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k-Neighborhood Template *A*-Type Two-Dimensional Bounded Cellular Acceptors

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Abstract

In this paper, we deal with three-dimensional computational model, *k*-neighborhood template *A*-type two-dimensional bounded cellular acceptor on three-dimensional tapes, and discuss some basic properties. This model consists of a pair of a converter and a configuration-reader. The former converts the given three-dimensional tape to two-dimensional configuration. The latter determines whether or not the derived two-dimensional configuration is accepted, and concludes the acceptance or non-acceptance of given three-dimensional tape. We mainly investigate some open problems about *k*-neighborhood template *A*-type two-dimensional bounded cellular acceptor on three-dimensional tapes whose configuration-readers are $L(m)$ space-bounded deterministic (nondeterministic) two-dimensional Turing machines.

Keywords: acceptor, configuration-reader, converter, neighbor, space-bounded, three-dimension, Turing machine.

1. Introduction

Due to the advances in many application areas such as computed tomography, robotics, and so on, the study of three-dimensional pattern processing has been of crucial importance. Thus, the study of three-dimensional automata as the computational models of three-dimensional pattern processing has been meaningful. From this point of view, we proposed several three-dimensional automata as computational models of three-dimensional pattern processing, and investigated their several accepting powers [2]. By the way, in the multi-dimensional pattern processing, designers often use a strategy whereby features are extracted by projecting high-dimensional space on low-dimensional space. So, from this viewpoint, a new computational model, *k-neighborhood template A-type two-dimensional bounded cellular acceptor* (abbreviated as A-2$BCA(k)$ (k $\in$ {1,5,9})) on three-dimensional tapes was introduced, and discussed some basic properties[3]. An A-2$BCA(k)$ is a three-dimensional automaton which consists of a pair of a *converter* and a *configuration-reader*. The former converts the given three-dimensional tape to the two-dimensional configuration and the latter determines the acceptance or non-acceptance of given three-dimensional tape whether or not the derived two-dimensional configuration is accepted. This paper mainly investigates some open problems about accepting powers of A-2$BCA(k)$'s whose configuration-readers are $L(m)$ space-bounded deterministic (nondeterministic) two-dimensional Turing machines [1]. We first show that a relationship between the accepting powers of deterministic A-2$BCA(1)$ and nondeterministic A-2$BCA(1)$. We then show that there exists a language accepted by any deterministic A-2$BCA(9)$.

2. Preliminaries

Definition 2.1. Let Σ be a finite set of symbols. A *three-dimensional tape* over Σ is a three-dimensional rectangular array of elements of Σ. The set of all three-dimensional tapes over Σ is denoted by $\Sigma^{(3)}$. Given a tape $x \in \Sigma^{(3)}$, for each integer $j(1 \leq j \leq 3)$, we let $m_j(x)$ be the length of x along the jth axis. The set of all $x \in \Sigma^{(3)}$ with $l_1(x)=m_1$, $l_2(x)=m_2$, and $l_3(x)=m_3$ is denoted by $\Sigma^{(m1,m2,m3)}$. If $1 \leq i_j \leq l_j(x)$ for each $j(1 \leq j \leq 3)$, let $x(i_1,i_2,i_3)$ denote the symbol in x with coordinates (i_1,i_2,i_3), as shown in Fig.1.

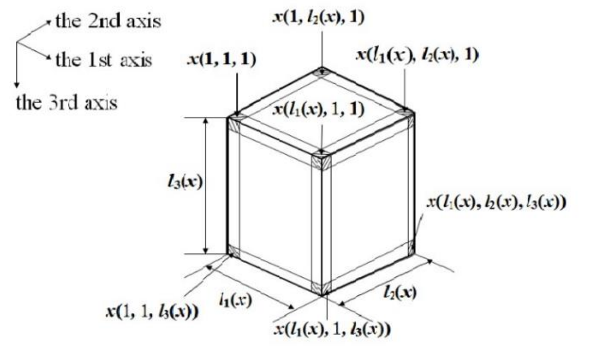

Fig.1. Three-dimensional tape.

Furthermore, we define

$$x[(i_1,i_2,i_3),(i'_1,i'_2,i'_3)],$$

when $1 \leq i_j \leq i'_j \leq l_j(x)$ for each integer $j(1 \leq j \leq 3)$, as the three-dimensional tape y satisfying the following (i) and (ii):

(i) for each $j(1 \leq j \leq 3)$, $l_j(y)=i'_j-i_j+1$;

(ii) for each r_1, r_2, $r_3(1 \leq r_1 \leq l_1(y)$, $1 \leq r_2 \leq l_2(y)$, $1 \leq r_3 \leq l_3(y))$, $y(r_1, r_2, r_3) = x(r_1+i_1-1, r_2+i_2-1, r_3+i_3-1)$. (We call $x[(i_1, i_2, i_3), (i'_1, i'_2, i'_3)]$ the $x[(i_1, i_2, i_3), (i'_1, i'_2, i'_3)]$-*segment* of x.).

We now introduce a k-neighborhood template A-type two-dimensional bounded cellular acceptor (A-2$BCA(k)$), which is a main object of discussion in this paper.

Definition 2.2. Let A be the class of an automaton moving on a two-dimensional configuration. Then, an A-2$BCA(k)$ M is defined by the 2-tuple $M=(R,B)$. R and B are said to be a converter and a configuration-reader in view of its property, respectively.

(1) R is a two-dimensional infinite array consists of the same finite state machines and is defined by the 6-tulpe

$$M = (\boldsymbol{Z}^2, \boldsymbol{N}^2, K, \Sigma, \sigma, q_0), \text{ where}$$

① $\boldsymbol{Z}$ is the set of all integer, and the finite state machines are assigned to each point of $\boldsymbol{Z}^2(=\boldsymbol{Z}\times\boldsymbol{Z})$. The finite state machine situated at coordinates $(i,j) \in \boldsymbol{Z}^2$ is called the (i,j)-th *cell* and denoted by A(i,j),

② $\boldsymbol{N}^2$ $(\subseteq Z^2)$ represents the *neighborhood template* of each cell and $\boldsymbol{N}^2 = \{(i,j) \mid -1 \leq i,j \leq 1\}$,

③ K is a finite set of states of each cell and contains $q_\#$ (the boundary state) and q_0 (the initial state),

④ Σ is a finite set of input symbols ($\# \notin \Sigma$ is the boundary symbol),

⑤ $\sigma: K^9 \times (\Sigma \cup \{\#\}) \rightarrow 2^K$ is the *cell state transition function*. Let $\mathbf{q}_{i,j}(t)$ be the state of the A(i,j) at time t. Then $q_{i,j}(t+1) \in \sigma(q_{i-1,j-1}(t), q_{i-1,j}(t), q_{i-1,j+1}(t), q_{i,j-1}(t), q_{i,j}(t), q_{i,j+1}(t), q_{i+1,j-1}(t), q_{i+1,j}(t), q_{i+1,j+1}(t), \text{a})$, where a is the symbol on the A(i,j) at time t. If $q_{i,j}(t) = q_\#$, however, $q_{i,j}(t+1) = \{q_\#\}$ for each $(i,j) \in \boldsymbol{Z}^2$ and each $t \geq 0$.

(2) A set of input symbols of B is $K-\{q_\#\}$ (where $B \in A$). Intuitively, $M=(R,B)$ moves as follows, given a three-dimensional input tape $x \in \Sigma^{(m1,m2,m3)}$ $(m_1, m_2, m_3 \geq 1)$ (x is surrounded by the boundary symbol #).

First, each cell A(i,j) of $R(1 \leq i \leq m_1, 1 \leq j \leq m_2,)$ reads each symbol on the first plane $x(i,j,1)$ in the initial state q_0, and all of the other cells read the boundary symbols #'s in the boundary state $q_\#$'s at time $t=0$. Starting from this condition, R keeps reading x according to the cell state transition function, and moving down the cell array by one plane, every time R reads one plane all.

Next, B starts to move regarding a *two-dimensional configuration* of R just after R finished reading x as a two-dimensional input tape and determines whether or not can accept the configuration. If B accepts it, x is said to be *accepted* by M. Let $T(M)$ be the set of all accepted three-dimensional tape by M (see Fig.2).

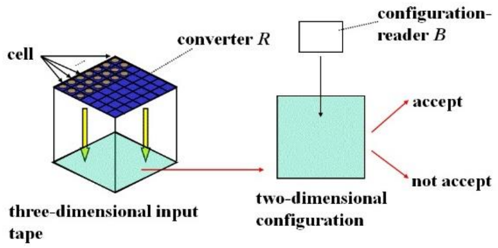

Fig.2. *A-2BCA*(k).

Definition 2.3. An *A*-2*BCA* in Dfinition 2.2 is called a 9-neighborhood template *A*-2*BCA*. If ② and ⑤ of (1) in Definition 2.2 in replaced with ②' and ⑤' in Table 1, each *A*-2*BCA* is called a 1-*neighborhood template A*-2*BCA* and a 5-*neighborhood template A*-2*BCA*, respectively. From now on, we denote k-neighborhood template *A*-2*BCA* by *A*-2*BCA*(k) ($k \in \{1,5,9\}$) (see Fig.3).

	1-neighbor	5-neighbor
②'	$N^2 = \{(0,0)\}$	$N^2 = \{(i,j)\} \mid i^2+j^2 \leq 1\}$
⑤'	$\cdot\ \sigma: k \times \Sigma \rightarrow 2^k$ $\cdot\ q_{i,j}(t+1)$ $\in \sigma(q_{i,j}(t), \mathrm{a})$	$\cdot\ \sigma: k^5 \times \Sigma \rightarrow 2^k$ $\cdot\ q_{i,j}(t+1)$ $\in \sigma(q_{i-1,j}(t), q_{i,j-1}(t),$ $q_{i,j}(t), q_{i,j+1}(t),$ $q_{i+1,j}(t), \mathrm{a})$

Table 1. 1- or 5-neighborhood template *A*-2*BCA*.

	$q_{i,j}$	

(a) 1-neighbor.

	$q_{i-1,j}$	
$q_{i,j-1}$	$q_{i,j}$	$q_{i,j+1}$
	$q_{i+1,j}$	

(b) 5-neighbor.

$q_{i-1,j-1}$	$q_{i-1,j}$	$q_{i-1,j+1}$
$q_{i,j-1}$	$q_{i,j}$	$q_{i,j+1}$
$q_{i+1,j-1}$	$q_{i+1,j}$	$q_{i+1,j+1}$

(c) 9-neighbor

Fig.3. Illustration of neighborhood templates.

Definition 2.4. If the image generated by σ in Definitions 2.2 and 2.3 is a singleton, the converter is said to be *deterministic*, and if not, it is said to be *nondeterministic*. An *A*-2*BCA*(k) ($k \in \{1,5,9\}$), which converter is deterministic (nondeterministic), is said to be a *deterministic* (*nondeterministic*). *A*-2*BCA*(k) and denoted by *A*-2*DBCA*(k) (*A*-2*NBCA*(k)).

We now consider the class of two-dimensional automata described by the following abbreviations as the class of the configuration-reader of *A*-2*BCA*(k) *A*.

In this paper, we assume that the reader is familiar with the definition of these automata. If necessary, see [1].

・2-$DTM^S(L(m))$⋯The class of $L(m)$ space-bounded deterministic two-dimensional Turing machine

・2-$NTM^S(L(m))$⋯The class of $L(m)$ space-bounded nondeterministic two-dimensional Turing machine

・DO^S⋯The class of deterministic two-dimensional on-line tessellation acceptor

・DB^S⋯The class of deterministic one-dimensional bounded cellular acceptor

For example 2-$DTM^S(L(m))$-2*DBCA*(9) represents such the class as its converter is deterministic and 9-neighborhood, and its configuration-reader is an $L(m)$ space-bounded deterministic two-dimensional Turing machine. Moreover, for any $A \in \{$2-$DTM^S(L(m))$, 2$NTM^S(L(m))\}$, for any $X \in \{D, N\}$ and for any $k \in \{1,5, 9\}$, the class of set of all three-dimensional tapes accepted by *A*-2*XBCA*(k) is denoted by L[*A*-2*XBCA*(k)]. Especially, the class of all three-dimensional tapes accepted by *A*-2*XBCA*(k) whose input tapes are restricted to cubic ones is denoted by L^C[*A*-2*XBCA*(k)], and the class of set of all square tapes accepted by A is denoted by $L^S[A]$. In this paper, we discuss regarding the three-dimensional input tapes as cubic ones all.

3. Main Results

In this section, we discuss some properties of A-$2BCA(k)$'s whose configuration-readers are $L(m)$ space-bounded deterministic (nondeterministic) two-dimensional Turing machines. First, we show that a relationship between determinism and nondeterminism.

Theorem 3.1. *For any* $X \in \{D,N\}$, $L^C[2\text{-}XTM^S(m^2)\text{-}2DBCA(1)] = L^C[2\text{-}XTM^S(m^2)\text{-}2NBCA(1)]$.

Proof: From above definition, it is obvious that $L^C[2\text{-}XTM^S(m^2)\text{-}2DBCA(1) \subseteq L^C[2\text{-}XTM^S(m^2)\text{-}2NBCA(1)]$. We below show that $L^C[2\text{-}XTM^S(m^2)\text{-}2NBCA(1)] \subseteq L^C[2\text{-}XTM^S(m^2)\text{-}2DBCA(1)]$. We now show only that $L^C[2\text{-}DTM^S(m^2)\text{-}2NBCA(1)] \subseteq L^C[2\text{-}DTM^S(m^2)\text{-}2DBCA(1)]$ (We can prove another case (i.e., $X=N$) in the same way).

Now, let $M=(R,B)$ ($R=(\mathbf{Z}^2, \mathbf{N}^2, K, \Sigma, \sigma, q_0)$, $B=(K, Q, \Sigma, \Gamma, \delta, p_0, F)$) be some $2\text{-}DTM^S(m^2)\text{-}2NBCA(1)$ (see [1-5,8], if you would like to know about the 7-tuple constructs B). Then, we consider an $M'=(R',B')$ ($R'=(\mathbf{Z}^2, \mathbf{N}^2, K', \Sigma, \sigma', q_0')$, $B'=(K', Q, \Sigma, \Gamma, \delta', p_0', F')$) constructed as follows.

(1) Construction of R'

① $K'=2^{(K-\{q_\#\})} \cup \{q_\#\}$, $q_0'=\{q_0\}$.

② For any $a \in \Sigma$ and any $K'' \in K'-\{q_\#\}$,

$\sigma'(K'', a) = \cup_{q \in K''} \sigma(q,a)$.

(2) Construction of B'

For any $p \in Q$ and any $K'' \in K'-\{q_\#\}$,

$\delta'(p, K'') = \bigcup_{r \in K''} \delta(p,r)$.

Intuitively, we explain the movement of $M'=(R', B')$ constructed in this way. Let us suppose that a three-dimensional tape $x \in \Sigma^{(3)+}$ is given to M'. R' is one-neighborhood, so we can consider each cell of R' as usual one-dimensional finite automata. Then, each cell of R' moves to store all states that each corresponding cell of R can enter in each state at each time by using the well-known subset construction method (see (1)).

B' nondeterministically chooses only one state from each state of each cell of R', and simulates the movement of B regarding the selected states as the input symbol. If B' can not accept the input, B' selects the next input and simulates the movement of B. From the way such as the above manner, B' checks the all input patterns, and if B' can accept one input, B' can accept the configuration of R' (see (2)).

It is clear that $T(M')=T(M)$ for $M'=(R',B')$ constructed in this manner. □

Next, we show that there exists a language accepted by a $2\text{-}DTM^S(0)\text{-}2NBCA(1)$, but not accepted by any $2\text{-}NTM^S(L(m))\text{-}2DBCA(9)$ for any $L(m)) = o(\ell\text{og } m)$.

Theorem 3.2. *For any function* $L(m)=o(\ell\text{og } m)$, $L^C[2DTM^S(0)\text{-}2NBCA(1)]$

$-L^C[2\text{-}NTM^S(L(m))\text{-}2DBCA(9)] \neq \varphi$.

Proof: Let $C=\{w_0 2 w_1 2 \cdots 2 w_k \mid k \geq 1 \ \& \ ^\forall i(0 \leq i \leq k)\ [w_1 \in \{0,1\}^+] \ \& \ ^\exists j(0 \leq j \leq k)\ [w_0 = w_j{}^r]\}$(where, for any one-dimensional tape w, w^r denotes the reversal of w), and $T_1=\{x \in \{0,1,2\}^{(3)+} \mid {}^\exists m \geq 3[l_1(x) = l_2(x) = l_3(x) = m \ \& \ x[(1,1,m), (1, m, m)] \in C]\}$. Then, by using a technique similar to that in the proof of Lemma 2(1) in [3], we show that T_1 is accepted by $2DTM^S(0)\text{-}2NBCA(1)$, but not accepted by any $2\text{-}NTM^S(L(m))\text{-}2DBCA(9)$ for any $L(m)=\text{o}(\ell\text{og m})$. □

From Theorem 3.2, we get the following.

Corollary 3.1. *For any* $L(m) = o(log\ m)$ *and any* $X \in \{D, N\}$, $L^C[2\text{-}XTM^S(L(m))\text{-}2DBCA(1)] \subsetneq L^C[2\text{-}XTM^S(L(m))\text{-}2NBCA(1)]$.

Remark 3.1. In Theorem 3 in [3], it has already shown that for any function $L(m)$ and any $k \in \{5,9\}$, $L^C[2\text{-}XTM^S(L(m))\text{-}2DBCA(k)] \subsetneq L^C[2\text{-}XTM^S(L(m))\text{-}2NBCA(k)]$.

Finally, by using the well-known technique, we can show that there exists a language accepted by a DO^S-$2NBCA(1)$ and a DB^S-$2NBCA(1)$, but not accepted by any $2\text{-}DTM^S(L(m)) - 2DBCA(9)$ for any function $L\ (m) = o(log\ m)$.

Theorem 3.3. *For any function* $L(m) = o(log\ m)$, $(L^C[DO^S\text{-}2NBCA(1)] \cap L^C[DB^S\text{-}2NBCA(1)])\text{-}L^C[2\text{-}DTM^S(L(m))\text{-}2DBCA(9)] \neq \varphi$.

4. Conclusion

In this paper, we showed some properties of A-$2BCA(k)$'s. We conclude this paper by giving the following problem. For any $X \in \{D,N\}$ and any $L(m)$ ($log\ m \leq L(m)$ and $L(m) = o(m^2)$), $L^C[XTM^S(L(m))\text{–}2DBCA(1)] \subsetneq L^C\ [XTM^S(L(m))\text{–}2NBCA(1)]$?

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