An FMCDM approach to purchasing decision-making based on cloud model and prospect theory in e-commerce

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Abstract

This paper presents a fuzzy multi-criteria decision-making (FMCDM) approach based on cloud model and prospect theory. In addition, a reference point selection method is developed according to the evaluations of the potential customer or similar consumers regarding certain items. An example of purchasing decision problems is provided in order to illustrate the applicability of the proposed approach. A comparative analysis of the proposed approach is also performed in order to verify its feasibility.

Keywords: Purchasing decision-making, cloud model, prospect theory, reference point.

1. Introduction

Purchasing decision problems involve the evaluations of certain products and their alternatives [1]. Both internal and environmental factors affect the choices of customers. Intuitive decision-making allows customers to bypass the conscious steps of product selection [2]. However, oftentimes potential customers are facing so much information, especially in e-commerce, that they find making intuitive purchasing decisions difficult. Such problems impel customers to use specific approaches that facilitate the appropriate selection of

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products or their alternatives. Up to now, some studies concerning purchasing frameworks and theories have been conducted. Nicosia [3] proposed a purchasing decision-making process model, in which it contained four stages: communication, searching, action and feedback. Engel et al. [4] put forward an Engel-Kollat-Blackwell (EKB) model, which was a framework of analyzing consumers' behaviors. Besides, some theories and models related to online purchasing decision-making problems were put forward [5-9]. However, these studies only provided a guiding framework for studying purchasing decision-making problems, but they hardly offered an approach with specific procedures to help customers make purchasing decisions. To overcome this deficiency, both theory and practicality should be considered when developing an approach to purchasing decisions.

Fuzzy multi-criteria decision-making (FMCDM) approach was introduced in the early 1970s [10] and was developed as a promising decision-making tool for complex problems with multi-criteria and fuzzy information [11-20]. In essence, a purchasing decision-making problem can be expressed as an FMCDM problem. In an e-commerce environment, customers make purchasing decisions according to the items' linguistic evaluations from several aspects [21, 22], which are consistent with the multi-criteria of decision-making problems. The evaluations provided by other consumers, or product reviews, are actually the criteria values. Furthermore, decision-making procedures involve the identification of compromises among several complex and conflicting items. Therefore, a purchasing decision-making problem can be tackled by FMCDM approaches.

Due to the complexity of products and the vagueness of human thinking, people used to give evaluations of items with linguistic information [23]. In some e-commerce websites, such as Taobao.com and Tmall.com, consumers are allowed to give evaluations of items under certain criteria after consuming, and these evaluations are often given with specific linguistic descriptors such as terrible, poor, medium, good and excellent, according to the websites. Although there are some methods, such as linguistic computational models based on membership functions [24-26], linguistic operators [27-29] and 2-tuple linguistic models [30-34], that can be used in decision-making problems with linguistic information, these methods have some limitations in

solving practical purchasing decision-making problems [35-37]. Cloud model was developed based on probability theory and fuzzy set theory [38], it allows for the transformation of linguistic terms into three associated numerical characteristics, which overcomes the limitations of aforementioned methods [39]. Recently, cloud models have been widely applied to practical problems with linguistic information [40-42], including some decision-making problems [43, 44].

In reality, because of the complexity of purchasing decisions and the cognitive limitations of customers, customers often exhibit bounded rationality [9]. Based on the "bounded rationality" principle [45], prospect theory was developed [46] and applied into decision-making problems for its capability of describing the decision-making behaviors of individuals at risk of loss [47]. In prospect theory, items are compared according to their prospect values, which reflect individuals' attitudes toward items under risk [46]. Due to online purchasing decisions containing numerous uncertainties and risks, customers make comparison among alternative products to reduce loss and select "most satisfied" items rather than "highest value" items. In order to calculate prospect values of items, reference point, which is considered as the equilibrium point of individual's psychological expectation, must be selected because different reference points result in different outcomes [46]. In some applications of prospect theory, five variables, including the zero point, expected value point, median value of an ordered sequence, worst point, and optimal point, are usually regarded as reference points [48]. In practical purchasing decision problems, different customers may have completely different expectations to the same item, let alone different items. Thus, reference point selection principles must be flexible, rather than fixed, in order to meet the needs of different customers.

In this study, an FMCDM approach to e-commerce purchasing decision problems was developed based on cloud model and prospect theory. Linguistic evaluations were processed using the cloud model and prospect theory was used to compensate for the bounded rationality of customers. In addition, reference point selection principles were investigated in order to improve the validity of the proposed approach.

The remainder of this paper is organized as follows. In Section 2, some basic concepts and definitions related to cloud model, prospect theory, and cloud

prospect values are introduced briefly. In Section 3, the selection principles of reference points are discussed. An FMCDM approach based on cloud model and prospect theory is proposed in Section 4. In Section 5, an illustrative example is provided, and the proposed FMCDM approach is compared to several other methods in order to confirm its validity and applicability. In Section 6, the conclusions of this paper and information concerning future studies are presented.

2. Preliminaries

In this section, we introduce some basic concepts and definitions related to cloud model, prospect theory, and cloud prospect values that will be used throughout this paper.

2.1. *Cloud model*

Definition 1 [39]. Let $U = \{x\}$ be the universe of discourse, and T is a linguistic term associated with U. The degree of membership $C_T(x)$ of x in U to the linguistic term T is a random number with a stable tendency. $C_T(x)$ takes the values in [0,1]. A normal compatibility cloud is a mapping from the universe of discourse U to the unit interval [0,1], and every x with a degree of membership $C_T(x)$ is defined as a cloud drop, such that

$$C_T(x):U \to [0,1],$$
$$\forall x \in U, \ x \to C_T(x).$$

The qualitative meaning of a linguistic term can be expressed by a normal compatibility cloud with three digital characteristics, and a cloud can be described as $Y(Ex, En, He)$, where the expected value Ex is the most representative and typical value of T. The entropy En, which describes the fuzziness of the linguistic term, reflects the acceptable range of that linguistic term in the universe of discourse. The hyper entropy He, the second-order entropy of the entropy En, reflects the dispersion and randomness of the degree of membership $C_T(x)$.

Definition 2 [49]. Let $Y_1 = (Ex_1, En_1, He_1)$ and $Y_2 = (Ex_2, En_2, He_2)$ be two arbitrary normal clouds. The distance $d(Y_1, Y_2)$ between Y_1 and Y_2 is defined as

$$d(Y_1,Y_2) = \sqrt{\frac{1}{2}\left(|Ex_1 - Ex_2|^2 + |En_1 - En_2|^2 + |He_1 - He_2|^2\right)} \quad (1)$$

Definition 3 [44]. Let $Y_1 = (Ex_1, En_1, He_1)$ and $Y_2 = (Ex_2, En_2, He_2)$ be any two normal clouds in the universe of discourse and $Y^* = \left(\max_i Ex_i, \min_i En_i, \min_i He_i\right)$ be the positive ideal cloud. When i=1, 2, the degree of possibility of $Y_1 \geq Y_2$ denoted as $P(Y_1 \geq Y_2)$ is defined as

$$P(Y_1 \geq Y_2) = 1 - \frac{d(Y^*, Y_1)}{d(Y^*, Y_1) + d(Y^*, Y_2)}, \quad (2)$$

where $d(Y^*, Y_1)$ represents the distance between Y_1 and Y^*, and $d(Y^*, Y_2)$ represents the distances between Y_2 and Y^*. If $P(Y_1 \geq Y_2) \geq 0.5$, then $Y_1 \geq Y_2$; otherwise, $Y_1 < Y_2$.

Definition 4 [50]. Assume that Ω is the set of all clouds and $Y_i = (Ex_i, En_i, He_i)(i = 1, 2, \cdots, n)$ is a subset of Ω. The mapping CWAA: $\Omega^n \to \Omega$ is defined as the cloud-weighted arithmetic averaging (CWAA) operator, such that

$$CWAA_\omega(Y_1,Y_2,\cdots,Y_n) = \sum_{i=1}^n \omega_i Y_i = \left(\sum_{i=1}^n \omega_i Ex_i, \sqrt{\sum_{i=1}^n \omega_i En_i^2}, \sqrt{\sum_{i=1}^n \omega_i He_i^2}\right). \quad (3)$$

In Equation (3), $\omega = (\omega_1, \omega_2 \cdots, \omega_n)$ represents the associated weight vector of $Y_i(Ex_i, En_i, He_i)$ $(i = 1, 2, \cdots, n)$, $\omega_i \in [0,1](i = 1, 2 \cdots, n)$, and $\sum_{i=1}^n \omega_i = 1$. In particular, when $\omega = \left(\frac{1}{n}, \frac{1}{n}, \cdots \frac{1}{n}\right)$, the CWAA operator will reduce to the cloud arithmetic average (CAA) operator, defined as

$$CAA_\omega(Y_1,Y_2,\cdots Y_n) = \sum_{i=1}^n \frac{1}{n} Y_i = \left(\sum_{i=1}^n \frac{1}{n} Ex_i, \sqrt{\sum_{i=1}^n \frac{1}{n} En_i^2}, \sqrt{\sum_{i=1}^n \frac{1}{n} He_i^2}\right). \quad (4)$$

2.2. *Prospect values and cloud prospect values*

Prospect values are codetermined by the value function and probability weight function [46], which can be expressed as

$$V = \sum_{i=1}^{n} \left(\pi\left(p_i\right) v\left(\Delta x_i\right) \right), \qquad (5)$$

where V is the prospect value, p_i represents the probability of the ith condition, and $\pi(p_i)$ is the probability weight function of the probability assessment, which reflects the attitude of a decision-maker regarding risk. In addition, $v\left(\Delta x_i\right)$ is the value function, which is based on the subjective feelings of a decision-maker regarding an item; and Δx_i represents the degree of deviation of a particular value from the reference point. The outcome is identified as a gain if the degree of deviation is a positive value; otherwise, the outcome is identified as a loss.

Definition 5 [44]. Assume that $Y_1\left(Ex_1, En_1, He_1\right)$ and $Y_2\left(Ex_2, En_2, He_2\right)$ are any two clouds in the universe of discourse, and let Y_2 be the reference point. The cloud prospect value of Y_1 can be calculated as

$$V\left(Y_1\right) = v\left(Y_1\right) \pi\left(p_1\right). \qquad (6)$$

The cloud value function $v\left(Y_1\right)$ shown in Equation (6) can be expressed as

$$v\left(Y_1\right) = \begin{cases} \left(d\left(Y_1, Y_2\right)\right)^{\alpha}, & Y_1 \geq Y_2, \\ -\lambda \left(d\left(Y_1, Y_2\right)\right)^{\beta}, & Y_1 < Y_2, \end{cases}$$

where $d\left(Y_1, Y_2\right)$ represents the distance between Y_1 and Y_2. It is identified as a gain when the value of Y_1 is greater than that of Y_2. In the same way, it is a loss when the value of Y_1 is smaller than that of Y_2. The values of Y_1 and Y_2 can be compared using the degree of possibility mentioned in Definition 3. In addition, α and β represent the degree of concavity or convexity of the value function within the regions of gain and loss, respectively and $\alpha, \beta \in \left(0,1\right)$. Higher values of α and β indicate that the decision-maker is more likely to seek risks. λ is used to represent the degree of severe feelings toward loss. Usually, $\lambda > 1$, which indicates aversion to loss of individuals.

The cloud probability weight function $\pi\left(p_1\right)$ in Equation (6) can be expressed as

$$\pi\left(p_1\right) = \begin{cases} \dfrac{p_1^{\gamma}}{\left(p_1^{\gamma} + \left(1 - p_1\right)^{\gamma}\right)^{1/\gamma}}, & Y_1 \geq Y_2, \\ \dfrac{p_1^{\delta}}{\left(p_1^{\delta} + \left(1 - p_1\right)^{\delta}\right)^{1/\delta}}, & Y_1 < Y_2, \end{cases}$$

where p_1 represents the probability of Y_1, γ and δ represent the decision-maker's attitude towards risk-gain and risk-loss, respectively.

3. Reference point selection principles

Selecting a proper reference point for each decision-maker is necessary in order to effectively construct a purchasing decision-making approach based on prospect theory. However, asking each decision-maker to provide a reference point is unpractical. A reference point can be inferred based on the item preferences of a customer. The item preferences of a customer can be identified via two methods. One is the method in which the preferences of a decision-maker can be directly inferred by collecting and analyzing previous decision-making experiences. The other is inferring the preferences of a decision-maker based on the decision-making experiences of decision-makers with similar preferences. Thus, reference points can be selected based on the previous decision-making experiences of a decision-maker or similar decision-makers.

A reference point represents the equilibrium point of decision-maker's psychological expectation for a particular item. The reference point of a decision-maker can be identified based on his or her previous decision-making experiences to determine whether that decision-maker will experience a gain or loss if he or she purchases particular item.

However, if a decision-maker does not have previous decision-making experiences, the customer's reference point of that item will be inferred based on the past decision-making experiences of other similar decision-makers.

In this section, two algorithms are put forward as the reference point selection principles of a particular item in a specific category in order to find proper reference points for purchasing decision-makers. Algorithm 1 is suitable for the situations in which decision-maker have previous evaluations of the items in a specific category; Algorithm 2 is available when the decision-maker does not have previous evaluations.

Algorithm 1. Reference point selection based on previous evaluations.

Input: Decision-maker's previous evaluations of the items in a specific category.

Output: Decision-maker's reference point for this type of item under each criterion.

Step 1: Collect the decision-maker's previous evaluations of the items in a specific category, as shown in Table 1.

Table 1 Linguistic evaluations of item criteria

Items	c_1	c_2	$\cdots$	c_j	$\cdots$	c_n
a_1	x_{11}^R	x_{12}^R	$\cdots$	x_{1j}^R	$\cdots$	x_{1n}^R
a_2	x_{21}^R	x_{22}^R	$\cdots$	x_{2j}^R	$\cdots$	x_{2n}^R
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
a_i	x_{i1}^R	x_{i2}^R	$\cdots$	x_{ij}^R	$\cdots$	x_{in}^R
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
a_m	x_{m1}^R	x_{m2}^R	$\cdots$	x_{mj}^R	$\cdots$	x_{mn}^R

In an e-commerce environment, customers usually do not buy the same one for several times, but they often buy various items, and they are allowed to give evaluations of items after consuming. These evaluations given by the customers are considered as the previous evaluations. In Table 1, a_i denotes the ith item, c_j denotes the jth criterion, and x_{ij}^R denotes the decision-maker's previous evaluation of the ith item under the jth criterion, expressed by a linguistic term. The superscript R means "reference point". We use this superscript to distinguish the evaluations that are used for finding reference points from those that are used for making purchasing decisions. This step can be problematic in that there may be no evaluations of an item under a certain criterion. In this paper, the evaluation is assumed to be the mode value of the customer's previous evaluations of the items in the category under specific criterion.

Step 2: Transform the linguistic evaluations into clouds.

Backward cloud generators (BCGs) and golden sections (GSs) are the approaches most frequently used to generate clouds [51]. Most decision-making approaches based on cloud model use GSs to generate clouds for simplicity. However, due to limitations of the applied mathematical calculation methodology, the clouds generated by GSs cannot accurately describe linguistic terms. In contrast, BCGs utilize statistical information to generate clouds, the clouds generated by BCGs describe linguistic terms more accurately than those generated by GSs [51].

In this step, linguistic evaluations can be aggregated and transformed into clouds using BCGs [52-54]. The results could be shown as $A = \left\{ Y_1^R, Y_2^R, \cdots, Y_j^R, \cdots, Y_n^R \right\}$, where A denotes the category of the item, and Y_j^R denotes the cloud evaluation aggregation result obtained under the jth criterion.

Step 3: Use the aggregation result as the reference point.

Use Y_j^R as the reference point of a decision-maker for item category A under the jth criterion.

Algorithm 2. Reference point selection based on the previous evaluations of other decision-makers.

Input: Similar decision-makers' evaluations of the items in a specific category.

Output: Decision-maker's reference point of this type of item under each criterion.

Step 1: Collect the evaluations of other similar decision-makers, and show them as Table 2.

Table 2 Linguistic evaluations of item criteria provided by other similar decision-makers

Decision-makers	Items	c_1	c_2	$\cdots$	c_j	$\cdots$	c_n
u_1	a_1^1	x_{111}^R	x_{112}^R	$\cdots$	x_{11j}^R	$\cdots$	x_{11n}^R
	a_2^1	x_{121}^R	x_{122}^R	$\cdots$	x_{12j}^R	$\cdots$	x_{12n}^R
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
	$a_{m'}^1$	$x_{1m'1}^R$	$x_{1m'2}^R$	$\cdots$	$x_{1m'j}^R$	$\cdots$	$x_{1m'n}^R$
u_2	a_1^2	x_{111}^R	x_{112}^R	$\cdots$	x_{11j}^R	$\cdots$	x_{11n}^R
	a_2^2	x_{221}^R	x_{222}^R	$\cdots$	x_{22j}^R	$\cdots$	x_{22n}^R
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
	$a_{m'}^2$	$x_{2m'1}^R$	$x_{2m'2}^R$	$\cdots$	$x_{2m'j}^R$	$\cdots$	$x_{2m'n}^R$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
u_i	a_1^i	x_{i11}^R	x_{i12}^R	$\cdots$	x_{i1j}^R	$\cdots$	x_{i1n}^R
	a_2^i	x_{i21}^R	x_{i22}^R	$\cdots$	x_{i2j}^R	$\cdots$	x_{i2n}^R
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
	a_k^i	x_{ik1}^R	x_{ik2}^R	$\cdots$	x_{ikj}^R	$\cdots$	x_{ikn}^R
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
	$a_{m'}^i$	$x_{im'1}^R$	$x_{im'2}^R$	$\cdots$	$x_{im'j}^R$	$\cdots$	$x_{im'n}^R$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
u_m	a_1^m	x_{m11}^R	x_{m12}^R	$\cdots$	x_{m1j}^R	$\cdots$	x_{m1n}^R
	a_2^m	x_{m21}^R	x_{m22}^R	$\cdots$	x_{m2j}^R	$\cdots$	x_{m2n}^R
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
	$a_{m'}^m$	$x_{mm'1}^R$	$x_{mm'2}^R$	$\cdots$	$x_{mm'j}^R$	$\cdots$	$x_{mm'n}^R$

In this step, all the similar decision-makers of the potential customer, who have purchased the items of the category and offered evaluations, are selected, and the degree of similarity between a potential customer and one of his similar customers can be obtained by using the similarity measurement [55-58] and the clustering technology [59-61].

In Table 2, u_i denotes the ith similar decision-maker, c_j denotes the jth criterion, a_k^i denotes the kth item of the ith decision-maker, and x_{ikj}^R denotes the ith similar decision-maker's evaluation of the kth item under the jth criterion, expressed as a linguistic term. The superscript R is utilized to distinguish the evaluations for finding reference points from those for making purchasing decisions.

Step 2: Transform the linguistic evaluations into clouds.

Using BCGs, linguistic evaluations can be aggregated and transformed into clouds, as displayed in Table 3.

Table 3 Cloud evaluations of item criteria provided by other similar decision-makers

Decision-makers	c_1	c_2	$\cdots$	c_j	$\cdots$	c_n
u_1	Y_{11}^R	Y_{12}^R	$\cdots$	Y_{1j}^R	$\cdots$	Y_{1n}^R
u_2	Y_{21}^R	Y_{22}^R	$\cdots$	Y_{2j}^R	$\cdots$	Y_{2n}^R
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
u_i	Y_{i1}^R	Y_{i2}^R	$\cdots$	Y_{ij}^R	$\cdots$	Y_{in}^R
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
u_m	Y_{m1}^R	Y_{m2}^R	$\cdots$	Y_{mj}^R	$\cdots$	Y_{mn}^R

In Table 3, u_i denotes the ith similar decision-maker, c_j denotes the jth criterion, and Y_{ij}^R denotes the ith similar customer's cloud evaluations of item category A under the jth criterion.

Step 3: Aggregate the cloud evaluations under each criterion.

The cloud evaluations of item category A provided by other similar decision-makers can be aggregated using the CWAA operator in Definition 4, and the results could be shown as $A = \{Y_1^R, Y_2^R, \cdots, Y_j^R, \cdots, Y_n^R\}$, where A denotes the category of the item, and Y_j^R denotes the cloud evaluation aggregation result under the jth

criterion and $Y_j^R = \sum_{i=1}^{m} \omega_{ij} Y_{ij}^R = \left(\sum_{i=1}^{m} \omega_{ij} Ex_{ij}^R, \right.$
$\left. \sqrt{\sum_{i=1}^{m} \omega_{ij} (En_{ij}^R)^2}, \sqrt{\sum_{i=1}^{m} \omega_{ij} (He_{ij}^R)^2} \right)$. In this paper, we assume all similar decision-makers have the same weight for the simplicity of calculation, thus the CWAA operator is reduced to CAA, and $Y_j^R =$
$$\sum_{i=1}^{m} \frac{1}{m} Y_{ij}^R = \left(\sum_{i=1}^{m} \frac{1}{m} Ex_{ij}^R, \sqrt{\sum_{i=1}^{m} \frac{1}{m} \left(En_{ij}^R \right)^2}, \sqrt{\sum_{i=1}^{m} \frac{1}{m} \left(He_{ij}^R \right)^2} \right).$$

Step 4: Use the aggregation result as the reference point.

Use Y_j^R as the reference point of the decision-maker for item category A under the jth criterion.

4. FMCDM approach based on cloud model and prospect theory

In this section, an FMCDM approach based on cloud model and prospect theory, in which the reference point selection principles proposed in Section 3 are applied, is developed.

Let $M = \{1, 2, \cdots, m\}$ and $N = \{1, 2, \cdots, n\}$, $A = \{a_1, a_2, \cdots, a_i, \cdots, a_m\}$ be a finite alternative set, where a_i denotes the ith alternative, and $C = \{c_1, c_2, \cdots, c_j, \cdots, c_n\}$ be a finite criteria set, where c_j denotes the jth criterion. Let $W = (w_1, w_2, \cdots, w_j, \cdots, w_n)^T$ be the criterion weight vector, where w_j denotes the weight or degree of importance of criterion c_j, and $\sum_{j=1}^{n} w_j = 1$, $0 \leq w_j \leq 1$, and $j = 1, 2, \cdots, n$. In addition, let $S = \{s_i | i = 1, 2, \cdots, 2t-1, t \in N\}$ be the linguistic term set, and let $X = \left[x_{ij} \right]_{m \times n}$ be a decision matrix, where x_{ij} is the linguistic evaluation of alternative a_i with respect to criterion c_j. Furthermore, let $x_{ij} \in S$, $i \in M$, and $j \in N$.

Since different decision-makers have different reference points, defining a fixed reference point for every decision-maker would be unreasonable. Thus, the reference point selection principles provided in Section 3 are used in the proposed approach to obtain an appropriate reference point for each decision-maker under every criterion.

In the remainder of this section, cloud model and prospect theory are used to develop an FMCDM approach with the predetermined reference point selection principles. The proposed approach is comprised of the following steps.

Step 1: Select a reference point for each decision-maker.

A reference point is selected for each decision-maker based on the criteria values of the linguistic terms. Let $S = \{s_i | i = 1, 2, \cdots, 2t-1, t \in N\}$ be a pre-established finite and totally ordered linguistic term set with odd cardinality, where s_i represents the ith linguistic variable of set S, and t represents the cardinality of the set [23].

According to the reference point selection principles described in Section 3, for a decision-maker with prior purchasing experiences, his or her reference point Y_j^R to an item under the jth criterion can be determined using Algorithm 1; otherwise, the reference point can be determined using Algorithm 2.

Step 2: Transform the linguistic terms into clouds.

Linguistic evaluations can be aggregated and transformed into clouds, which are described by three digital characteristics, by utilizing the BCGs. For example, the linguistic evaluations $X = \left[x_{ij} \right]_{m \times n}$ of an item a_i under criterion c_j can be aggregated and transformed into the cloud evaluation $Y_{ij}\left(Ex_{ij}, En_{ij}, He_{ij} \right)$, where Ex_{ij} is the expected value of X, En_{ij} is the entropy of Ex_{ij}, and He_{ij} is the hyper entropy. These three digital characteristics reflect the overall evaluations of item a_i under criterion c_j.

Step 3: Calculate the cloud prospect values of each item under each criterion.

The cloud prospect value v_{ij} of the ith item under the jth criterion can be calculated using Equation (6) provided in Definition 5. First, the cloud evaluation Y_{ij} and the decision-maker's reference point Y_j^R are compared using Definition 3. Next, the distance between Y_{ij} and Y_j^R, expressed as $d\left(Y_{ij}, Y_j^R\right)$, can be calculated using Definitions 2. The value of $d\left(Y_{ij}, Y_j^R\right)$ reflects the deviation of an item value from the reference point. Then, $\pi\left(p_{ij}\right)$, which reflects the decision-maker's risk attitude to risk, can also be calculated using Equation (6). Finally, the cloud prospect value v_{ij} of the ith item under the jth criterion can be calculated using Equation (6).

Step 4: Calculate the comprehensive cloud prospect value of each item.

$$V_i = \sum_{j=1}^{n} w_j v_{ij}, i = 1, 2, \cdots, m. \tag{7}$$

The comprehensive cloud prospect value of an item can be calculated using Equation (7), in which w_j denotes the weight of the jth criterion, v_{ij} denotes the cloud prospect value of the ith item under the jth criterion, and V_i denotes the comprehensive cloud prospect value of the ith item.

Step 5: Rank the items.

The items can be ranked according to their comprehensive cloud prospect values V_i. Higher values of V_i are associated with higher-satisfaction items.

5. Illustrative example and comparative analysis

In this section, a purchasing decision problem is used to demonstrate the applicability of the proposed FMCDM approach based on cloud model and prospect theory. In addition, a comparative study and discussion is performed in order to verify the applicability and accuracy of the proposed approach.

5.1. *Illustrative example*

As mentioned previously, consumer reviews can significantly influence the purchasing decisions of potential customers. However, as a consequence of the large number of evaluations available to potential customers, selecting alternatives and making effective purchasing decisions can be difficult. Consumers who purchase a single lens reflex (SLR) camera may choose to provide an evaluation of that item on an e-commerce website, and a potential customer who is considering purchasing an SLR camera may make a decision based on these evaluations.

Taobao.com, a network retail business platform, was founded by Alibaba group on May 10, 2003. Taobao.com has become the largest C2C website in China, with more than 60 million visits each day. Currently, there are nearly 5 billion users and more than 8 billion items available for purchase on this website. Approximately 48 thousand items are sold on Taobao.com per minute. Although the same item can be listed for sale by multiple sellers, the quality of that item can differ significantly depending on its source. Thus, e-commerce can be risky for the customer. In order to

combat this riskiness, potential customers oftentimes rely heavily on information provided by other consumers.

In the following example, linguistic evaluations of SLR cameras provided by a potential customer are collected in order to determine his reference point. Linguistic evaluations of alternative SLR cameras provided by other consumers are also collected in order to further assist the customer with his or her purchasing decision.

First, a potential customer was asked to select a few SLR cameras from Taobao.com. Then, the characteristics of these SLR cameras were analyzed using the evaluations provided by other customers in order to determine which SLR cameras might be preferred by the potential customer. The terms a_1, a_2, a_3, and a_4 were used to denote the alternative SLR camera options. Three criteria were considered during the analysis, including battery performance (c_1), screen performance (c_2), and image quality (c_3). The weight vector of the three criteria was given as w=(0.3, 0.25, 0.45)$^{\mathrm{T}}$. The linguistic term evaluations were obtained from Taobao.com.

The purchasing-making decision procedure based on cloud model and prospect theory was comprised of the following steps.

Step 1: Determine the reference point of the potential customer.

On Taobao.com, item evaluations are provided in linguistic terms with a linguistic term set of
$$S = \{s_1 = \text{VP:very poor}, s_2 = \text{P:poor}, s_3 = \text{M:medium},$$
$$s_4 = \text{H:high}, s_5 = \text{VH:very high}\} .$$

As shown in Table 4, the customer in this example purchased and provided linguistic evaluations for 20 SLR cameras. As shown in Table 5, the reference points for the SLR camera under each criterion were obtained using Algorithm 1.

Table 5 Reference points for the SLR camera under each criterion

Item	c_1	c_2	c_3
SLR camera	(3.8, 0.78, 0.95)	(3.95, 0.77, 0.96)	(3.85, 0.61, 0.75)

Step 2: Transform the linguistic terms into clouds.

The linguistic evaluations of the alternative items a_1, a_2, a_3, and a_4 are shown in Tables 6-9, respectively.

Table 4 Linguistic evaluations of the SLR camera under each criterion

Items	c_1	c_2	c_3
Canon 100D	VH	VH	VH
Canon 1000D	H	M	M
Nikon D5100	P	H	H
Canon 350D	M	VH	H
Nikon D300S	M	M	VH
Canon 600D	M	VH	H
Nikon D800	H	H	M
Canon 700D	VH	M	H
Canon 10D	H	VH	VH
Nikon D7100	M	H	M
Canon 550D	P	M	H
Nikon D5200	VH	VH	H
Canon 650D	H	H	VH
Nikon D2HS	VH	M	M
Canon 20Da	H	VH	H
Nikon D100	H	M	M
Canon 7D2	H	H	M
Canon 400D	H	VH	M
Nikon D40X	M	P	H
Nikon D3000	VH	H	H

Table 6 Linguistic evaluations of a_1

c_1	c_2	c_3
H	VH	H
H	H	VH
M	H	VH
VH	H	H
M	VH	H
H	M	M
VH	H	H
H	M	H
M	M	VH
M	VH	M
VH	VH	VH
H	H	H
VH	H	H
M	M	M
H	VH	H
VH	H	H
H	M	M
M	H	H
H	VH	H
H	M	H

Table 7 Linguistic evaluations of a_2

c_1	c_2	c_3
M	H	H
M	H	H
H	M	VH
H	H	M
M	M	VH
VH	M	VH
H	M	M
M	H	M
H	H	M
H	M	VH
M	H	M
M	M	VH
H	M	H
H	M	H
H	H	M
VH	H	VH
M	H	M
M	H	VH
M	VH	M
M	M	VH

Table 9 Linguistic evaluations of a_4

c_1	c_2	c_3
H	H	M
H	H	VH
VH	M	M
M	M	M
VH	M	VH
VH	H	M
M	H	M
M	VH	VH
M	VH	VH
VH	M	M
M	M	M
VH	VH	M
H	M	VH
H	VH	VH
M	VH	M
VH	VH	VH
M	M	VH
VH	M	M
M	VH	VH
VH	VH	VH

Table 8 Linguistic evaluations of a_3

c_1	c_2	c_3
VH	VH	M
VH	H	VH
VH	M	H
H	M	H
M	M	H
M	H	M
M	VH	M
VH	M	M
M	M	M
VH	M	M
VH	M	M
M	H	M
M	M	VH
M	M	VH
M	M	M
M	VH	M
M	M	M
M	VH	VH
M	H	H
M	H	M

The linguistic evaluations of each alternative SLR camera under the three criteria were transformed into aggregated cloud values using BCGs. The results are shown in Table 10.

Table 10 Aggregated cloud evaluations of the alternative SLR cameras

Items	c_1	c_2	c_3
a_1	(4, 0.67, 0.75)	(4, 0.67, 0.75)	(4, 0.58, 0.50)
a_2	(3.65, 0.45, 0.65)	(3.6 0.52, 0.75)	(3.6, 0.45, 0.68)
a_3	(3.65, 0.71, 1.06)	(3.6, 0.68, 0.95)	(3.6, 0.63, 0.90)
a_4	(4, 0.7, 1.00)	(4, 0.71, 1.00)	(4, 0.73, 1.25)

Step 3: Calculate the cloud prospect value of each item under each criterion.

The cloud prospect value of each alternative SLR camera under each criterion was calculated using Equation (6). In this process, p_i was identified as the positive review rate of the ith item and we adopted $\alpha = \beta = 0.88$, $\lambda = 2.25$, $\gamma = 0.61$ and $\delta = 0.72$ of Equation (6) based on the results of numerous experiments conducted by Kahneman and Tversky [46, 62]. The calculation results are shown in Table 11.

Table 11 Cloud prospect values of the alternatives under each criterion

Items	c_1	c_2	c_3
a_1	0.08371	0.06921	0.08502
a_2	0.11397	-0.232	-0.16351
a_3	-0.0866	-0.16953	-0.12871
a_4	0.05784	0.02572	-2.38372

Step 4: Calculate the comprehensive cloud prospect value of each item.

The comprehensive cloud prospect value of each alternative SLR camera was obtained using Equation (7) as follows:

$V_1 = 0.0807, V_2 = -0.0974, V_3 = -0.1263, V_4 = -0.1039$.

Step 5: Rank the items.

Since the cloud prospect values of the SLR cameras were ranked as $V_1 > V_2 > V_4 > V_3$, the rankings of the items can be obtained as $a_1 \succ a_2 \succ a_4 \succ a_3$, the customer would be most satisfied with option a_1.

5.2. *Comparative study and discussion*

In order to verify the applicability of the proposed FMCDM approach, a comparative study was performed using two other commonly used decision-making approaches that utilize linguistic information. The SLR camera example presented in Subsection 5.1 was used to conduct the comparative analysis. The ranking results are shown in Table 12.

Table 12 Ranking results obtained by the three approaches

Approaches	The final ranking	The best item(s)	The worst item(s)
The approach developed in Ref. [50]	$a_4 \succ a_1 \succ a_3 \succ a_2$	a_4	a_2
The approach developed in Ref. [63]	$a_1 \succ a_4 \succ a_2 \succ a_3$	a_1	a_3
The proposed approach	$a_1 \succ a_2 \succ a_4 \succ a_3$	a_1	a_3

The proposed approach was compared to two other approaches. First of all, the approach was compared to the approach developed in Ref. [50]. In that approach, the linguistic terms were transformed into clouds using the golden section method and aggregated using cloud aggregation operators based on expected utility theory.

The alternatives were then ranked according to their final aggregated values. According to the results, the proposed method and the method developed in Ref. [50] yielded different final rankings. These results were attributed to two differences in these methods. The approach developed in Ref. [50] is based on expected utility theory, conventional arithmetic weighted average operators are used as cloud aggregation operators. Thus, items with more positive linguistic evaluations are ranked higher than other items. However, based on prospect theory, reference points and loss aversion are considered during the decision-making process in the proposed approach. Hence, a consumer may select an item that may not have the highest aggregated value if it satisfies his or her subjective expectations and risk preferences. In addition, in the approach developed in Ref. [50], clouds are generated via golden section. Generally, items with a larger number of s_2 or s_4 evaluations may be assigned lower orders because the Ex of the cloud is smaller than it should be according to gold section method. However, in the proposed approach, clouds are generated with BCGs, so it avoids the problems associated with the golden section method.

Next, the proposed approach was also compared to the approach developed by Liu in Ref. [63]. In the approach developed by Liu, prospect theory was utilized to handle decision-making problems under risk with uncertain linguistic information, and the probability is defined as the interval value. According to Liu's approach, the linguistic evaluations were transformed into uncertain linguistic variables, and the statistical probability of each item receiving rave reviews was determined. Using the resulting information, the prospect values of the alternative items were calculated, and the items were ranked, as shown in Table 12. Also, the proposed method and the method developed in Ref. [63] yielded identical maximum and minimum rankings. However, items a_2 and a_4 received different rankings. These rankings were not the same due to the differences in the reference point selection principles of the two approaches. In Liu's approach, a fixed reference point, calculated as the median of the linguistic variables, is employed. In our approach, novel reference point selection principles are utilized in order to determine the reference point of that particular customer.

Unlike the approaches developed in Ref. [50] and Ref. [63], the subjective expectations and risk preferences of a customer, as well as the specific reference point for that

particular customer, are considered in the proposed approach. The final item rankings obtained via the proposed approach, $a_1 \succ a_2 \succ a_4 \succ a_3$, were more precise and reliable than the rankings obtained via the other methods. Thus, the proposed purchasing decision approach could be effectively used by consumers in order to maximize customer satisfaction and minimize risk.

6. Conclusion

In this paper, an FMCDM purchasing decision-making approach that takes into account the customer's subjective expectations and aversion to loss in an e-commerce environment is developed. In the proposed approach, linguistic evaluations are transformed into clouds, and prospect theory and reference point selection principles are used to calculate the cloud prospect values of alternative items. The advantage of the approach is that it considers the unique subjective expectation and loss aversion of a customer in real-life purchasing decision-making process in e-commerce environment. Combining cloud model and prospect theory, the proposed approach could be applied to some scenarios with unique potential customers. Thus, the rankings obtained via the proposed approach reflect the preferences of customers more properly than the rankings obtained via other approaches.

Further studies concerning the proposed approach should be conducted. First of all, the proposed approach only considers linguistic evaluations, limiting the consumer's ability to express his or her feelings regarding an item. These linguistic evaluations could be transformed into fuzzy linguistic sets, such as intuitionistic fuzzy sets and neutrosophic sets, in order to maximize the potential of FMCDM approaches. In addition, the criteria weights used for the purposes of this paper were provided directly by a consumer in the illustrative example. However, in an actual e-commerce environment, asking customers to provide criteria weights would be unpractical. Thus, a method to purchasing problems employing objectively calculating weights should be developed. Moreover, the similarity between decision-makers needs to be carefully studied, because it will affect the final rankings of alternatives when using Algorithm 2. Some similarity measurements should be discussed and selected to find similar decision-makers, and the number and weights of similar decision-makers should also be discussed and identified.

Besides, further studies can also focus on improving the effectiveness of the proposed approach.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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