard optimization problems. However, almost all existing EO-based algorithms have overlooked the inherent structural properties behind the optimization problems, e.g., the backbone information. This paper presents a novel stochastic local search method called *Backbone Guided Extremal Optimization* (BGEO) to solve the hard maximum satisfiability (MAX-SAT) problem, one of typical NP-hard problems. The key idea behind the proposed method is to incorporate the backbone information into a recent developed optimization algorithm termed extremal optimization (EO) to guide the entire search process approach the optimal solutions. The superiority of BGEO to the reported BE-EEO algorithm without backbone information is demonstrated by the experimental results on the hard Max-SAT instances.**

Keywords-Backbone, Extremal optimization, Maximum satisfiability problem, Phase transition

I. INTRODUCTION

Extremal optimization [1,2] was originally inspired by far-from-equilibrium dynamics of Bak-Sneppen model [3] of biological co-evolution, showing the features of self-organized criticality (SOC) [4]. This method provides a novel insight into the optimization domain as its novel evolutionary mechanism that merely selects against the bad, instead of favoring the good, randomly or according to a power-law distribution. Beneficial from this evolutionary mechanism, the search dynamics of the EO-based algorithms has non-equilibrium feature. During the past decade, the basic EO algorithm and its modified versions have been successfully applied to a variety of NP-hard optimization problems.The more details are investigated in the surveys on extremal optimization [5,6]. However, almost all existing EO-based algorithms have overlooked the inherent structural properties behind the optimization problems, e.g., backbone information.

In fact, the computational complexity of an optimization problem depends on not only its dimension, but also some inherent structural properties, e.g., backbone. As one of the most interesting and important structures, backbone has been used to explain the difficulty of problem instances [7-9]. The problems with larger backbone is generally harder for local search algorithms to solve because the clustered solutions in these problems result in making mistakes more easily and wasting time for searching empty subspaces before correcting the bad assignments [7]. On the other hand, the utilization of the backbone information may help the design of effective and efficient optimization algorithms [10-12].

In this paper, we focus on a novel method called *backbone guided extremal optimization* (BGEO) for the hard MAX-SAT problem, a well-known NP-hard optimization problem. The key idea behind the proposed method is to incorporate the backbone information into a recent developed optimization algorithm termed extremal optimization (EO) to guide the entire search process approach the optimal solutions. The remaining of this paper is organized as follows. Section 2 introduces MAX-SAT problem. Then, we present the framework of BGEO in section 3. Section 4 gives the experimental results on the hard MAX-SAT instances. Finally, the conclusion and the future work are given in section 5.

II. MAX-SAT PROBLEM

As the optimization counterpart of the Boolean satisfiability (SAT) problem, the MAX-SAT problem is one of well-studied NP-hard optimization problems [13]. The Max-SAT problem can be described as quadruple (**X**, **C**, **W**, W_c).

--X=$\{x_1, x_2, \ldots, x_n\}$ is a set of Boolean variables, where x_i can take the value 0 (false) or the value 1 (true).

--C={C_1, C_2, …, C_m} is a set of clauses, each of which is a disjunction of literals l_{ij} and l_{ij} is a Boolean variable x_i or its negation $\overline{x_i}$.

--W=(w_i)∈ N^m is an integer vector and w_i is the weight of the clause C_i.

The weighted MAX-SAT problem can be defined to find an assignment S to maximize the total weight of the satisfied clauses, i.e., minimize the total weight of unsatisfied clauses,

$$W_c(\mathbf{C}, \mathbf{W}, S) = \max \sum_{C_i(S)=1} w_i = \min \sum_{C_j(S)=0} w_j \quad (1)$$

If each clause consists of K literals, where K is a positive constant, we call this problem as MAX-K-SAT. Specially, when w_i=1 for each clause, the problem is called unweighted MAX-SAT. It is obvious that MAX-K-SAT is more general than K-SAT, because its solution can be used to answer the question of K-SAT problem, but not vice versa [12]. The parameter controlling the satisfiability of an instance is $\alpha=m/n$, the ratio of clauses to variables. Extensive empirical and analytical research [9,10] have shown that for K-SAT (K≥3), the solving-cost of many local search algorithms can be characterized as "easy-hard-easy" phase transition. There exists a similar feature for MAX-K-SAT but with "easy-hard" phase transition [9].

III. BACKBONE GUIDED EXTREMAL OPTIMIZATION

A. *Bose-Einstein Distribution based Initial Configuration*

The research work [14] has shown that for the Max-SAT problem, only the Bose-Einstein (BE) distribution [15], which describes the statistical behavior of bosons (integer spin particles) in quantum physics, can guarantee that an initial assignment set is generated with an arbitrary proportion of 1s and 0s. Therefore, a BE-based assignment [16] will be used as the initial configuration of the proposed framework in this paper. It is constructed by the following procedure called "BE-based Initial Configuration Generator (BEICG)".

B. *Definition of Fitness*

According to the seminal work [1,2], one of the most important issues for designing the EO-based algorithms is the appropriate definition of local and global fitness. For a given configuration S of a weighted MAX-SAT problem, the local fitness λ_i of each variable x_i is defined as follows:

$$\lambda_i = \frac{-\sum_{x_i \in C_j \text{ and } C_j(S)=0} w_j}{\sum_{x_i \in C_k} w_k} \quad (2)$$

In other words, the local fitness is defined as the fraction of the sum of weights of unsatisfied clauses in which the variable x_i appears by the sum of weights of clauses connected to this variable.

The global fitness $C(S)$ is defined as the sum of the contribution from each variable, i.e.,

$$C(S) = -\sum_{i=1}^{n} (\lambda_i \sum_{x_i \in C_k} w_k) = -\sum_{i=1}^{n} (c_i \lambda_i) \quad (3)$$

where $c_i = \sum_{x_i \in C_k} w_k$ and is a constant for a given problem. Clearly, the global fitness $C(S)$ is a linear combination of the local fitness λ_i.

C. *The BGEO Framework*

For the MAX-SAT problem, the exact backbone is the set of variables having the same assignments in all the global optimal solutions. Nevertheless, the exact backbone information of a given problem instance are even more difficult to obtain than actual problem solutions [26]. An approximate approach to estimate the backbone information is considering the local minima as "real" optimal ones. The quasi-backbone $\mathbf{X_B}$ is the set of variables having the same assignments in a set of local optimal solutions $\mathbf{S}_{\text{local}}$. Its formal definition is given as follows:

$$\mathbf{X_B} = \{x_i \,|\, \forall s_j, s_k \in \mathbf{S}_{\text{local}}, s_j(x_i) = s_k(x_i)\} \quad (4)$$

In this paper, we incorporate the quasi-backbone information into the extremal optimization algorithm, and obtain the framework of *Backbone Guided Extremal Optimization* (*BGEO*).

The BGEO framework can be viewed as an iterative process, which consists of *backbone estimation phase* and *backbone-guided optimization phase*. In the first iteration, i.e., l=1, the BGEO collects R_l local optimum solutions starting from pure randomly generated initial solutions without any backbone information. In the following iterations, BGEO explores the complex landscape by utilizing the backbone information obtained in the last iteration. When the evolutionary probability distribution $P_l(k_1)$ adopted in BGEO algorithm is chosen as power-law $P_l(k_1) \propto k_1^{-\tau_l}$, exponential distribution $P_l(k_1) \propto e^{-\mu_l k_1}$, and hybrid distribution $P_l(k_1) \propto e^{-h_l k_1} k_1^{-h_l}$, respectively according to our recent research [16], the corresponding algorithm is called BG-PEO, BG-EEO, and BG-HEO, respectively, so the corresponding parameter p_l is τ_l , μ_l, and h_l, respectively.

Obviously, the performance of BGEO depends on these parameters including MI, R_l, SS_l, MS_l and p_l. Therefore, how to determine appropriate values of these parameters to make BGEO achieve the best performance is a critical issue. Here, MI and R_l are all positive constants. According to the study on BE-EO [17], the parameters SS_l, MS_l are as follows:

$$SS_l = C_{l1} \times |\mathbf{X_{NB}}(l)| \quad (5)$$

$$MS_l = C_{l2} \times |\mathbf{X_{NB}}(l)| \quad (6)$$

where C_{l1}、C_{l2} are all positive constants and $|\mathbf{X_{NB}}(l)|$ is the number of non-backbone variables in the l-th iteration .

Now we focus on the adjustable parameter p_l for controlling the evolutionary probability distribution of BGEO. Due to the different features of the first and the remaining iterations, p_l is given as the following form:

$$p_l = \begin{cases} p_c, & l = 1 \\ p_c + d * |\mathbf{X_B}(l-1)|, & 2 \le l \le MI \end{cases} \quad (7)$$

where p_c is the initial value of the parameter p_l and d is a positive constant.

IV. EXPERIMENTAL RESULTS

In order to demonstrate the effectiveness of the BGEO, we choose the hard MAX-SAT problem instances from SATLIB [19] as a testbed. The tested MAX-3-SAT unsatisfiable instances are represented as 'uuf-n:m'here, in which n is the number of the variables and m is the number of the clauses. These instances with α=m/n ranges from 4.260 to 4.360, which are close to the critical threshold of phase transition α_c≈4.267. It should be remarked that the experimental results [10] on random and structured MAXSAT instances have shown that BE-EO can provide better or at least competitive performance than more elaborate stochastic optimization methods, such as SA [20], GSAT [21], WALKSAT [18] and Tabu search [22]. Furthermore, the superiority of the BE-EEO and BE-HEO algorithms under EOSAT framework to the BE-EO algorithm is demonstrated by our recent research [23]. This paper concentrates on comparing BG-EEO with BE-EEO [23] by the experiments on the hard MAXSAT instances near the phase transition. The experimental results are shown in Tab. 1. Clearly, BG-EEO outperforms BE-EEO for these hard instances. Especially, the BG-EEO algorithm reaches the optimal solutions for some instances shown as the bold.

V. CONCLUSION

This paper presents a novel optimization method, *backbone guided extremal optimization (BGEO)*, for hard SAT and MAX-SAT problems.The experimental results on the hard MAX-SAT problem instances have shown that BGEO outperforms the reported BE-EO algorithm. Nevertheless, it should be stressed that the main propose of this research is to develop a new optimization method rather than provide the best algorithm for a particular problem. As a consequence, the performance of BGEO is possible to further improve by fine-tuning the control parameters or adapting other backbone-guided search strategies, which is well-studied in our future work.

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REFERENCES

[1] S. Boettcher; A.G. Percus. Nature's way of optimizing. Artificial Intelligence[J],vol.119, 2000, PP275-286.

[2] S. Boettcher; A.G. Percus. Optimization with extremal dynamics. Phys. Rev. Lett.[J], vol.86, no.23, 2001, PP5211-5214.

[3] P. Bak; K. Sneppen. Punctuated equilibrium and criticality in a simple model of evolution. Phys. Rev. Lett.[J], 1993,vol.71, no.24, PP4083-4086.

[4] P. Bak; C. Tang; K. Wiesenfeld. Self-Organized Criticality. Phys. Rev. Lett.[J], vol.59, 1987, PP381-384.

[5] S. Boettcher. Evolutionary dynamics of extremal optimization. Lecture Notes on Computer Science, vol.5581, 2009, PP1-14.

[6] ZENG Guo-qiang; LU Yong-zai. Survey on computational complexity with phase transitions and extremal optimization. *Proc. 48th IEEE conf. Control and Decision&28th Chinese Control Conf.*, 2009, PP4352-4359.

[7] J. Slaney; T. Walsh. Backbones in optimization and approximation. Proc. IJCAI-01, Seattle, WA, Morgan Kaufman, San Mateo, CA, 2001, PP254-259.

[8] R. Monasson; R. Zecchina; S. Kirkpatrick; B. Selman; L. Troyansky. Determining computational complexity from characteristic 'phase transition'. Nature[J], vol.400, 1999, PP133-137.

[9] ZHANG Wei-xiong. Phase transitions and backbones of 3-SAT and Maximum 3-SAT. Proc. CP, 2001, PP153-167.

[10] J. Schneider; C. Froschhammer; I. Morgenstern; T. Husslein; J. M. Singer. Searching for backbones-efficient parallel algorithm for the traveling salesman problem. Comput. Phys. Commun.[J], vol.96, 1996, PP173-188.

[11] O. Telelis; P. Stamatopoulos. Heuristic backbone sampling for maximum satisfability. Proceedings of the 2nd Hellenic Conference on Artificial Intelligence, 2002, PP129-139.

[12] ZHANG Wei-xiong. Configuration landscape analysis and backbone guided local search. Part I: satisfiability and maximum satisfiability. Artificial Intelligence[J], vol.158, 2004, PP1-26.

[13] A. Biere; M. Heule; H. Maaren; T. Walsh, Handbook of satisfiability, IOS Press, 2009.

[14] S. Szedmak. How to find more efficient initial solution for searching, RUTCOR Research Report 49-2001, Rutgers Center for Operations Research, Rutgers University, Piscataway, New Jersey, USA.

[15] H. Haken; H. C. Wolf. The physics of atoms and quanta, 5th edn., Springer-Verlag, 1996.

[16] ZENG Guo-qiang; LU Yong-zai; MAO Wei-jie; CHU Jian. Study on probability distributions for evolution in modified extremal optimization. Physica A[J], vol.389, 2010, PP1922-1930.

[17] M. B. Menaï; M. Batouche. An effective heuristic algorithm for the maximum satisfiability problem. Appl. Intell. [J], vol.24, 2006, PP227-239.

[18] SATLIB: http://www.cs.ubc.ca/~hoos/SATLIB/benchm.html

[19] B. Selman; H. A. Kautz; B. Cohen. Noise strategies for improving local search. Proceedings of the 12th National Conference on Artificial Intelligence, 1994, PP337-343.

[20] P. Hansen; B. Jaumard. Algorithms for maximum satisfiability problems. Computing, vol. 44, 1990, PP279-303.

[21] B. Selman; H A Kautz. An empirical study of greedy local search for satisfiability testing. Proceeding of 11th National Conference on Aritificial Intelligence, 1993, PP46-51.

[22] F. Glover. Tabu search: Part I. ORSA J. Comp.[J], vol.1, no.3, pp.190-206,1989.

[23] ZENG Guo-qiang; LU Yong-zai; MAO Wei-jie. Modified extremal optimization for the maximum satisfiability problem. J. Zhejiang Univ-Sci C[J] (Comput & Electron), to appear.

BE-based Initial Configuration Generator

Input: a set of n Boolean variables x_i

Output: a random BE-based assignment of the n Boolean variables

u=0 //number of 1s
for i=1 to n

$$p_i = \frac{u+1}{(i-1)+2}$$

Generate randomly a real number a in [0, 1]
if $p_i > a$
then x_i =1, u=u+1
else x_i =0
end if
end for

Algorithm: Backbone Guided Extremal Optimization

Input: a weighted SAT instance; MI: the maximum iterations; Rl: the maximum independent runs of the l-th iteration; SSl: the maximum sample size in the l-th iteration; MSl: the maximum steps of EO algorithm in the l-th iteration; pl: the adjustable parameter for evolutionary probability distribution of EO algorithm in the l-th iteration;

Output: SB: the best configurations found; C(SB): the total weights of unsatisfied clauses.

1: Initialization: set the backbone set XB=0, non-backbone set XNB=X.
2: for l=1: MI
3: for j=1: Rk
4: for i=1: SSk
5: Fix the values of XB, initialize XNB=X-XB by BEICG, and construct the initial solution Si, set Sbest= Si
6: For the current solution Si,
(a). Evaluate λi for each variable xi and rank all the variables according to λi , i.e., find a permutation Π1 of the labels i such that $\lambda_{\Pi_1(1)} \ge \lambda_{\Pi_1(2)} \ge \ldots \ge \lambda_{\Pi_1(n)}$ *;*
(b). Select a rank Π(k) according to a probability distribution Pl(k1), 1≤k1≤n and denote the corresponding variable as xj ;
(c). Flip the value of xj and set Snew= Si in which xj value is flipped;
(d). If C(Snew) ≤ C(Sbest), then Sbest=Snew;
(e). Accept Si =Snew unconditionally;
7: Repeat the step 6 until the maximum steps MSl, and obtain the best solution Sif= Sbest.
8: end for
9: Choose the best solution Sbj from the solution set{Sif}.
10: end for
11: Extract the backbone information from the solution set Sl={Sbj}, and update XB and XNB.
12: end for
13: Choose the best solution SB from $\mathbf{S} = \bigcup_{l=1}^{MI} \mathbf{S}_l$ *, and obtain the cost C(SB).*

TABLE I. COMPARSION OF BG-EEO AND REPORTED BE-EEO ALGORITHM FOR THE HARD MAX 3-SAT INSTANCES NEAR PHASE TRANSTION

		BE-EEO			BG-EEO		
problems	α	e_b(%)	e_m(%)	e_w(%)	e_b(%)	e_m(%)	e_w(%)
uuf-50:218	4.360	1.38	2.38	3.21	**0.00**	0.00	0.00
uuf-75:325	4.333	1.85	2.65	3.37	**0.00**	0.17	0.31
uuf-100:430	4.300	1.86	2.63	2.80	**0.00**	0.26	0.46
uuf-125:538	4.304	1.86	2.70	3.35	0.19	0.30	0.37
uuf-150:645	4.300	2.17	2.71	3.41	0.16	0.32	0.47
uuf-175:753	4.303	2.92	3.33	3.98	0.13	0.34	0.40
uuf-200:860	4.300	3.49	3.85	4.30	0.12	0.16	0.23
uuf-225:960	4.267	2.81	3.48	4.17	0.10	0.44	0.62
uuf-250:1065	4.260	3.09	3.51	4.38	0.09	0.45	0.66