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An efficient computational approach for solving type-2 intuitionistic fuzzy numbers based Transportation Problems

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Abstract

This paper is concerned with the solution procedure of a Transportation Problem in which costs are triangular intuitionistic fuzzy numbers (TIFN) and availabilities and demands are taken as exact numerical values. According to the existing solution approach, TIFN are first ordered by using an accuracy function defined on score functions for membership and non-membership functions of TIFN. Then this ordering is used to develop methods for finding an initial basic feasible solution and the optimal solution of intuitionistic fuzzy Transportation Problems in terms of triangular intuitionistic fuzzy numbers. This solution approach, in spite of its merits, requires a lot of fuzzy arithmetic operations, such as additions and subtractions of TIFN, as well as a lot of comparisons on TIFN. In this paper an efficient computational solution approach is proposed for solving intuitionistic fuzzy Transportation Problems based on classical transportation algorithms to overcome the shortcomings of the aforementioned solution approach. In the approach here presented, the comparison of triangular intuitionistic fuzzy costs is done once and all arithmetic operations are done on real numbers. Finally, for the sake of illustration, two intuitionistic fuzzy Transportation Problems are solved herein to demonstrate the usages and advantages of the proposed solution approach.

Keywords: Intuitionistic fuzzy transportation problem; Triangular intuitionistic fuzzy number; Accuracy function.

1. Introduction

A useful, effective and operative tool to handle imprecise data is that of fuzzy sets as defined by Zadeh [1]. In such a case, the main drawback to be pointed out is the fact that the accomplishment of the property defining the postulated fuzzy set is to be measured by means of an "exact" and unique real number. Therefore using fuzzy sets is not suitable in cases where there may be a hesitation or uncertainty about the accomplishment degree of the element in a set. To tackle this drawback, Atanassov [2] introduced the concept of Intuitionistic Fuzzy Set (IFS) that seems to suitably describe an imprecise concept and incorporates the mentioned hesitation in the membership degrees. In IFS, not only the degree of acceptance is defined by a membership function, but

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also the degree of rejection is considered by a non-membership function so that the sum of both degrees should be smaller than one. Because of these facts, it is expected that IFS can cope with the presence of vagueness and hesitancy originating from imprecise knowledge or information. IFS Theory has been applied in many important and essential areas, including multi attribute decision-making models [3,4], multi attribute group decision-making problems [5,6], image restoration [7], medical diagnosis [8], game theory [9,10] and pattern recognitions [11] among others.

As is well known, nowadays Transportation Problems (TP) because of their recent relevant applications (optimization problems associated to different models of smart cities, multimodal transports, etc.) are of upmost importance and in almost all the cases imprecise data are omnipresent. Hence, Fuzzy Sets Theory and IFS Theory could be used to capture linguistic uncertainty in optimization problems (see Refs. 12–18), particularly in TP.

Basically, the central question in TP is to find the least total transportation cost of a commodity in order to satisfy demands at destinations using available supplies at origins. In classical TP it is assumed that the transportation costs and values of supplies and demands are exactly known. However, in many situations the decision maker has no precise information about the coefficients defining the TP. In these cases, the corresponding coefficients or elements defining the problem can be specified by means of fuzzy sets, and a Fuzzy Transportation Problem (FTP) appears in a natural way.

From this point of view, numerous researchers have devoted their efforts to using fuzzy numbers in real life TP. Thus, for instance, Jimenez and Verdegay [19,20] studied Fuzzy Solid TP in which supplies, demands and conveyance capacities are represented by trapezoidal fuzzy numbers and presented a parametric approach for finding a fuzzy solution. Also, an FTP in which supplies and demands are trapezoidal fuzzy numbers was formulated by Gani and Razak [21] and a parametric approach for finding a fuzzy solution with the aim of minimizing the sum of the transportation costs in the two stages was analysed by them. Dinagar and Palanivel [22] considered an FTP with trapezoidal fuzzy numbers and proposed a fuzzy modified distribution method to obtain the optimal solution in terms of fuzzy numbers. A new algorithm, namely the fuzzy zero point method for finding the fuzzy optimal solution for FTP, in which the transportation cost, supplies and demands are represented by trapezoidal fuzzy numbers was introduced by Pandian and Natarajan [23]. A systematic procedure for solving all types of FTP whether to maximize or to minimize the objective function was proposed by Basirzadeh [24], and a new method to find the solution of a linear multi-objective TP, by representing all the parameters as interval-valued fuzzy numbers, was considered by Gupta and Kumar [25]. Shanmugasundari and Ganesan [26] developed the fuzzy version of Vogel's and MODI methods for obtaining the fuzzy initial basic feasible solution and the fuzzy optimal solution respectively, and also Kaur and Kumar [27] proposed a new method based on ranking functions for solving FTP by assuming that the parameters of the TP are represented by generalized trapezoidal fuzzy numbers. Moreover, Kaur and Kumar [28] approached a special type of FTP where transportation costs are represented by generalized trapezoidal fuzzy numbers. More recently, Ebrahimnjead [29] presented a simplified approach to find the optimal solution of the FTP studied by Kaur and Kumar [28]. Sudhagar and Ganesan [30] proposed an algorithm to find an optimal solution of a FTP with all parameters represented by fuzzy numbers. It was shown by Ebrahimnjead [31] that the algorithm considered by Sudhagar and Ganesan [30] does not always lead to a fuzzy optimal solution. In addition, Ebrahimnejad [32] proposed a two-step method for solving FTP where all of the parameters are represented by non-negative triangular fuzzy numbers.

In spite of this, although fuzzy numbers are commonly used for modeling imprecise data when one has to cope with real TP, this may not be suitable for situations where one has to deal with uncertainty as well as with hesitation. In such situations, Intuitionistic Fuzzy Numbers are used to represent the imprecise parameters of the TP under consideration. The resulting problem is therefore referred to as an Intuitionistic Fuzzy Transportation Programming Problem (IFTP). Despite the importance of the problem there are few studies in the current literature facing the practical solving of IFTP.

Hussain and Kumar [33] focused on TP in which supplies and demands are intuitionistic fuzzy numbers. Then, they analysed an intuitionistic fuzzy zero point method to find the optimal solution in terms of triangular intuitionistic fuzzy numbers. Nagoorgani and Abbas [34] proposed another method based on ranking functions for finding an optimal solution of the same IFTP. Singh and Yadav [35] presented intuitionistic fuzzy methods to find the starting basic feasible solution in terms of triangular intuitionistic fuzzy numbers, and also proposed an intuitionistic fuzzy modified distribution method to find optimal solution of the same IFTP. Antony et al. [36] considered solving TP with triangular intuitionistic fuzzy numbers using Vogel's approximation method. Finally, a new method for solving TP has been approached by Singh and Yadav [37] in which transportation costs are triangular intuitionistic fuzzy numbers (TIFNs) and availabilities and demands are taken as exact numerical values. The method, first ranks TIFNs using an accuracy function defined on score functions for membership and non-membership functions of TIFNs. Then, one uses this ordering to develop methods for finding an initial basic feasible solution and an optimal solution of IFTP in terms of triangular intuitionistic fuzzy numbers. However the method proposed by Singh and Yadav [37], in spite of its merits, requires a lot of fuzzy arithmetic operations such as additions and subtractions of TIFNs and a lot of comparisons on TIFNs. For that reason in this paper one proposes an efficient computational solution approach for solving the same problem based on classical transportation algorithms. In the proposed approach, comparison of triangular intuitionistic fuzzy costs is done once, and all arithmetic operations are done on real numbers.

Consequently the rest of the paper is organized as follows: In Section 2, some basic concepts of fuzzy sets theory and intuitionistic fuzzy sets theory are reviewed. In Section 3, the TP with intuitionistic fuzzy transportation costs is formulated and the aforementioned approach to find the optimal solution is summarized. In Section 4, an efficient computational solution approach based on classical transportation algorithms is proposed for solving IFTP and to diminish the amount of computations of the existing approach. Finally, in Section 5, the application of the proposed method is illustrated by using two numerical examples and the obtained results are discussed. Section 6, including the main conclusions as well as some interesting future research lines, ends the paper.

2. Preliminaries

This section briefly introduces some basic concepts including fuzzy sets theory and intuitionistic fuzzy sets which are applied throughout this paper (see Refs. 2 and 37).

Definition 1: Let X denote the universe set. A fuzzy set $\widetilde{A}$ in X is defined by a set of ordered pairs $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x))\,; x \in X\}$ where $\mu_{\widetilde{A}}(x) \in [0, 1]$ represents the membership degree of x in $\widetilde{A}$, and is called the membership function of $\widetilde{A}$.

Definition 2: Let X denote the universe set. An intuitionistic fuzzy set (IFS) $\widetilde{A}^I$ in X is defined by a set of ordered triple $\widetilde{A}^I = \{\langle x, \mu_{\widetilde{A}^I}(x), \upsilon_{\widetilde{A}^I}(x)\rangle\,; x \in X\}$ where the functions $\mu_{\widetilde{A}^I}(x) : X \to [0, 1]$ and $\upsilon_{\widetilde{A}^I}(x) : X \to [0, 1]$, respectively, represent the membership degree and non-membership degree of x in $\widetilde{A}$ such that for each element $x \in X$, $0 \leqslant \mu_{\widetilde{A}^I}(x) + \upsilon_{\widetilde{A}^I}(x) \leqslant 1$.

Definition 3: For each intuitionistic fuzzy set $\widetilde{A}^I = \{\langle x, \mu_{\widetilde{A}^I}(x), \upsilon_{\widetilde{A}^I}(x)\rangle\,; x \in X\}$ in X, the value $h_{\widetilde{A}^I}(x) = 1 - \mu_{\widetilde{A}^I}(x) - \upsilon_{\widetilde{A}^I}(x)$ is called degree of hesitancy of x to $\widetilde{A}^I$.

Definition 4: An intuitionistic fuzzy set $\widetilde{A}^I = \{\langle x, \mu_{\widetilde{A}^I}(x), \upsilon_{\widetilde{A}^I}(x)\rangle\,; x \in X\}$ is called intuitionistic fuzzy normal if there is any $x_\circ \in X$ such that $\mu_{\widetilde{A}^I}(x_\circ) = 1$ (so $\upsilon_{\widetilde{A}^I}(x_\circ) = 0$).

Definition 5: An intuitionistic fuzzy set $\widetilde{A}^I = \{\langle x, \mu_{\widetilde{A}^I}(x), \upsilon_{\widetilde{A}^I}(x)\rangle\,; x \in X\}$ is called intuitionistic fuzzy convex if its membership function is fuzzy convex and its non-membership function is concave, i.e. $\forall x_1, x_2 \in X$, $\forall \lambda \in [0, 1]$, $\mu_{\widetilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \geqslant$

$\min\left\{\mu_{\widetilde{A}^I}(x_1),\mu_{\widetilde{A}^I}(x_2)\right\}$ and $\upsilon_{\widetilde{A}^I}(\lambda x_1+(1-\lambda)x_2)\leqslant \max\left\{\upsilon_{\widetilde{A}^I}(x_1),\upsilon_{\widetilde{A}^I}(x_2)\right\}$.

Definition 6: An intuitionistic fuzzy set $\widetilde{A}^I=\left\{\left\langle x,\mu_{\widetilde{A}^I}(x),\upsilon_{\widetilde{A}^I}(x)\right\rangle; x\in \mathrm{R}\right\}$ of the real number R is called an intuitionistic fuzzy number if

(i) $\widetilde{A}^I$ is intuitionistic fuzzy normal and intuitionistic fuzzy convex.
(ii) $\mu_{\widetilde{A}^I}$ is upper semi continuous and $\upsilon_{\widetilde{A}^I}$ is semi lower continuous.
(iii) $Supp\widetilde{A}^I=\left\{x\in\mathrm{R};\ \upsilon_{\widetilde{A}^I}(x)<1\right\}$ is bounded.

Definition 7: A triangular intuitionistic fuzzy number (TIFN) $\widetilde{A}^I$ is a special IFN with the membership function non-membership function defined as follows:

$$\mu_{\widetilde{A}^I}(x)=\begin{cases}\frac{x-a_1}{a_2-a_1}, & a_1<x\leqslant a_2\\ \frac{a_3-x}{a_3-a_2}, & a_2\leqslant x<a_3\\ 0, & Otherwise.\end{cases}$$

and

$$\upsilon_{\widetilde{A}^I}(x)=\begin{cases}\frac{a_2-x}{a_2-a_1'}, & a_1'<x\leqslant a_2\\ \frac{x-a_2}{a_3'-a_2}, & a_2\leqslant x<a_3'\\ 1, & Otherwise.\end{cases}$$

where $a_1'\leqslant a_1<a_2<a_3\leqslant a_3'$. This TIFN is denoted by $\widetilde{A}^I=(a_1,a_2,a_3;a_1',a_2,a_3')$.

Definition 8: Given two TIFNs $\widetilde{A}^I=(a_1,a_2,a_3;a_1',a_2,a_3')$ and $\widetilde{B}^I=(b_1,b_2,b_3;b_1',b_2,b_3')$, some arithmetic operations between $\widetilde{A}^I$ and $\widetilde{B}^I$ can be defined as follows:

(i) $\widetilde{A}^I\oplus\widetilde{B}^I=(a_1+b_1,a_2+b_2,a_3+b_3;a_1'+b_1',a_2+b_2,a_3'+b_3')$,
(ii) $\widetilde{A}^I\ominus\widetilde{B}^I=(a_1-b_3,a_2-b_2,a_3-b_1;a_1'-b_3',a_2-b_2,a_3'-b_1')$,
(iii) $k\widetilde{A}^I(ka_1,ka_2,ka_3;ka_1',ka_2,ka_3'),\ k>0$,
(iv) $k\widetilde{A}^I=(ka_3,ka_2,ka_1;ka_3',ka_2,ka_1'),\ k<0$.

Definition 9: For TIFN $\widetilde{A}^I=(a_1,a_2,a_3;a_1',a_2,a_3')$, its accuracy function is defined as follows:

$$f(\widetilde{A}^I)=\frac{(a_1+2a_2+a_3)+(a_1'+2a_2+a_3')}{8} \quad (1)$$

Singh and Yadav [37] proved that the accuracy function $f:IF(\mathrm{R})\rightarrow\mathrm{R}$, where $IF(\mathrm{R})$ is a set of TIFNs defined on a set of real numbers, is a linear function. They used this linear function for comparing two TIFNs $\widetilde{A}^I=(a_1,a_2,a_3;a_1',a_2,a_3')$ and $\widetilde{B}^I=(b_1,b_2,b_3;b_1',b_2,b_3')$.

Definition 10: Assume two TIFNs $\widetilde{A}^I=(a_1,a_2,a_3;a_1',a_2,a_3')$ and $\widetilde{B}^I=(b_1,b_2,b_3;b_1',b_2,b_3')$, with $f(\widetilde{A}^I)$ and $f(\widetilde{B}^I)$ as their accuracy functions, respectively. Then

(i) $\widetilde{A}^I\geqslant\widetilde{B}^I$ if $f(\widetilde{A}^I)\geqslant f(\widetilde{B}^I)$,
(ii) $\widetilde{A}^I\leqslant\widetilde{B}^I$ if $f(\widetilde{A}^I)\leqslant f(\widetilde{B}^I)$,
(iii) $\widetilde{A}^I=\widetilde{B}^I$ if $f(\widetilde{A}^I)=f(\widetilde{B}^I)$.

3. Intuitionistic Fuzzy Balanced Transportation Problems

Singh and Yadav [37] applied an interesting methodology in solving a TP having uncertainty as well as hesitation in prediction of the transportation costs. One defines a TP having intuitionistic fuzzy transportation costs but crisp availabilities and demands as an intuitionistic Fuzzy Balanced Transportation Problem of type-2 (IFBTP-2).

The IFBTP-2, in which a decision maker considers the cost as TIFN to deal efficiently with the uncertainty as well as hesitation arising in prediction of transportation cost, but (s)he is sure about the availability and demand of the product, can be formulated as follows (see Ref. 37):

$$\begin{aligned}\min\quad & \widetilde{Z}^I=\sum_{i=1}^{m}\sum_{j=1}^{n}\widetilde{c}_{ij}^I x_{ij}\\ s.t.\quad & \sum_{j=1}^{n}x_{ij}=a_i,\quad i=1,2,...,m,\\ & \sum_{i=1}^{m}x_{ij}=b_j,\quad j=1,2,...,n,\\ & x_{ij}\geqslant 0,\ i=1,2,...,m,\ j=1,2,...,n.\end{aligned} \quad (2)$$

where a_i is: the total availability of the product at i^{th} source; b_j: the total demand of the product at j^{th} destination; $\widetilde{c}_{ij}^I=(c_1^{ij},c_2^{ij},c_3^{ij};c_1^{ij'},c_2^{ij},c_3^{ij'})$: the intuitionistic cost for transporting one unit quantity of the product from the i^{th} source to the j^{th} destination; x_{ij}: the quantity transported from the i^{th} source to the j^{th} destination or decision variables; $\sum_{i=1}^{m}\sum_{j=1}^{n}\widetilde{c}_{ij}^{I}x_{ij}$: total intuitionistic fuzzy transportation cost.

Singh and Yadav [37] considered the linear ordering given in (1) to develop methods for obtaining the initial basic feasible solution (IBFS) of the IFBTP-2 given in (2). Then one generalizes an intuitionistic fuzzy method to test the optimality of the obtained IBFS using the same ordering. To do so, they rewrote the IFBTP-2 given in (2) as an Intuitionistic Fuzzy Linear Programming problem and proved some theorems concerning optimality conditions and duality properties. Let $\widetilde{u}_i^I = (u_1^i, u_2^i, u_3^i; u_1^{i'}, u_2^i, u_3^{i'})$ and $\widetilde{v}_j^I = (v_1^j, v_2^j, v_3^j; v_1^{j'}, v_2^j, v_3^{j'})$ be the intuitionistic fuzzy dual variables associated with i^{th} row and j^{th} column constraints, respectively, then the intuitionistic fuzzy dual of the IFBTP-2 given in Eq. (2) is defined as follows (see Ref. 37):

$$\begin{array}{ll} \max & \widetilde{Z}_D^I = \sum_{i=1}^{m} a_i \widetilde{u}_i^I \oplus \sum_{j=1}^{n} b_j \widetilde{v}_j^I \\ s.t. & \widetilde{u}_i^I \oplus \widetilde{v}_j^I \leqslant \widetilde{c}_{ij}^I \quad i=1,2,\ldots,m,\ j=1,2,\ldots,n. \end{array} \tag{3}$$

Based on the linear function given in (1), Singh and Yadav [37] introduced three methods, namely intuitionistic fuzzy North West corner method (IFNWCM), intuitionistic fuzzy least-cost method (IFLCM) and intuitionistic fuzzy Vogel's approximation method (IFVAM) to find the initial IBFS for the IFBTP-2 given in Eq. (2). Then, they utilized the intuitionistic fuzzy modified distribution method (IFMODIM) to find the fuzzy optimal solution for the IFBTP-2 (2) with the help of IBFS. They also proved some theorems to provide optimality criteria for the obtained IBFS.

The resulting theorem is summarized as follows (see Ref. 37):

Theorem 1. *Let IFBTP-2 (2) has a basic feasible solution (BFS) with B as a basis matrix. If $\widetilde{d}_{ij}^I = \widetilde{u}_i^I \oplus \widetilde{v}_j^I \leqslant \widetilde{c}_{ij}^I$ for all non-basic variables, the current BFS is optimal.*

In the proposed approach by Singh and Yadav [37] all arithmetic operations are performed on the triangular intuitionistic fuzzy numbers, i.e., $\widetilde{u}_i^I$, $\widetilde{v}_j^I$ and $\widetilde{c}_{ij}^I$. In the following section, we show that it is possible to find the same solution as the IFBTP-2 (2) without solving any intuitionistic fuzzy problems and so all arithmetic operations are done on real numbers instead of triangular intuitionistic fuzzy numbers.

4. An efficient computational approach

In this section an efficient computational solution approach is proposed for solving intuitionistic fuzzy TP, based on classical transportation algorithms to diminish the amount of computations of the Sing and Yadav's solution approach.

In the algorithm proposed by Singh and Yadav [37], the linear accuracy function (1) has been used to compare between triangular intuitionistic fuzzy numbers. In such a case by using this linear ranking function it is possible to define a rank for each triangular intuitionistic fuzzy number. In fact, assuming that $\widetilde{A}^I = (a_1, a_2, a_3; a_1', a_2, a_3')$ is a triangular intuitionistic fuzzy number, then $f(\widetilde{A}^I) = \frac{(a_1+2a_2+a_3)+(a_1'+2a_2+a_3')}{8}$. This equation enables us to convert the IFBTP-2 (2) into a crisp TP. To do this, we substitute the rank of each triangular intuitionistic fuzzy number instead of the corresponding triangular intuitionistic fuzzy number in the problem under consideration. This leads to an equivalent crisp TP that can be solved by traditional transportation algorithms. In summary, once the ranking function is chosen, the intuitionistic fuzzy TP under consideration is converted into a crisp one, which is easily solved by the existing transportation simplex methods. Therefore, it is possible to obtain the optimal solution of the IFBTP-2 (2) problem, without the need for a fuzzy approach. As a result, the computational amount is decreased significantly in our proposed approach.

Our main contribution in this study is the reduction of the amount of computations for solving the IFBTP-2 (2) compared to the intuitionistic fuzzy transportation algorithm proposed by Singh and Yadav [37]. In particular, in what follows it is shown in detail that our method needs fewer elementary operations such as additions, subtractions and comparisons than the aforementioned method.

Two main steps of the intuitionistic fuzzy transportation algorithm proposed by Singh and Yadav [37] for solving IFBTP-2 (2) are summarized as follows:
Step 1: Find an initial BFS using IFNWCM, IFLCM or IFVAM.
Step 2: Find the optimal solution of IFBTP-2 using the intuitionistic fuzzy modified distribution method (IFMODIM).

First, the computation effort required for finding an initial BFS using the proposed method and the method of Singh and Yadav [37] are compared.

It should be noted that in the IFBTP-2 (2) only the transportation cost is represented by triangular intuitionistic fuzzy numbers. It can be seen that in the IFNWCM proposed by Singh and Yadav [37], the intuitionistic fuzzy costs have no role on finding initial BFS. Thus this method needs the same amount of computations compared to the standard North West corner method for finding an initial BFS. In addition, in the IFLCM proposed by them it is required to determine the smallest intuitionistic fuzzy cost in the IFBTP table using the accuracy function given in (1) for each iteration. Thus in this approach no addition and subtraction operations are done on intuitionistic fuzzy costs to obtain an initial BFS. In this approach, the comparison between intuitionistic fuzzy costs is only done once using the accuracy function given in (1) to determine the smallest one.

However, to find an initial BFS according to IFVAM, it is required to

(1) Compute the intuitionistic penalty for each row and each column of the IFBTP table of order $m \times n$,
(2) Select the highest intuitionistic penalty using the accuracy function,
(3) Determine the cell with the smallest intuitionistic cost in the selected row or column using the accuracy function,
(4) Allocate as much as possible to the variable corresponding to the determined cell, adjust the supply and demand and delete the satisfied row or column,
(5) Repeat the process for the reduced table until the table is reduced to1×1.

Remark 1: The penalty measure for each row (column) is determined by subtracting the smallest unit intuitionistic fuzzy cost element in the row (column) from the next smallest unit intuitionistic fuzzy cost element in the same row (column).

Regarding Remark 1 and the process of IFVAM, the main drawback of this method is that it requires a lot of fuzzy subtractions of TIFNs and a lot of comparisons based on accuracy function on TIFNs to compute the intuitionistic penalty measures of each row and each column and to identify the row or column with the largest intuitionistic penalty.

In sum, our proposed method and the existing method of Singh and Yadav [37] have a same computation effort if IFNWCM or IFLCM is used to obtain the initial BFS of IFBTP-2 (2). However, in our approach all the triangular intuitionistic fuzzy costs are changed to crisp numbers according to the accuracy function. Thus, comparison of triangular intuitionistic fuzzy costs is done once and all arithmetic operations are performed on real numbers. Due to these facts, our method here needs less computation effort in comparison to the method proposed by Singh and Yadav [37] if IFVAM is used to obtain the initial BFS of IFBTP-2 (2).

Now, the computation effort required for finding the optimal solution of IFBTP-2 (2) using the presented method and the method proposed by Singh and Yadav [37] is compared.

Let us suppose that an initial BFS obtained using IFNWCM, IFLCM or IFVAM with basis B is at hand. The main steps of the intuitionistic fuzzy modified distribution method (IFMODIM) proposed by Singh and Yadav [37] to find the intuitionistic fuzzy optimal solution for the IFBTP-2 (2) are summarized as follows:
Step 1: For each cell (i, j), define intuitionistic fuzzy dual variables $\widetilde{u}_i^I = (u_1^i, u_2^i, u_3^i; u_1^{i'}, u_2^i, u_3^{i'})$ and $\widetilde{v}_j^I = (v_1^j, v_2^j, v_3^j; v_1^{j'}, v_2^j, v_3^{j'})$ associated with i^{th} row and j^{th} column, respectively.
Step 2: Solve the intuitionistic fuzzy system $\widetilde{u}_i^I \oplus \widetilde{v}_j^I = \widetilde{c}_{ij}^I$ for each basic cell (i, j).
Step 3: Compute $\widetilde{d}_{ij} = \widetilde{u}_i^I \oplus \widetilde{v}_j^I \ominus \widetilde{c}_{ij}^I$ for each non-basic cell (i, j). If $f(\widetilde{d}_{ij}) \leqslant 0$ for each non-basic cell

$(i,\ j)$, then stop, the current BFS is optimal. Otherwise select an entering cell (a non-basic cell with the most positive $f(\widetilde{d}^I_{ij})$).

Step 4: Determine an existing cell, obtain the new BFS using the standard transportation methods and repeat Step 1.

Our main contribution here is the reduction of the computational complexity of the method proposed by Singh and Yadav [37]. In particular, it is shown that our proposed method needs a lower number of elementary operations such as additions, multiplications, and comparisons as compared to the method proposed by Singh and Yadav [37] for testing the optimality conditions.

According to the method proposed by Singh and Yadav [37], to carry out Step 2 of the IFMODIM it is required to solve the intuitionistic fuzzy system $\widetilde{u}^I_i + \widetilde{v}^I_j = \widetilde{c}^I_{ij}$ with $m+n-1$ intuitionistic fuzzy equations corresponding to basic cells. After solving this intuitionistic fuzzy system, the intuitionistic fuzzy value $\widetilde{d}^I_{ij}$ for each non-basic cell is obtained based on $\widetilde{d}_{ij} = \widetilde{u}^I_i \oplus \widetilde{v}^I_j \ominus \widetilde{c}^I_{ij}$. Finally the entering cell is determined according to the most positive rank of $\widetilde{d}^I_{ij}$. These facts ensure that this step requires a lot of intuitionistic fuzzy additions and subtractions on TIFNs. While based on our proposed methods, the optimality criteria are checked without solving any intuitionistic fuzzy system, without any intuitionistic fuzzy arithmetic operations and without any comparison of TIFNs. These results confirm that the proposed method is simpler and computationally more efficient than the method proposed by Singh and Yadav [37].

As a final point, in the next theorem we mathematically prove that the results of the method proposed by Singh and Yadav [37] and the proposed method for solving IFBTP-2 (2) are the same.

Theorem 2. *The optimal solution of the IFBTP-2 (2) according to the existing method and the proposed method is the same.*

Proof. According to the proposed approach, using the linear accuracy function (1) we substitute the rank of each triangular intuitionistic fuzzy transportation cost instead of the corresponding triangular intuitionistic fuzzy transportation cost in IFBTP-2 (2). This leads to the following crisp TP:

$$\begin{aligned} \min \quad & f(\widetilde{Z}^I) = \sum_{i=1}^{m}\sum_{j=1}^{n} f(\widetilde{c}^I_{ij})x_{ij} \\ s.t. \quad & \sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2,...,m, \\ & \sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2,...,n, \\ & x_{ij} \geqslant 0,\ i = 1,2,...,m,\ j = 1,2,...,n. \end{aligned} \tag{4}$$

Let $f(\widetilde{u}^I_i)$ and $f(\widetilde{v}^I_j)$ be the dual variables associated with i^{th} row and j^{th} column constraints, respectively. In this case, the dual of the TP (4) is given as follows:

$$\begin{aligned} \max \quad & f(\widetilde{Z}^I_D) = \sum_{i=1}^{m} a_i f(\widetilde{u}^I_i) + \sum_{j=1}^{n} b_j f(\widetilde{v}^I_j) \\ s.t. \quad & f(\widetilde{u}^I_i) + f(\widetilde{v}^I_j) \leqslant f(\widetilde{c}^I_{ij}), \\ & i = 1,2,...,m,\ j = 1,2,...,n. \end{aligned} \tag{5}$$

Therefore, if the basis B is the optimal basis of the crisp TP (4), then we have $f(\widetilde{u}^I_i) \oplus f(\widetilde{v}^I_j) \leqslant f(\widetilde{c}^I_{ij})$ as the optimality conditions of TP (4).

It should be noted that in order to obtain the optimal solution according to the method proposed by Singh and Yadav [37] and our proposed method, the IFBTP-2 (2) and the crisp TP (4) are solved, respectively. If we show that these problems have the same optimal solution, we conclude that the results of our proposed approach are matched with those obtained based on the method proposed by Singh and Yadav [37]. Note that both problems have a same feasible space. Thus, it is sufficient to show that both problems have the same optimality conditions. In fact, if the basis B is the optimal basis of the IFBTP-2 (2), then it will be the optimal basis of the equivalent crisp TP (4).

To do this, suppose that $x^* = (x^*_{ij})_{1\times mn}$ is an optimal solution of the IFBTP-2 (2) with B as the optimal basis. Thus, according to Theorem 1 we have $\widetilde{u}^I_i \oplus \widetilde{v}^I_j \leqslant \widetilde{c}^I_{ij}$ for all non-basic variables. With regard to Definition 10, these conditions are equivalent to $f(\widetilde{u}^I_i \oplus \widetilde{v}^I_j) \leqslant f(\widetilde{c}^I_{ij})$. Since the accuracy function is linear, we have $f(\widetilde{u}^I_i \oplus \widetilde{v}^I_j) = f(\widetilde{u}^I_i) +$

Table 1. Summary of the intuitionistic FTP.

	D_1	D_2	D_3	D_4	a_i
S_1	(2, 4, 5; 1, 4, 6)	(2, 5, 7; 1, 5, 8)	(4, 6, 8; 3, 6, 9)	(4, 7, 8; 3, 7, 9)	11
S_2	(4, 6, 8; 3, 6, 9)	(3, 7, 12; 2, 7, 13)	(10, 15, 20; 8, 15, 22)	(11, 12, 13; 10, 12, 14)	11
S_3	(3, 4, 6; 1, 4, 8)	(8, 10, 13; 5, 10, 16)	(2, 3, 5; 1, 3, 6)	(6, 10, 14; 5, 10, 15)	11
S_4	(2, 4, 6; 1, 4, 7)	(3, 9, 10; 2, 9, 12)	(3, 6, 10; 2, 6, 12)	(3, 4, 5; 2, 4, 8)	12
b_j	16	10	8	11	45

$f(\tilde{v}^I_j)$. This means that $\tilde{u}^I_i \oplus \tilde{v}^I_j \leqslant \tilde{c}^I_{ij}$ if and only if $f(\tilde{u}^I_i) + f(\tilde{v}^I_j) \leqslant f(\tilde{c}^I_{ij})$. Thus, we conclude that $x^* = (x^*_{ij})_{1\times mn}$ is the optimal solution of the TP (4). This completes the proof. □

5. Numerical examples

In this section, in order to demonstrate the effectiveness of the proposed method, two intuitionistic fuzzy TP taken from Singh and Yadav [37] are considered.

Example 5.1: Table 1 gives the crisp availability (a_i) of the product available at four origins $S_i, i = 1,2,3,4$ and the crisp demand (b_j) at four destinations D_j, $j = 1,2,3,4$. The transportation costs from origins to destinations are represented by intuitionistic fuzzy triangular fuzzy numbers. The aim is to find the least total intuitionistic fuzzy transportation cost of the commodity in order to satisfy demands at destinations using available availabilities at origins.

According to the accuracy function given in (1), we substitute the rank order of each intuitionistic fuzzy transportation cost (given in Table 1) with its corresponding intuitionistic fuzzy number to obtain the classical transportation problem. The results are given in Table 2.

Table 2. Summary of the classical transportation problem.

	D_1	D_2	D_3	D_4	a_i
S_1	$\frac{30}{8}$	$\frac{38}{8}$	$\frac{48}{8}$	$\frac{52}{8}$	11
S_2	$\frac{48}{8}$	$\frac{58}{8}$	$\frac{120}{8}$	$\frac{96}{8}$	11
S_3	$\frac{34}{8}$	$\frac{82}{8}$	$\frac{26}{8}$	$\frac{80}{8}$	11
S_4	$\frac{32}{8}$	$\frac{63}{8}$	$\frac{51}{8}$	$\frac{34}{8}$	12
b_j	16	10	8	11	45

In what follows, we derive the optimal solution of the given intuitionistic fuzzy transportation problem based on the method proposed by Singh and Yadav [37] according to Table 1 and our proposed method according to Table 2.

The initial BFS of Table 1 can be found by any one of the IFNWCM, IFLCM or IFVAM methods proposed by Singh and Yadav [37]. Also, the initial BFS of Table 2 can be found by any one of the classical North West corner method (NWCM), least-cost method (LCM) or Vogel's approximation method (VAM) that correspond to our proposed method.

5.1. Results based on IFNWCM and NWCM

Both the IFNWCM proposed by Singh and Yadav [37] and the classical NWCM give the initial BFS given in Table 3.

Table 3. Initial BFS by IFNWCM.

	D_1	D_2	D_3	D_4	a_i
S_1	11	-	-	-	11
S_2	5	6	-	-	11
S_3	-	4	7	-	11
S_4	-	-	1	11	12
b_j	16	10	8	11	45

5.1.1. Iteration 1(IFNWCM)

According to Step 1 and Step 2 of the IFMODIM proposed by Singh and Yadav [37] the following intuitionistic fuzzy system should be solved to test the optimality of the initial BFS given in Table 3:

$$
\begin{aligned}
&(u_1^1,u_2^1,u_3^1;u_1^{1'},u_2^1,u_3^{1'})+(v_1^1,v_2^1,v_3^1;v_1^{1'},v_2^1,v_3^{1'})=\widetilde{c}_{11}^I\\
&=(2,\ 4,\ 5;\ 1,\ 4,\ 6)\\
&(u_1^2,u_2^2,u_3^2;u_1^{2'},u_2^2,u_3^{2'})+(v_1^1,v_2^1,v_3^1;v_1^{1'},v_2^1,v_3^{1'})=\widetilde{c}_{21}^I\\
&=(4,\ 6,\ 8;\ 3,\ 6,\ 9)\\
&(u_1^2,u_2^2,u_3^2;u_1^{2'},u_2^2,u_3^{2'})+(v_1^2,v_2^2,v_3^2;v_1^{2'},v_2^2,v_3^{2'})=\widetilde{c}_{22}^I\\
&=(3,\ 7,\ 12;\ 2,\ 7,\ 13)\\
&(u_1^3,u_2^3,u_3^3;u_1^{3'},u_2^3,u_3^{3'})+(v_1^2,v_2^2,v_3^2;v_1^{2'},v_2^2,v_3^{2'})=\widetilde{c}_{32}^I\\
&=(8,\ 10,\ 13;\ 5,\ 10,\ 16)\\
&(u_1^3,u_2^3,u_3^3;u_1^{3'},u_2^3,u_3^{3'})+(v_1^3,v_2^3,v_3^3;v_1^{3'},v_2^3,v_3^{3'})=\widetilde{c}_{33}^I\\
&=(2,\ 3,\ 5;\ 1,\ 3,\ 6)\\
&(u_1^4,u_2^4,u_3^4;u_1^{4'},u_2^4,u_3^{4'})+(v_1^3,v_2^3,v_3^3;v_1^{3'},v_2^3,v_3^{3'})=\widetilde{c}_{43}^I\\
&=(3,\ 6,\ 10;\ 2,\ 6,\ 12)\\
&(u_1^4,u_2^4,u_3^4;u_1^{4'},u_2^4,u_3^{4'})+(v_1^4,v_2^4,v_3^4;v_1^{4'},v_2^4,v_3^{4'})=\widetilde{c}_{44}^I\\
&=(3,\ 4,\ 5;\ 2,\ 4,\ 8)
\end{aligned}
\tag{6}
$$

The intuitionistic fuzzy solution of this system is given as follows (see Ref. 37):

$$
\begin{aligned}
\widetilde{u}_1^I&=(u_1^1,u_2^1,u_3^1;u_1^{1'},u_2^1,u_3^{1'})\\
&=(-24,-8,7;-33,-8,15),\\
\widetilde{v}_1^I&=(v_1^1,v_2^1,v_3^1;v_1^{1'},v_2^1,v_3^{1'})\\
&=(-2,12,26;-9,12,34)\\
\widetilde{u}_2^I&=(u_1^2,u_2^2,u_3^2;u_1^{2'},u_2^2,u_3^{2'})\\
&=(-18,-6,6;-25,-6,12),\\
\widetilde{v}_2^I&=(v_1^2,v_2^2,v_3^2;v_1^{2'},v_2^2,v_3^{2'})\\
&=(6,13,21;1,13,27)\\
\widetilde{u}_3^I&=(u_1^3,u_2^3,u_3^3;u_1^{3'},u_2^3,u_3^{3'})\\
&=(-8,-3,2;-11,-3,4),\\
\widetilde{v}_3^I&=(v_1^3,v_2^3,v_3^3;v_1^{3'},v_2^3,v_3^{3'})\\
&=(3,6,10;2,6,12),\\
\widetilde{u}_4^I&=(u_1^4,u_2^4,u_3^4;u_1^{4'},u_2^4,u_3^{4'})\\
&=(0,0,0;0,0,0),\\
\widetilde{v}_4^I&=(v_1^4,v_2^4,v_3^4;v_1^{4'},v_2^4,v_3^{4'})\\
&=(3,4,5;2,4,8)
\end{aligned}
\tag{7}
$$

According to Step 3 of the IFMODIM proposed by Singh and Yadav [37], it is required to compute $\widetilde{d}_{ij}^I=\widetilde{u}_i^I\oplus\widetilde{v}_j^I\ominus\widetilde{c}_{ij}^I$ for each non-basic cell$(i,\ j)$. Thus, we have:

$$
\begin{aligned}
&\widetilde{d}_{12}^I=\widetilde{u}_1^I\oplus\widetilde{v}_2^I\ominus\widetilde{c}_{12}^I=(-24,-8,7;-33,-8,15)\oplus\\
&(6,13,21;1,13,27)\ominus(2,\ 5,\ 7;\ 1,\ 5,\ 8)\\
&=(-25,0,26;-40,0,41),\\
&\widetilde{d}_{13}^I=\widetilde{u}_1^I\oplus\widetilde{v}_3^I\ominus\widetilde{c}_{13}^I=(-24,-8,7;-33,-8,15)\oplus\\
&(3,6,10;2,6,12)\ominus(4,\ 6,\ 8;\ 3,\ 6,\ 9)\\
&=(-29,-8,13;-40,-8,24),\\
&\widetilde{d}_{14}^I=\widetilde{u}_1^I\oplus\widetilde{v}_4^I\ominus\widetilde{c}_{14}^I=(-24,-8,7;-33,-8,15)\oplus\\
&(3,4,5;2,4,8)\ominus(4,\ 7,\ 8;\ 3,\ 7,\ 9)\\
&=(-29,-11,8;-40,-11,20),\\
&\widetilde{d}_{23}^I=\widetilde{u}_2^I\oplus\widetilde{v}_3^I\ominus\widetilde{c}_{23}^I=(-18,-6,6;-25,-6,12)\oplus\\
&(3,6,10;2,6,12)\ominus(10,\ 15,\ 20;\ 8,\ 15,\ 22)\\
&=(-35,-15,6;-45,-15,16),\\
&\widetilde{d}_{24}^I=\widetilde{u}_2^I\oplus\widetilde{v}_4^I\ominus\widetilde{c}_{24}^I=(-18,-6,6;-25,-6,12)\oplus\\
&(3,4,5;2,4,8)\ominus(11,\ 12,\ 13;\ 10,\ 12,\ 14)\\
&=(-28,-14,0;-37,-14,10),\\
&\widetilde{d}_{31}^I=\widetilde{u}_3^I\oplus\widetilde{v}_1^I\ominus\widetilde{c}_{31}^I=(-8,-3,2;-11,-3,4)\oplus\\
&(-2,12,26;-9,12,34)\ominus(3,\ 4,\ 6;\ 1,\ 4,\ 8)\\
&=(-16,5,25;-28,5,37),\\
&\widetilde{d}_{34}^I=\widetilde{u}_3^I\oplus\widetilde{v}_4^I\ominus\widetilde{c}_{34}^I=(-8,-3,2;-11,-3,4)\oplus\\
&(3,4,5;2,4,8)\ominus(6,\ 10,\ 14;\ 5,\ 10,\ 15)\\
&=(-19,\ -9,\ 1;\ -24,\ -9,\ 7),\\
&\widetilde{d}_{41}^I=\widetilde{u}_4^I\oplus\widetilde{v}_1^I\ominus\widetilde{c}_{41}^I=(0,0,0;0,0,0)\oplus\\
&(-2,12,26;-9,12,34)\ominus(2,\ 4,\ 6;\ 1,\ 4,\ 7)\\
&=(-8,8,24;-16,8,33),\\
&\widetilde{d}_{42}^I=\widetilde{u}_4^I\oplus\widetilde{v}_2^I\ominus\widetilde{c}_{42}^I=(0,0,0;0,0,0)\oplus\\
&(6,13,21;1,13,27)\ominus(3,\ 9,\ 10;\ 2,\ 9,\ 12)\\
&=(-4,4,18;-11,4,25).
\end{aligned}
\tag{8}
$$

Also,

$$
\begin{aligned}
&f(\widetilde{d}_{12}^I)=\tfrac{2}{8},f(\widetilde{d}_{13}^I)=-8,f(\widetilde{d}_{14}^I)=\tfrac{-85}{8},\\
&f(\widetilde{d}_{23}^I)=\tfrac{-118}{8},f(\widetilde{d}_{24}^I)=\tfrac{-111}{8},f(\widetilde{d}_{31}^I)=\tfrac{38}{8},\\
&f(\widetilde{d}_{34}^I)=\tfrac{-71}{8},f(\widetilde{d}_{41}^I)=\tfrac{65}{8},f(\widetilde{d}_{42}^I)=\tfrac{44}{8},
\end{aligned}
\tag{9}
$$

Since $f(\widetilde{d}_{ij}^I)\nleq 0$ for all non-basic cells $(i,\ j)$, then the current BFS is not optimal. Thus, a non-basic cell with the most positive $f(\widetilde{d}_{ij}^I)$, i.e., x_{41} is selected as the entering variable. According to the classical transportation algorithm, x_{43} is selected as the leaving variable and the new BFS is found as given in Table 4.

Table 4. Improved solution-1.

	D_1	D_2	D_3	D_4	a_i
S_1	11	-	-	-	11
S_2	4	7	-	-	11
S_3	-	3	8	-	11
S_4	1	-	-	11	12
b_j	16	10	8	11	45

5.1.2. Iteration 1(NWCM)

According to the classical modified distribution method (MODIM) applied by our proposed approach the following crisp system should be solved to test the optimality of the initial BFS given in Table 3:

$$\begin{array}{l} u_1+v_1=c_{11}=\frac{30}{8}, u_2+v_1=c_{21}=6,\\ u_2+v_2=c_{22}=\frac{58}{8}, u_3+v_2=c_{32}=\frac{82}{8},\\ u_3+v_3=c_{33}=\frac{26}{8}, u_4+v_3=c_{43}=\frac{51}{8},\\ u_4+v_4=c_{44}=\frac{34}{8} \end{array} \tag{10}$$

The solution of this crisp system is given as follows:

$$\begin{array}{l} u_1=\frac{-67}{8}, u_2=\frac{-49}{8}, u_3=\frac{-25}{8}, u_4=0\\ v_1=\frac{97}{8}, v_2=\frac{107}{8}, v_3=\frac{51}{8}, v_4=\frac{34}{8} \end{array} \tag{11}$$

To test the optimality of the solution given in Table 4 it is required to compute $d_{ij}=u_i+v_j-c_{ij}$ for each non-basic cell $(i,\, j)$. Thus, we have:

$$\begin{array}{l} d_{12}=u_1+v_2-c_{12}=\frac{-67}{8}+\frac{107}{8}-\frac{38}{8}=\frac{2}{8},\\ d_{13}=u_1+v_3-c_{13}=\frac{-67}{8}+\frac{51}{8}-\frac{48}{8}=8,\\ d_{14}=u_1+v_4-c_{14}=\frac{-67}{8}+\frac{34}{8}-\frac{52}{8}=\frac{-85}{8},\\ d_{23}=u_2+v_3-c_{23}=\frac{-49}{8}+\frac{51}{8}-\frac{120}{8}=\frac{-118}{8},\\ d_{24}=u_2+v_4-c_{24}=\frac{-49}{8}+\frac{34}{8}-\frac{96}{8}=\frac{-111}{8},\\ d_{31}=u_3+v_1-c_{31}=\frac{-25}{8}+\frac{97}{8}-\frac{34}{8}=\frac{38}{8},\\ d_{34}=u_3+v_4-c_{34}=\frac{-25}{8}+\frac{34}{8}-\frac{80}{8}=\frac{-71}{8},\\ d_{41}=u_4+v_1-c_{41}=0+\frac{97}{8}-\frac{32}{8}=\frac{65}{8},\\ d_{42}=u_4+v_2-c_{42}=0+\frac{107}{8}-\frac{63}{8}=\frac{-44}{8}. \end{array} \tag{12}$$

Since $d_{ij} \nleq 0$ for all non-basic cells $(i,\, j)$, then the current BFS is not optimal. Thus, according to the classical transportation algorithm in crisp environment, x_{41} and x_{43} are selected as the entering variable and the leaving variable, respectively, and thus the new BFS is found as given in Table 4 matching with the result of the improved solution-1 of the method proposed by Singh and Yadav [37]. However, to find the improved solution-1 given in Table 4 it is necessary to solve the intuitionistic fuzzy system (6). After solving this system, the intuitionistic fuzzy value $\widetilde{d}^I_{ij}=\widetilde{u}^I_i \oplus \widetilde{v}^I_j \ominus \widetilde{c}^I_{ij}$ for each non-basic cell is calculated according to (8). As we see, these two steps require a large number of fuzzy additions and subtractions on TIFNs. While based on our proposed approach, these steps are done with solving the crisp system (10) and with calculating the crisp values given in (12) using the elementary operation on real numbers. Moreover, to choose the entering variable according to the method proposed by Singh and Yadav [37] it is necessary to compare the intuitionistic fuzzy value $\widetilde{d}^I_{ij}$ for each non-basic cell as done in (9). While according to our proposed method, the entering variable is selected without any comparison of TIFNs. Due to these facts, our proposed method is preferred to the method proposed by Singh and Yadav [37] from the computational attempt point of view.

5.1.3. Iteration 2(IFNWCM)

According to Step 1 and Step 2 of the IFMODIM proposed by Singh and Yadav [37] the following intuitionistic fuzzy system should be solved to test the optimality of the improved solution-1 given in Table 4:

$$\begin{array}{l} (u^1_1,u^1_2,u^1_3;u^{1'}_1,u^1_2,u^{1'}_3)+(v^1_1,v^1_2,v^1_3;v^{1'}_1,v^1_2,v^{1'}_3)=\widetilde{c}^I_{11}\\ =(2,\,4,\,5;\,1,\,4,\,6),\\ (u^2_1,u^2_2,u^2_3;u^{2'}_1,u^2_2,u^{2'}_3)+(v^1_1,v^1_2,v^1_3;v^{1'}_1,v^1_2,v^{1'}_3)=\widetilde{c}^I_{21}\\ =(4,\,6,\,8;\,3,\,6,\,9),\\ (u^2_1,u^2_2,u^2_3;u^{2'}_1,u^2_2,u^{2'}_3)+(v^2_1,v^2_2,v^2_3;v^{2'}_1,v^2_2,v^{2'}_3)=\widetilde{c}^I_{22}\\ =(3,\,7,\,12;\,2,\,7,\,13),\\ (u^3_1,u^3_2,u^3_3;u^{3'}_1,u^3_2,u^{3'}_3)+(v^2_1,v^2_2,v^2_3;v^{2'}_1,v^2_2,v^{2'}_3)=\widetilde{c}^I_{32}\\ =(8,\,10,\,13;\,5,\,10,\,16),\\ (u^3_1,u^3_2,u^3_3;u^{3'}_1,u^3_2,u^{3'}_3)+(v^3_1,v^3_2,v^3_3;v^{3'}_1,v^3_2,v^{3'}_3)=\widetilde{c}^I_{33}\\ =(2,\,3,\,5;\,1,\,3,\,6),\\ (u^4_1,u^4_2,u^4_3;u^{4'}_1,u^4_2,u^{4'}_3)+(v^1_1,v^1_2,v^1_3;v^{1'}_1,v^1_2,v^{1'}_3)=\widetilde{c}^I_{41}\\ =(2,\,4,\,6;\,1,\,4,\,7),\\ (u^4_1,u^4_2,u^4_3;u^{4'}_1,u^4_2,u^{4'}_3)+(v^4_1,v^4_2,v^4_3;v^{4'}_1,v^4_2,v^{4'}_3)=\widetilde{c}^I_{44}\\ =(3,\,4,\,5;\,2,\,4,\,8). \end{array} \tag{13}$$

The intuitionistic fuzzy solution of intuitionistic fuzzy system (13) is given as follows (see Ref. 37):

$$\begin{aligned}
&\widetilde{u}_1^I = (-6,-2,1;-8,-2,3), \widetilde{v}_1^I = (4,6,8;3,6,9)\\
&\widetilde{u}_2^I = (0,0,0;0,0,0), \widetilde{v}_2^I = (3,7,12;2,7,13)\\
&\widetilde{u}_3^I = (-4,-3,10;-8,3,14), \widetilde{v}_3^I = (-8,0,9;-13,0,14)\\
&\widetilde{u}_4^I = (-6,-2,2;-8,-2,4), \widetilde{v}_4^I = (1,6,11;-2,6,16)
\end{aligned} \tag{14}$$

Now, according to Step 3 of the IFMODIM proposed by Singh and Yadav [37], the intuitionistic fuzzy value $\widetilde{d}_{ij}^I = \widetilde{u}_i^I \oplus \widetilde{v}_j^I \ominus \widetilde{c}_{ij}^I$ for each non-basic cell is calculated as follows

$$\begin{aligned}
&\widetilde{d}_{12}^I = \widetilde{u}_1^I \oplus \widetilde{v}_2^I \ominus \widetilde{c}_{12}^I =\\
&(-6,-2,1;-8,-2,3) \oplus (3,7,12;2,7,13)\\
&\ominus(2,\ 5,\ 7;\ 1,\ 5,\ 8) = (-10,0,11;-14,0,15),\\
&\widetilde{d}_{13}^I = \widetilde{u}_1^I \oplus \widetilde{v}_3^I \ominus \widetilde{c}_{13}^I\\
&= (-6,-2,1;-8,-2,3) \oplus (-8,0,9;-13,0,14)\\
&\ominus(4,\ 6,\ 8;\ 3,\ 6,\ 9) = (-22,-8,6;-30,-8,14),\\
&\widetilde{d}_{14}^I = \widetilde{u}_1^I \oplus \widetilde{v}_4^I \ominus \widetilde{c}_{14}^I\\
&= (-6,-2,1;-8,-2,3) \oplus (1,6,11;-2,6,16)\\
&\ominus(4,\ 7,\ 8;\ 3,\ 7,\ 9) = (-13,-3,8;-19,-3,16),\\
&\widetilde{d}_{23}^I = \widetilde{u}_2^I \oplus \widetilde{v}_3^I \ominus \widetilde{c}_{23}^I\\
&= (0,0,0;0,0,0) \oplus (-8,0,9;-13,0,14)\\
&\ominus(10,\ 15,\ 20;\ 8,\ 15,\ 22)\\
&= (-28,-15,-1;-35,-15,6),\\
&\widetilde{d}_{24}^I = \widetilde{u}_2^I \oplus \widetilde{v}_4^I \ominus \widetilde{c}_{24}^I\\
&= (0,0,0;0,0,0) \oplus (1,6,11;-2,6,16)\\
&\ominus(11,\ 12,\ 13;\ 10,\ 12,\ 14)\\
&= (-12,-6,0;-16,-6,6),\\
&\widetilde{d}_{31}^I = \widetilde{u}_3^I \oplus \widetilde{v}_1^I \ominus \widetilde{c}_{31}^I\\
&= (-4,3,10;-8,3,14) \oplus (4,6,8;3,6,9)\\
&\ominus(3,\ 4,\ 6;\ 1,\ 4,\ 8) = (-6,5,15;-13,5,22),\\
&\widetilde{d}_{34}^I = \widetilde{u}_3^I \oplus \widetilde{v}_4^I \ominus \widetilde{c}_{34}^I\\
&= (-4,3,10;-8,3,14) \oplus (1,6,11;-2,6,16)\\
&\ominus(6,\ 10,\ 14;\ 5,\ 10,\ 15)\\
&= (-17,-1,15;-25,-1,25),\\
&\widetilde{d}_{42}^I = \widetilde{u}_4^I \oplus \widetilde{v}_2^I \ominus \widetilde{c}_{42}^I\\
&= (-6,-2,2;-8,-2,4) \oplus (3,7,12;2,7,13)\\
&\ominus(3,\ 9,\ 10;\ 2,\ 9,\ 12) = (-13,-4,11;-18,-4,15),\\
&\widetilde{d}_{43}^I = \widetilde{u}_4^I \oplus \widetilde{v}_3^I \ominus \widetilde{c}_{43}^I\\
&= (-6,-2,2;-8,-2,4) \oplus (-8,0,9;-13,0,14)\\
&\ominus(3,\ 6,\ 10;\ 2,\ 6,\ 12) = (-24,-8,8;-33,-8,16).
\end{aligned} \tag{15}$$

To test the optimality of the improved solution-1, the rank order of the intuitionistic fuzzy value $\widetilde{d}_{ij}^I$ given in (13) should be obtained using the accuracy function as follows:

$$\begin{aligned}
&f(\widetilde{d}_{12}^I) = \tfrac{2}{8}, f(\widetilde{d}_{13}^I) = -8, f(\widetilde{d}_{14}^I) = \tfrac{-20}{8},\\
&f(\widetilde{d}_{23}^I) = \tfrac{-118}{8}, f(\widetilde{d}_{24}^I) = \tfrac{-46}{8}, f(\widetilde{d}_{31}^I) = \tfrac{38}{8},\\
&f(\widetilde{d}_{34}^I) = \tfrac{-6}{8}, f(\widetilde{d}_{42}^I) = \tfrac{-21}{8}, f(\widetilde{d}_{43}^I) = \tfrac{-65}{8}.
\end{aligned} \tag{16}$$

Since $f(\widetilde{d}_{ij}^I) \not\leq 0$ for all non-basic cells $(i,\ j)$, then the improved solution-1 is not optimal. According to the IFMODIM proposed by Singh and Yadav [37] the improved solution-2 given in Table 5 is obtained.

Table 5. Improved solution-2.

	D_1	D_2	D_3	D_4	a_i
S_1	11	-	-	-	11
S_2	1	10	-	-	11
S_3	3	-	8	-	11
S_4	1	-	-	11	12
b_j	16	10	8	11	45

5.1.4. Iteration 2(NWCM)

Again it is demonstrated that the same improved solution can be obtained by our proposed approach and without solving any intuitionistic fuzzy system, without doing an arithmetic operations on the intuitionistic triangular fuzzy numbers and without any comparison of ITFNs.

According to the classical modified distribution method (MODIM) applied by our proposed approach the following crisp system should be solved to test the optimality of the improved solution-1 given in Table 4:

$$\begin{aligned}
&u_1 + v_1 = c_{11} = \tfrac{30}{8}, u_2 + v_1 = c_{21} = 6,\\
&u_2 + v_2 = c_{22} = \tfrac{58}{8}, u_3 + v_2 = c_{32} = \tfrac{82}{8},\\
&u_3 + v_3 = c_{33} = \tfrac{26}{8}, u_4 + v_1 = c_{41} = \tfrac{32}{8},\\
&u_4 + v_4 = c_{44} = \tfrac{34}{8}.
\end{aligned} \tag{17}$$

The solution of this crisp system is given as follows:

$$\begin{aligned}
&u_1 = \tfrac{-18}{8}, u_2 = 0, u_3 = 3, u_4 = -2\\
&v_1 = 6, v_2 = \tfrac{58}{8}, v_3 = \tfrac{2}{8}, v_4 = \tfrac{50}{8}
\end{aligned} \tag{18}$$

The crisp value of $d_{ij} = u_i + v_j - c_{ij}$ for each non-basic cell $(i,\ j)$ is calculated as follows:

$$
\begin{aligned}
d_{12} &= u_1+v_2-c_{12} = \tfrac{-18}{8}+\tfrac{58}{8}-\tfrac{38}{8} = \tfrac{2}{8},\\
d_{13} &= u_1+v_3-c_{13} = \tfrac{-18}{8}+\tfrac{2}{8}-\tfrac{48}{8} = 8,\\
d_{14} &= u_1+v_4-c_{14} = \tfrac{-18}{8}+\tfrac{50}{8}-\tfrac{52}{8} = \tfrac{-20}{8},\\
d_{23} &= u_2+v_3-c_{23} = 0+\tfrac{2}{8}-\tfrac{120}{8} = \tfrac{-118}{8},\\
d_{24} &= u_2+v_4-c_{24} = 0+\tfrac{50}{8}-\tfrac{96}{8} = \tfrac{-46}{8},\\
d_{31} &= u_3+v_1-c_{31} = 3+6-\tfrac{34}{8} = \tfrac{38}{8},\\
d_{34} &= u_3+v_4-c_{34} = 3+\tfrac{50}{8}-\tfrac{80}{8} = \tfrac{-6}{8},\\
d_{42} &= u_4+v_2-c_{42} = -2+\tfrac{58}{8}-\tfrac{63}{8} = \tfrac{-21}{8},\\
d_{43} &= u_4+v_3-c_{43} = -2+\tfrac{2}{8}-\tfrac{51}{8} = \tfrac{-65}{8}.
\end{aligned}
\tag{19}
$$

Since $f(\widetilde{d}^I_{ij}) \nleq 0$ for all non-basic cells (i, j), then the improved solution-1 is not optimal. According to the classical MODIM applied by our proposed approach, the improved solution-2 given in Table 5 is obtained. Thus, the results of our proposed approach are matched with those obtained based on the method proposed by Singh and Yadav [37]. However, regarding the process of finding the improved solutions, the method proposed in this study is far simpler and computationally much more efficient than the corresponding one proposed by Singh and Yadav [37].

It should be noted that in the next iteration, both our proposed method and the method proposed by Singh and Yadav [37] give the same optimal solution shown in Table 6 and the same total intuitionistic fuzzy transportation cost $\widetilde{Z}^I =$ (126, 204, 282; 78, 204, 352).

Table 6. Optimal solution.

	D_1	D_2	D_3	D_4	a_i
S_1	1	10	-	-	11
S_2	11	-	-	-	11
S_3	3	-	8	-	11
S_4	1	-	-	11	12
b_j	16	10	8	11	45

5.2. Results based on IFLCM and LCM

According to the IFLCM proposed by Singh and Yadav [37] and the classical LCM applied by our proposed approach the initial BFS given in Table 7 is obtained.

Table 7. Initial BFS by IFLCM and LCM.

	D_1	D_2	D_3	D_4	a_i
S_1	11	-	-	-	11
S_2	-	10	-	1	11
S_3	-	-	8	3	11
S_4	5	-	-	7	12
b_j	16	10	8	11	45

5.2.1. Iteration 1(IFLCM)

According to Step 1 and Step 2 of the IFMODIM proposed by Singh and Yadav [37] the following intuitionistic fuzzy system is solved to test the optimality of the initial BFS given in Table 7:

$$
\begin{aligned}
&(u_1^1,u_2^1,u_3^1;u_1^{1'},u_2^1,u_3^{1'})+(v_1^1,v_2^1,v_3^1;v_1^{1'},v_2^1,v_3^{1'})\\
&=\widetilde{c}^I_{11}=(2,\ 4,\ 5;\ 1,\ 4,\ 6),\\
&(u_1^2,u_2^2,u_3^2;u_1^{2'},u_2^2,u_3^{2'})+(v_1^2,v_2^2,v_3^2;v_1^{2'},v_2^2,v_3^{2'})\\
&=\widetilde{c}^I_{22}=(3,\ 7,\ 12;\ 2,\ 7,\ 13),\\
&(u_1^2,u_2^2,u_3^2;u_1^{2'},u_2^2,u_3^{2'})+(v_1^4,v_2^4,v_3^4;v_1^{4'},v_2^4,v_3^{4'})\\
&=\widetilde{c}^I_{24}=(11,\ 12,\ 13;\ 10,\ 12,\ 14),\\
&(u_1^3,u_2^3,u_3^3;u_1^{3'},u_2^3,u_3^{3'})+(v_1^3,v_2^3,v_3^3;v_1^{3'},v_2^3,v_3^{3'})\\
&=\widetilde{c}^I_{33}=(2,\ 3,\ 5;\ 1,\ 3,\ 6),\\
&(u_1^3,u_2^3,u_3^3;u_1^{3'},u_2^3,u_3^{3'})+(v_1^4,v_2^4,v_3^4;v_1^{4'},v_2^4,v_3^{4'})\\
&=\widetilde{c}^I_{34}=(6,\ 10,\ 14;\ 5,\ 10,\ 15),\\
&(u_1^4,u_2^4,u_3^4;u_1^{4'},u_2^4,u_3^{4'})+(v_1^1,v_2^1,v_3^1;v_1^{1'},v_2^1,v_3^{1'})\\
&=\widetilde{c}^I_{41}=(2,\ 4,\ 6;\ 1,\ 4,\ 7),\\
&(u_1^4,u_2^4,u_3^4;u_1^{4'},u_2^4,u_3^{4'})+(v_1^4,v_2^4,v_3^4;v_1^{4'},v_2^4,v_3^{4'})\\
&=\widetilde{c}^I_{44}=(3,\ 4,\ 5;\ 2,\ 4,\ 8).
\end{aligned}
\tag{20}
$$

The intuitionistic fuzzy solution of this system is given as follows:

$$
\begin{aligned}
&\widetilde{u}^I_1=(u_1^1,u_2^1,u_3^1;u_1^{1'},u_2^1,u_3^{1'})=(-1,4,8;-4,4,13),\\
&\widetilde{v}^I_1=(v_1^1,v_2^1,v_3^1;v_1^{1'},v_2^1,v_3^{1'})=(-3,0,3;-7,0,5),\\
&\widetilde{u}^I_2=(u_1^2,u_2^2,u_3^2;u_1^{2'},u_2^2,u_3^{2'})\\
&\quad=(11,\ 12,\ 13;\ 10,\ 12,\ 14),\\
&\widetilde{v}^I_2=(v_1^2,v_2^2,v_3^2;v_1^{2'},v_2^2,v_3^{2'})\\
&\quad=(-10,-5,1;-12,-5,3),\\
&\widetilde{u}^I_3=(u_1^3,u_2^3,u_3^3;u_1^{3'},u_2^3,u_3^{3'})=(6,\ 10,\ 14;\ 5,\ 10,\ 15),\\
&\widetilde{v}^I_3=(v_1^3,v_2^3,v_3^3;v_1^{3'},v_2^3,v_3^{3'})\\
&\quad=(-12,-7,-1;-14,-7,1),\\
&\widetilde{u}^I_4=(u_1^4,u_2^4,u_3^4;u_1^{4'},u_2^4,u_3^{4'})=(3,\ 4,\ 5;\ 2,\ 4,\ 8),\\
&\widetilde{v}^I_4=(v_1^4,v_2^4,v_3^4;v_1^{4'},v_2^4,v_3^{4'})=(0,0,0;0,0,0).
\end{aligned}
\tag{21}
$$

According to Step 3 of the IFMODIM proposed by Singh and Yadav [37], the intuitionistic fuzzy value of $\widetilde{d}^I_{ij} = \widetilde{u}^I_i \oplus \widetilde{v}^I_j \ominus \widetilde{c}^I_{ij}$ for each non-basic cell $(i,\, j)$ is computed as follows:

$$
\begin{aligned}
&\widetilde{d}^I_{12} = \widetilde{u}^I_1 \oplus \widetilde{v}^I_2 \ominus \widetilde{c}^I_{12} = (-1,4,8;-4,4,13)\oplus \\
&(-10,-5,1;-12,-5,3)\ominus(2,\ 5,\ 7;\ 1,\ 5,\ 8) \\
&= (-18,-6,7;-24,-6,15), \\
&\widetilde{d}^I_{13} = \widetilde{u}^I_1 \oplus \widetilde{v}^I_3 \ominus \widetilde{c}^I_{13} = (-1,4,8;-4,4,13)\oplus \\
&(-12,-7,-1;-14,-7,1)\ominus(4,\ 6,\ 8;\ 3,\ 6,\ 9) \\
&= (-21,-9,3;-27,-9,11), \\
&\widetilde{d}^I_{14} = \widetilde{u}^I_1 \oplus \widetilde{v}^I_4 \ominus \widetilde{c}^I_{14} = (-1,4,8;-4,4,13)\oplus \\
&(0,0,0;0,0,0)\ominus(4,\ 7,\ 8;\ 3,\ 7,\ 9) \\
&= (-9,-3,4;-13,-3,10), \\
&\widetilde{d}^I_{21} = \widetilde{u}^I_2 \oplus \widetilde{v}^I_1 \ominus \widetilde{c}^I_{21} = (11,\ 12,\ 13;\ 10,\ 12,\ 14)\oplus \\
&(-3,0,3;-7,0,5)\ominus(4,\ 6,\ 8;\ 3,\ 6,\ 9) \\
&= (0,6,12;-6,6,16), \\
&\widetilde{d}^I_{23} = \widetilde{u}^I_2 \oplus \widetilde{v}^I_3 \ominus \widetilde{c}^I_{23} = (11,\ 12,\ 13;\ 10,\ 12,\ 14)\oplus \\
&(-12,-7,-1;-14,-7,1)\ominus(10,\ 15,\ 20;\ 8,\ 15,\ 22) \\
&= (-21,-10,2;-26,-10,7), \\
&\widetilde{d}^I_{31} = \widetilde{u}^I_3 \oplus \widetilde{v}^I_1 \ominus \widetilde{c}^I_{31} = (6,\ 10,\ 14;\ 5,\ 10,\ 15)\oplus \\
&(-3,0,3;-7,0,5)\ominus(3,\ 4,\ 6;\ 1,\ 4,\ 8) \\
&= (-3,6,14;-10,6,19), \\
&\widetilde{d}^I_{32} = \widetilde{u}^I_3 \oplus \widetilde{v}^I_2 \ominus \widetilde{c}^I_{32} = (6,\ 10,\ 14;\ 5,\ 10,\ 15)\oplus \\
&(-10,-5,1;-12,-5,3)\ominus(8,\ 10,\ 13;\ 5,\ 10,\ 16) \\
&= (-17,\ -5,\ 7;\ -23,\ -5,\ 13), \\
&\widetilde{d}^I_{42} = \widetilde{u}^I_4 \oplus \widetilde{v}^I_2 \ominus \widetilde{c}^I_{42} = (3,\ 4,\ 5;\ 2,\ 4,\ 8)\oplus \\
&(-10,-5,1;-12,-5,3)\ominus(3,\ 9,\ 10;\ 2,\ 9,\ 12) \\
&= (-17,-10,3;-22,-10,9), \\
&\widetilde{d}^I_{43} = \widetilde{u}^I_4 \oplus \widetilde{v}^I_3 \ominus \widetilde{c}^I_{43} = (3,\ 4,\ 5;\ 2,\ 4,\ 8)\oplus \\
&(-12,-7,-1;-14,-7,1)\ominus(3,\ 6,\ 10;\ 2,\ 6,\ 12) \\
&= (-19,-9,1;-24,-9,7).
\end{aligned}
\tag{22}
$$

Now, the rank order of the intuitionistic fuzzy value of $\widetilde{d}^I_{ij}$ given in (22) should be computed to test the optimality of the initial BFS shown in Table 7. So, we have

$$
\begin{aligned}
&f(\widetilde{d}^I_{12}) = \tfrac{-44}{8}, f(\widetilde{d}^I_{13}) = \tfrac{-70}{8}, f(\widetilde{d}^I_{14}) = \tfrac{-20}{8}, \\
&f(\widetilde{d}^I_{21}) = \tfrac{46}{8}, f(\widetilde{d}^I_{24}) = \tfrac{-78}{8},, f(\widetilde{d}^I_{31}) = \tfrac{44}{8}, \\
&f(\widetilde{d}^I_{32}) = \tfrac{-40}{8}, f(\widetilde{d}^I_{42}) = \tfrac{-67}{8}, f(\widetilde{d}^I_{43}) = \tfrac{-71}{8}.
\end{aligned}
\tag{23}
$$

Since $f(\widetilde{d}^I_{ij}) \not\leqslant 0$ for all non-basic cells $(i,\, j)$, then the current BFS is not optimal. Thus, a non-basic cell with the most positive $f(\widetilde{d}^I_{ij})$, i.e., x_{21} is selected as the entering variable. According to the classical transportation algorithm, x_{24} is selected as the leaving variable and the new BFS is found as given in Table 8.

Table 8. The improved BFS-1.

	D_1	D_2	D_3	D_4	a_i
S_1	11	-	-	-	11
S_2	1	10	-	-	11
S_3	-	-	8	3	11
S_4	4	-	-	8	12
b_j	16	10	8	11	45

5.2.2. Iteration 1(LCM)

According to the classical modified distribution method (MODIM) applied by our proposed approach the following crisp system is solved to test the optimality of the initial BFS given in Table 7:

$$
\begin{aligned}
&u_1 + v_1 = c_{11} = \tfrac{30}{8}, u_2 + v_2 = c_{22} = \tfrac{58}{8}, \\
&u_2 + v_4 = c_{24} = \tfrac{96}{8}, u_3 + v_3 = c_{33} = \tfrac{26}{8}, \\
&u_3 + v_4 = c_{34} = \tfrac{80}{8}, u_4 + v_1 = c_{41} = \tfrac{32}{8}, \\
&u_4 + v_4 = c_{44} = \tfrac{34}{8}.
\end{aligned}
\tag{24}
$$

The solution of this crisp system is given as follows:

$$
\begin{aligned}
&u_1 = \tfrac{32}{8}, u_2 = \tfrac{96}{8}, u_3 = \tfrac{80}{8}, u_4 = \tfrac{34}{8}, \\
&v_1 = \tfrac{-2}{8}, v_2 = \tfrac{-38}{8}, v_3 = \tfrac{-54}{8}, v_4 = 0.
\end{aligned}
\tag{25}
$$

To test the optimality of the solution given in Table 7 it is required to compute $d_{ij} = u_i + v_j - c_{ij}$ for each non-basic cell$(i,\, j)$. Thus, we have:

$$
\begin{aligned}
&d_{12} = u_1 + v_2 - c_{12} = \tfrac{32}{8} + \tfrac{-38}{8} - \tfrac{38}{8} = \tfrac{-44}{8}, \\
&d_{13} = u_1 + v_3 - c_{13} = \tfrac{32}{8} + \tfrac{-54}{8} - \tfrac{48}{8} = \tfrac{-70}{8}, \\
&d_{14} = u_1 + v_4 - c_{14} = \tfrac{32}{8} + 0 - \tfrac{52}{8} = \tfrac{-20}{8}, \\
&d_{21} = u_2 + v_1 - c_{21} = \tfrac{96}{8} + \tfrac{-2}{8} - \tfrac{48}{8} = \tfrac{46}{8}, \\
&d_{23} = u_2 + v_3 - c_{23} = \tfrac{96}{8} + \tfrac{-54}{8} - \tfrac{120}{8} = \tfrac{-78}{8}, \\
&d_{31} = u_3 + v_1 - c_{31} = \tfrac{80}{8} + \tfrac{-2}{8} - \tfrac{34}{8} = \tfrac{44}{8}, \\
&d_{32} = u_3 + v_2 - c_{32} = \tfrac{80}{8} + \tfrac{-38}{8} - \tfrac{82}{8} = \tfrac{-40}{8}, \\
&d_{42} = u_4 + v_2 - c_{42} = \tfrac{34}{8} + \tfrac{-38}{8} - \tfrac{63}{8} = \tfrac{-67}{8}, \\
&d_{43} = u_4 + v_3 - c_{43} = \tfrac{34}{8} + \tfrac{-54}{8} - \tfrac{51}{8} = \tfrac{-71}{8}.
\end{aligned}
\tag{26}
$$

Table 9. A real life intuitionistic FTP.

	D_1	D_2	D_3	D_4	a_i
S_1	(210,250,270;200,250,280)	(600,700,750;600,700,800)	(950,1000,1050;900,1000,1100)	(3500, 3700, 3900;3400, 3700,4100)	4500
S_2	(650,750, 800;600,750,850)	(350,400,450;340,400,480)	(1000,1050,1100;950,1050,1150)	(3600, 3900, 4600;3500, 3900,4600)	3500
S_3	(2600,2800,3000; 2500,2800,3100)	(2100,2200,2300;2100,2200,2350)	(2900,3100,3300;2800,3100,3400)	(5400, 5600, 5800;5300, 5600,6000)	2000
b_j	3500	3000	2000	1500	10000

Since $d_{ij} \nleq 0$ for all non-basic cells (i, j), then the current BFS is not optimal. Thus, according to the classical transportation algorithm in crisp environment, x_{21} and x_{24} are selected as the entering variable and the leaving variable, respectively and thus the new BFS is found as given in Table 8 matching with the result of the improved solution-1 of themethod proposed by Singh and Yadav [37]. However, to find the improved solution-1 given in Table 8 it is necessary to solve the intuitionistic fuzzy system (20), to calculate the intuitionistic fuzzy value $\widetilde{d}^I_{ij} = \widetilde{u}^I_i \oplus \widetilde{v}^I_j \ominus \widetilde{c}^I_{ij}$ for each non-basic cell according to (22) and to compare the intuitionistic fuzzy value $\widetilde{d}^I_{ij}$ for each non-basic cell as done in (23). It can be seen that these steps require a large number of intuitionistic fuzzy additions, intuitionistic fuzzy subtractions and comparison on TIFNs. However, based on our proposed approach, these steps are done with solving crisp system (24) and using the elementary operations and comparison on real numbers. These results confirm that our proposed approach is more effective than the method proposed by Singh and Yadav [37] from the computational point of view.

It is worthwhile to note that on using IFMODIM after three more iterations, the optimal solution given in Table 5 is obtained. This means that to get the optimal solution of the IFBTP given in Table 1 by the help of initial BFS obtained based on IFLCM it is required to solve three more intuitionistic fuzzy systems similar to (20), to calculate the intuitionistic fuzzy value $\widetilde{d}^I_{ij} = \widetilde{u}^I_i \oplus \widetilde{v}^I_j \ominus \widetilde{c}^I_{ij}$ for each non-basic cell similar to (22) three more times and to compare the intuitionistic fuzzy value $\widetilde{d}^I_{ij}$ for each non-basic cell three more times. The same optimal solution is found easily using our proposed approach and according to classical transportation algorithms.

A similar discussion can be done by comparing the results obtained from using IFMODIM proposed by Singh and Yadav [37] on initial BFS found by IFVAM and MODIM applied by our proposed approach on initial BFS found by VAM.

In the next example a real life intuitionistic fuzzy TP given in Ref. 37 is solved and the results obtained are discussed and compared in details.

Example 5.2: The data shown in Table 9 are collected from a trader of Chandigarh, India, which supplies the commodity TMT (Thermo mechanically treated) steel from three plants S_1, S_2 and S_3 to four different companies D_1, D_2, D_3 and D_4. The trader is certain about the availabilities and demands of the materials, but (s)he is uncertain about the transportation cost from different sources to different destinations due to some uncontrollable factors such as weather in Hilly areas. In this kind of situation, the usual way is to obtain the triangular intuitionistic fuzzy numbers after a thorough discussion based upon past experience or expert advice. The aim is to determine the optimal transportation of products so that the total intuitionistic fuzzy transportation cost is minimized.

According to the accuracy function given in (1), the rank order of each intuitionistic fuzzy transportation cost (given in Table 9) is substituted by the corresponding intuitionistic fuzzy numbers to obtain the classical TP. The results are given in Table 10.

Table 10. A real life crisp TP.

	D_1	D_2	D_3	D_4	a_i
S_1	245	693.75	1000	3712.5	4500
S_2	737.5	402.5	1050	3987.5	3500
S_3	2800	2206.25	3100	5612.5	2000
b_j	3500	3000	2000	1500	10000

Now, we obtain the initial BFS for the problems given in Table 9 and Table 10 according to IFVAM and VAM, respectively and explore the obtained results.

Table 11. First assignment in IFVAM.

	D_1	D_2	D_3	D_4	$RI\widetilde{P}_i$
S_1	(210,250,270;200,250,280)	(600,700,750;600,700,800)	(950,1000,1050;900,1000,1100)	(3500, 3700, 3900;3400, 3700,4100)	(330,450,540;320,450,600)
S_2	(650,750, 800;600,750,850)	(350,400,450;340,400,480)	(1000,1050,1100;950,1050,1150)	(3600, 3900, 4600;3500, 3900,4600)	(200,350,450;120,350,510)
$CI\widetilde{P}_j$	(380,500,590;320,500,650)	(150,300,400;120,300,460)	(-50,50,150;-150,50,250)	(-300,200,1100;-600,200,1200)	

Table 12. Second assignment in IFVAM.

	D_2	D_3	D_4	$RI\widetilde{P}_i$
S_1	(600,700,750;600,700,800)	(950,1000,1050;900,1000,1100)	(3500, 3700, 3900;3400, 3700,4100)	(330,450,540;320,450,600)
S_2	(350,400,450;340,400,480)	(1000,1050,1100;950,1050,1150)	(3600, 3900, 4600;3500, 3900,4600)	(200,350,450;120,350,510)
$CI\widetilde{P}_j$	(150,300,400;120,300,460)	(-50,50,150;-150,50,250)	(-300,200,1100;-600,200,1200)	

5.3. *Initial BFS based on IFVAM and VAM*

According to Step (1) of the IFVAM proposed by Singh and Yadav [37], it is required to compute the intuitionistic penalty regarding to Remark 1 for each row and each column of Table 9. The row intuitionistic penalty (RIP) and the column intuitionistic penalty (CIP) of Table 9 are obtained as follows:

$RI\widetilde{P}_1^I = \tilde{c}_{12}^I \ominus \tilde{c}_{11}^I = (600,700,750;600,700,800) \ominus$
$(210,250,270;200,250,280)$
$= (330,450,540;320,450,600)$
$RI\widetilde{P}_2^I = \tilde{c}_{21}^I \ominus \tilde{c}_{22}^I = (650,750,\ 800;600,750,850) \ominus$
$(350,400,450;340,400,480)$
$= (200,350,450;120,350,510)$
$RI\widetilde{P}_3^I = \tilde{c}_{31}^I \ominus \tilde{c}_{32}^I = (2600,2800,3000;2500,2800,3100) \ominus$
$(2100,2200,2300;2100,2200,2350)$
$= (300,600,900;150,600,1000),$

$CI\widetilde{P}_1^I = \tilde{c}_{21}^I \ominus \tilde{c}_{11}^I = (650,750,\ 800;600,750,850) \ominus$
$(210,250,270;200,250,280)$
$= (380,500,590;320,500,650),$
$CI\widetilde{P}_2^I = \tilde{c}_{12}^I \ominus \tilde{c}_{22}^I = (600,700,750;600,700,800) \ominus$
$(350,400,450;340,400,480)$
$= (150,300,400;120,300,460),$
$CI\widetilde{P}_3^I = \tilde{c}_{23}^I \ominus \tilde{c}_{13}^I = (1000,1050,1100;950,1050,1150) \ominus$
$(950,1000,1050;900,1000,1100)$
$= (-50,50,150;-150,50,250),$
$CI\widetilde{P}_4^I = \tilde{c}_{24}^I \ominus \tilde{c}_{14}^I = (3600,\ 3900,\ 4600;3500,\ 3900,4600) \ominus$
$(3500,\ 3700,\ 3900;3400,\ 3700,4100)$
$= (-300,200,1100;-600,200,1200).$

Because row 3 has the highest intuitionistic penalty based on the accuracy function, and the cell (3, 2) has the smallest intuitionistic cost in this row, the amount 2000 is assigned to $x_{32} = 2000$. Now, row 3 is satisfied and should be deleted. The reduced table is given as Table 11. Now, in a similar way the new intuitionistic penalties are recomputed as in Table 11.

Now according to the accuracy function, column 1 in Table 11 has the highest intuitionistic penalty and cell (1, 1) has the smallest intuitionistic cost in this column. Thus, the amount 3500 is assigned to $x_{11} = 3500$. In this case, column 1 is satisfied and should be deleted. The reduced table and new computed intuitionistic penalties are given as Table 12.

This process is continued and after four more iterations the initial BFS given in Table 13 is found.

Table 13. Initial BFS by IFVAM.

	D_1	D_2	D_3	D_4	a_i
S_1	3500	–	–	1000	4500
S_2	–	1000	2000	500	3500
S_3	–	2000	–	–	2000
b_j	3500	3000	2000	1500	10000

It should be noted that the same initial BFS (given in Table 13) can be obtained for Table 10 by Vogel's approximation method (VAM) corresponding to our proposed method without doing any arithmetic operations on the intuitionistic triangular fuzzy numbers and without any comparison of ITFNs. This confirms that our proposed method needs less computational effort for finding initial BFS compared to the method proposed by Singh and Yadav [37]. In what follows, it is shown that the same result can be concluded for finding the optimal solution.

5.4. *Optimal solution based on IFMODIM*

According to Step 1 and Step 2 of the IFMODIM given in Ref. 37 the following intuitionistic fuzzy system is solved to test the optimality of the initial BFS given in Table 13:

$$\begin{aligned}
&(u_1^1,u_2^1,u_3^1;u_1^{1'},u_2^1,u_3^{1'})+(v_1^1,v_2^1,v_3^1;v_1^{1'},v_2^1,v_3^{1'})=\widetilde{c}_{11}^I\\
&=(210,250,270;200,250,280),\\
&(u_1^1,u_2^1,u_3^1;u_1^{1'},u_2^1,u_3^{1'})+(v_1^4,v_2^4,v_3^4;v_1^{4'},v_2^4,v_3^{4'})=\widetilde{c}_{14}^I\\
&=(3500,\ 3700,\ 3900;3400,\ 3700,4100),\\
&(u_1^2,u_2^2,u_3^2;u_1^{2'},u_2^2,u_3^{2'})+(v_1^2,v_2^2,v_3^2;v_1^{2'},v_2^2,v_3^{2'})=\widetilde{c}_{22}^I\\
&=(350,400,450;340,400,480),\\
&(u_1^2,u_2^2,u_3^2;u_1^{2'},u_2^2,u_3^{2'})+(v_1^3,v_2^3,v_3^3;v_1^{3'},v_2^3,v_3^{3'})=\widetilde{c}_{23}^I\\
&=(1000,1050,1100;950,1050,1150),\\
&(u_1^2,u_2^2,u_3^2;u_1^{2'},u_2^2,u_3^{2'})+(v_1^4,v_2^4,v_3^4;v_1^{4'},v_2^4,v_3^{4'})=\widetilde{c}_{24}^I\\
&=(3600,\ 3900,\ 4600;3500,\ 3900,4600),\\
&(u_1^3,u_2^3,u_3^3;u_1^{3'},u_2^3,u_3^{3'})+(v_1^2,v_2^2,v_3^2;v_1^{2'},v_2^2,v_3^{2'})=\widetilde{c}_{32}^I\\
&=(2100,2200,2300;2100,2200,2350)
\end{aligned}\tag{27}$$

The intuitionistic fuzzy solution of this system is given as follows:

$$\begin{aligned}
\widetilde{u}_1^I&=(u_1^1,u_2^1,u_3^1;u_1^{1'},u_2^1,u_3^{1'})=\\
&\quad(-1100,-200,300;-1200,-200,600),\\
\widetilde{v}_1^I&=(v_1^1,v_2^1,v_3^1;v_1^{1'},v_2^1,v_3^{1'})=\\
&\quad(-90,450,1370;-400,450,1480),\\
\widetilde{u}_2^I&=(u_1^2,u_2^2,u_3^2;u_1^{2'},u_2^2,u_3^{2'})=(0,0,0;0,0,0),\\
\widetilde{v}_2^I&=(v_1^2,v_2^2,v_3^2;v_1^{2'},v_2^2,v_3^{2'})=\\
&\quad(350,400,450;340,400,480)\\
\widetilde{u}_3^I&=(u_1^3,u_2^3,u_3^3;u_1^{3'},u_2^3,u_3^{3'})=\\
&\quad(1650,1800,1950;1620,1800,2010),\\
\widetilde{v}_3^I&=(v_1^3,v_2^3,v_3^3;v_1^{3'},v_2^3,v_3^{3'})=\\
&\quad(1000,1050,1100;950,1050,1150),\\
\widetilde{v}_4^I&=(v_1^4,v_2^4,v_3^4;v_1^{4'},v_2^4,v_3^{4'})=\\
&\quad(3600,\ 3900,\ 4600;3500,\ 3900,4600).
\end{aligned}\tag{28}$$

According to Step 3 of the IFMODIM given in Ref. 37, the intuitionistic fuzzy value of $\widetilde{d}_{ij}^I=\widetilde{u}_i^I\oplus\widetilde{v}_j^I\ominus\widetilde{c}_{ij}^I$ for each non-basic cell $(i,\ j)$ is computed as follows:

$$\begin{aligned}
\widetilde{d}_{12}^I&=(-1500,-500,150;-1660,-500,480),\\
\widetilde{d}_{13}^I&=(-1150,-150,450;-1350,-150,850),\\
\widetilde{d}_{21}^I&=(-890,-300,720;-1250,-300,880),\\
\widetilde{d}_{31}^I&=(-1440,-550,720;-1880,-550,990),\\
\widetilde{d}_{33}^I&=(-650,-250,150;-830,-250,360)\\
\widetilde{d}_{34}^I&=(-550,100,1150;-880,100,1310).
\end{aligned}\tag{29}$$

Now, the rank order of the intuitionistic fuzzy value of $\widetilde{d}_{ij}^I$ given in (29) should be computed to test the optimality of the initial BFS shown in Table 13. So, we have

$$\begin{aligned}
&f(\widetilde{d}_{12}^I)=-566.25, f(\widetilde{d}_{13}^I)=-225,\\
&f(\widetilde{d}_{21}^I)=-217.5, f(\widetilde{d}_{31}^I)=-476.25,\\
&f(\widetilde{d}_{33}^I)=-246.25, f(\widetilde{d}_{34}^I)=178.75.
\end{aligned}\tag{30}$$

Since $f(\widetilde{d}_{34}^I)=178.75\nleq 0$, then the current BFS is not optimal and x_{34} is selected as the entering variable. According to Step 5 of IFMODIM proposed by Singh and Yadav [37], x_{24} is selected as the leaving variable and the new BFS is found as given in Table 14.

Table 14. Improved BFS-1 (optimal solution).

	D_1	D_2	D_3	D_4	a_i
S_1	3500	–	–	1000	4500
S_2	–	1500	2000	–	3500
S_3	–	1500	–	500	2000
b_j	3500	3000	2000	1500	10000

According to Step 1 and Step 2 of the IFMODIM given in Ref. 37, the following intuitionistic fuzzy system is solved to test the optimality of the improved solution-1 given in Table 14:

$$\begin{aligned}
&(u_1^1,u_2^1,u_3^1;u_1^{1'},u_2^1,u_3^{1'})\oplus(v_1^1,v_2^1,v_3^1;v_1^{1'},v_2^1,v_3^{1'})=\widetilde{c}_{11}^I\\
&\quad=(210,250,270;200,250,280),\\
&(u_1^1,u_2^1,u_3^1;u_1^{1'},u_2^1,u_3^{1'})\oplus(v_1^4,v_2^4,v_3^4;v_1^{4'},v_2^4,v_3^{4'})=\widetilde{c}_{14}^I\\
&\quad=(3500,\ 3700,\ 3900;3400,\ 3700,4100),\\
&(u_1^2,u_2^2,u_3^2;u_1^{2'},u_2^2,u_3^{2'})\oplus(v_1^2,v_2^2,v_3^2;v_1^{2'},v_2^2,v_3^{2'})=\widetilde{c}_{22}^I\\
&\quad=(350,400,450;340,400,480),\\
&(u_1^2,u_2^2,u_3^2;u_1^{2'},u_2^2,u_3^{2'})\oplus(v_1^3,v_2^3,v_3^3;v_1^{3'},v_2^3,v_3^{3'})=\widetilde{c}_{23}^I\\
&\quad=(1000,1050,1100;950,1050,1150),\\
&(u_1^3,u_2^3,u_3^3;u_1^{3'},u_2^3,u_3^{3'})\oplus(v_1^2,v_2^2,v_3^2;v_1^{2'},v_2^2,v_3^{2'})=\widetilde{c}_{32}^I\\
&\quad=(2100,2200,2300;2100,2200,2350),\\
&(u_1^3,u_2^3,u_3^3;u_1^{3'},u_2^3,u_3^{3'})\oplus(v_1^4,v_2^4,v_3^4;v_1^{4'},v_2^4,v_3^{4'})=\widetilde{c}_{34}^I\\
&\quad=(5400,\ 5600,\ 5800;5300,\ 5600,6000).
\end{aligned}\tag{31}$$

The intuitionistic fuzzy solution of this system is given as follows:

$$\begin{aligned}
&\widetilde{u}_1^I = (u_1^1, u_2^1, u_3^1; u_1^{1'}, u_2^1, u_3^{1'}) \\
&= (-650, -100, 450; -980, -100, 810), \\
&\widetilde{v}_1^I = (v_1^1, v_2^1, v_3^1; v_1^{1'}, v_2^1, v_3^{1'}) \\
&= (-240, 350, 920; -610, 350, 1260), \\
&\widetilde{u}_2^I = (u_1^2, u_2^2, u_3^2; u_1^{2'}, u_2^2, u_3^{2'}) = (0,0,0;0,0,0), \\
&\widetilde{v}_2^I = (v_1^2, v_2^2, v_3^2; v_1^{2'}, v_2^2, v_3^{2'}) \\
&= (350, 400, 450; 340, 400, 480) \\
&\widetilde{u}_3^I = (u_1^3, u_2^3, u_3^3; u_1^{3'}, u_2^3, u_3^{3'}) \\
&= (1650, 1800, 1950; 1620, 1800, 2010), \\
&\widetilde{v}_3^I = (v_1^3, v_2^3, v_3^3; v_1^{3'}, v_2^3, v_3^{3'}) \\
&= (1000, 1050, 1100; 950, 1050, 1150), \\
&\widetilde{v}_4^I = (v_1^4, v_2^4, v_3^4; v_1^{4'}, v_2^4, v_3^{4'}) \\
&= (3450,\ 3800,\ 4150; 3290,\ 3800, 4380).
\end{aligned} \tag{32}$$

According to Step 3 of the IFMODIM proposed by Singh and Yadav [37], the intuitionistic fuzzy value of $\widetilde{d}_{ij}^I = \widetilde{u}_i^I \oplus \widetilde{v}_j^I \ominus \widetilde{c}_{ij}^I$ for each non-basic cell $(i,\, j)$ is computed as follows:

$$\begin{aligned}
&\widetilde{d}_{12}^I = \widetilde{u}_1^I \oplus \widetilde{v}_2^I \ominus \widetilde{c}_{12}^I \\
&= (-1050, -400, 300; -1440, -400, 690), \\
&\widetilde{d}_{13}^I = \widetilde{u}_1^I \oplus \widetilde{v}_3^I \ominus \widetilde{c}_{13}^I \\
&= (-700, -50, 600; -1130, -50, 1060), \\
&\widetilde{d}_{21}^I = \widetilde{u}_2^I \oplus \widetilde{v}_1^I \ominus \widetilde{c}_{21}^I \\
&= (-1040, -400, 270; -1460, -400, 660), \\
&\widetilde{d}_{24}^I = \widetilde{u}_2^I \oplus \widetilde{v}_4^I \ominus \widetilde{c}_{24}^I \\
&= (-1150, -100, 550; -1310, -100, 880), \\
&\widetilde{d}_{31}^I = \widetilde{u}_3^I \oplus \widetilde{v}_1^I \ominus \widetilde{c}_{31}^I \\
&= (-1590, -650, 270; -2090, -650, 770), \\
&\widetilde{d}_{33}^I = \widetilde{u}_3^I \oplus \widetilde{v}_3^I \ominus \widetilde{c}_{33}^I \\
&= (-650, -250, 150; -830, -250, 360).
\end{aligned} \tag{33}$$

Now, the rank order of the intuitionistic fuzzy value of $\widetilde{d}_{ij}^I$ given in (33) is obtained to test the optimality of the improved solution-1 shown in Table 14 as follows:

$$\begin{aligned}
&f(\widetilde{d}_{12}^I) = -387.5, f(\widetilde{d}_{13}^I) = -46.25, \\
&f(\widetilde{d}_{21}^I) = -396.25, f(\widetilde{d}_{24}^I) = -178.75, \\
&f(\widetilde{d}_{31}^I) = -655, f(\widetilde{d}_{33}^I) = -246.25.
\end{aligned} \tag{34}$$

Since $f(\widetilde{d}_{ij}^I) \leqslant 0$ for all non-basic cells (i, j), the improved solution-1, shown in Table 14 is optimal. This means that the optimal solution is given by

$$\begin{aligned}
&x_{11} = 3500, x_{14} = 1000, x_{22} = 1500, \\
&x_{23} = 2000, x_{32} = 1500, x_{34} = 500.
\end{aligned} \tag{35}$$

Putting the values of the optimal solution (35) in the objective function of the intuitionistic fuzzy TP in Table 9, the total intuitionistic fuzzy transportation cost achieved is:

$$\begin{aligned}
&\widetilde{Z}_{OPI}^I = \sum_{i=1}^{3}\sum_{j=1}^{4} \widetilde{c}_{ij}^I x_{ij} = \\
&(12610000, 13375000, 14070000; \\
&\qquad 12310000, 13375000, 14625000)
\end{aligned} \tag{36}$$

It is worthwhile to note that the optimal solution and total intuitionistic fuzzy transportation cost of the real life intuitionistic fuzzy TP (Table 9) obtained by the method proposed by Singh and Yadav [37], are as follows:

$$\begin{aligned}
&\text{Optimal Solution of the method given in Ref. 37}: \\
&x_{11} = 3500, x_{13} = 1000, \\
&x_{22} = 2500, x_{23} = 1000, \\
&x_{32} = 500,\ x_{34} = 1500.
\end{aligned} \tag{37}$$

$$\begin{aligned}
&\widetilde{Z}_{OPI-\text{existing method proposed by Singh and Yadav } 37:}^I = \\
&(12710000,\ 13425000,\ 14070000; \\
&\qquad 12400000,\ 13425000,\ 14605000)
\end{aligned} \tag{38}$$

By comparing the intuitionistic fuzzy objective value (36) with the intuitionistic fuzzy objective value given in (38), we conclude that the solution obtained by Singh and Yadav [37] is not the optimal solution. If fact, we have

$$\begin{aligned}
&\widetilde{Z}_{OPI}^I = (12610000, 13375000, 14070000; \\
&\qquad 12310000, 13375000, 14625000) \prec \\
&\widetilde{Z}_{OPI-method\ proposed\ by\ Singh\ and\ Yadav^{37}}^I = \\
&(12710000,\ 13425000,\ 14070000; \\
&\qquad 12400000,\ 13425000,\ 14605000)
\end{aligned}$$

5.5. *Optimal solution based on MODIM*

Here, it is shown that the same optimal solution can be found for the real life intuitionistic fuzzy TP (Table 9) according to our proposed approach and using traditional MODIM on initial BFS given in Table 13.

According to classical MODIM applied by our proposed approach, the following crisp system is solved to test the optimality of the initial BFS given in Table 13:

$$\begin{aligned}&u_1+v_1=c_{11}=245,\ u_1+v_4=c_{24}=3712.5,\\&u_2+v_2=c_{22}=402.5, u_2+v_3=c_{23}=1050,\\&u_2+v_4=c_{24}=3987.5, u_3+v_2=c_{32}=2206.25.\end{aligned} \tag{39}$$

The solution of this crisp system is given as follows:

$$\begin{aligned}&u_1=-275,\ u_2=0, u_3=1803.75,\\&v_1=520, v_2=402.5, v_3=1050, v_4=3987.5.\end{aligned} \tag{40}$$

To test the optimality of the solution given in Table 13, it is required to compute $d_{ij}=u_i+v_j-c_{ij}$ for each non-basic cell $(i,\ j)$. Thus, we have:

$$\begin{aligned}&d_{12}=u_1+v_2-c_{12}=-275+402.5-693.75\\&\quad=-566.25,\\&d_{13}=u_1+v_3-c_{13}=-275+1050-1000\\&\quad=-225,\\&d_{21}=u_2+v_1-c_{21}=0+520-737.5\\&\quad=-217.5,\\&d_{31}=u_3+v_1-c_{31}=1803.75+520-2800\\&\quad=-476.25,\\&d_{33}=u_3+v_3-c_{33}=1803.75+1050-3100\\&\quad=-246.25,\\&d_{34}=u_3+v_4-c_{34}=1803.75+3987.5-5612.5\\&\quad=178.75.\end{aligned} \tag{41}$$

Since d_{34}= 178.75 $\nleq 0$, then the current BFS is not optimal. Thus, according to the classical transportation algorithm in crisp environment, x_{34} and x_{24} are selected as the entering variable and the leaving variable, respectively, and thus the new solution is found as given in Table 14 matching with the result of the improved solution-1 of the method proposed by Singh and Yadav [37]. Now, the following crisp system is solved to test the optimality of the improved solution-1 given in Table 14:

$$\begin{aligned}&u_1+v_1=c_{11}=245,\ u_1+v_4=c_{24}=3712.5,\\&u_2+v_2=c_{22}=402.5, u_2+v_3=c_{23}=1050,\\&u_3+v_2=c_{32}=2206.25, u_3+v_4=c_{34}=5612.5.\end{aligned} \tag{42}$$

The intuitionistic fuzzy solution of this system is given as follows:

$$\begin{aligned}&u_1=-96.25,\ u_2=0, u_3=1803.75,\\&v_1=341.25, v_2=402.5, v_3=1050, v_4=3808.75.\end{aligned} \tag{43}$$

The value of $d_{ij}=u_i+v_j-c_{ij}$ for each non-basic cell $(i,\ j)$ is obtained as follows:

$$\begin{aligned}&d_{12}=u_1+v_2-c_{12}=-387.5,\\&d_{13}=u_1+v_3-c_{13}=-46.25,\\&d_{21}=u_2+v_1-c_{21}=-396.25,\\&d_{24}=u_2+v_4-c_{24}=-178.75,\\&d_{31}=u_3+v_1-c_{31}=-655,\\&d_{33}=u_3+v_3-c_{33}=-246.25.\end{aligned} \tag{44}$$

Thus, the improved-solution 1 (Table 14) is optimal. This means that our proposed method and the method proposed by Singh and Yadav [37] have the same results. However, the method proposed by Singh and Yadav [37] requires a large number of intuitionistic fuzzy additions, intuitionistic fuzzy subtractions and comparison on TIFNs. While based on our proposed approach, all elementary operations and comparison are done on real numbers. These results confirm that our proposed approach should be preferred to the method proposed by Singh and Yadav [37] in terms of the computational point of view.

6. Conclusions and future work

In this paper, a TP having uncertainty as well as hesitation in prediction of the transportation cost has been investigated. In the TP considered in this study, the values of transportation costs are represented by triangular intuitionistic fuzzy numbers and the values of supply and demand of the products are represented by real numbers. Here, we proposed an efficient computational solution approach for solving intuitionistic fuzzy TP based on classical transportation algorithms. In contrast to the method proposed by Singh and Yadav [37] where all elementary operations and comparisons are done on triangular intuitionistic fuzzy numbers, in the proposed algorithm in this paper all arithmetic calculations and comparisons are performed on crisp numbers. Therefore, the complexity of computation is reduced very much compared to the method proposed by Singh and Yadav [37]. Here, we shall point out that the IFTP studied in this paper is not in the form of a problem whose demands and supplies are as triangular intuitionistic fuzzy numbers too. Therefore, further research on extending the proposed method to overcome these shortcomings is an interesting stream of future research. We shall report the significant results of these ongoing projects in the near future by extending the proposed methods by Ebrahimnjead [38,39] utilized for solving fully FTP and interval-valued FTP.

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