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A joint optimization strategy for scale-based product family positioning

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Abstract

With the development of modern technologies and global manufacturing, it becomes more difficult for companies to distinguish themselves from their competitors. In order to keep their competitive advantages, companies must properly position their product families by offering a right product portfolio to each target market. To evaluate competitive advantages for a scale-based product family, this paper takes product family competitive advantage (PFCA) as a measure metric which is consisted of customer choice probability, sales, and profit. Meanwhile, to keep lower manufacturing costs, a commonality index of scale-based product family is proposed based on product design technology parameters in a product family. A multi-objective joint optimization model that balances the competitive advantages and the commonality is proposed. Based on a case study of motor product family positioning, Pareto frontier solutions are generated by genetic algorithm, and the results show that the joint optimization model excels in supporting product family positioning.

Keywords: Scale-based product family, product portfolio, product family positioning, joint optimization

1. Introduction

Many companies turn to mass customization in response to the tremendous changes in today's global marketplace [1][2], due to the fact that mass customization is able to satisfy individual customer requirements with the efficiency of mass production [3][4]. Several strategies have been proposed to help producers to build mass customized products and maintain a lower cost. Among these, product family strategy is one of the most popular operations management approaches, in which each product has unique functions or features to meet specific customer requirements [5][6]. To stimulate sales and maximize revenue, manufacturers intend to provide a variety of product variants in a product family [5]. However, high variety will result in inefficiencies in manufacturing, and accordingly the costs increase exponentially [7]. In this regard, it is important for a company to resolve the tradeoffs between product diversity and engineering costs [8].

Generally, it is believed that satisfying more market requirements will bring more sales and accordingly more profits. Therefore, a company often assesses to what extent one product variant is able to satisfy market requirements, and then determine an optimal portfolio to satisfy as many market requirements as possible. However, based on a marketing analysis report by Simpson et al. [9], meeting more market requirements is not always optimal for a small or medium-sized company. Although its products may be equally possible to satisfy customer requirements of individual market segments, it will have a smaller market share and fewer profits because of its weaker product competition ability. Based on the theory of competitive intensity, targeting some specific market segments by satisfying their customers may help gain competitive advantages and increase sales and profits. Accordingly, it is more reasonable that the product family competitive advantage (PFCA) should be regarded as a metric during the optimization of product diversity.

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To control engineering costs, increasing commonality of a product family has been widely considered as an effective method. Commonality can be obtained by minimizing the non-value added variations across the product variants within a family without limiting the choices of customers in each market segment, i.e., make each product within a family distinct in ways customers notice and identical in ways customers cannot see [11]. High commonality means reduced product line complexity, decreased setup and retooling time, and increased productivity [10]. However, if commonality is too high, variants lack distinctiveness and individual performance of products of a family is not optimized. PFCA will accordingly be low, and vice versa [11]. Therefore, PFCA and commonality need to be well leveraged by proper positioning, and the relevant tradeoff analysis between PFCA and commonality essentially necessitates a formulation of joint optimization of both PFCA and commonality.

There are two types of product families: module-based product family and scale-based product family. Accordingly, there are two kinds of metrics of PFCA and commonality. For the module-based product family, product diversity is derived by adding, substituting, and/or removing one or more functional modules [12][13]. The positioning method of module-based product family is essentially a combinatorial optimization problem of various modules to tradeoff PFCA and commonality. However, the product diversity of scale-based product family is obtained by "stretching" or "shrinking" scaling parameters of a product in one or more dimensions [14]. As a result, the positioning method of scale-based product family is essentially a combinatorial optimization problem of various parameters to tradeoff PFCA and commonality. This paper takes scale-based product family as the research objective and focuses on the issue of product family positioning using a joint optimization method. The rest of the paper proceeds as follows. Section 2 reviews related work regarding product family positioning. Customer requirements, market segments, and parameters in product family are described in Section 3. Metrics of PFCA and commonality index are established in Section 4 and Section 5, respectively. Section 6 elaborates the optimization model for product family positioning and develops a genetic algorithm (GA) for multi-objective optimization. A case study is reported in Section 7. The paper concludes in Section 8 with discussions on further research.

2. Related Work

2.1. *Evaluation metrics*

Product family design involves the challenges of how to balance the commonality of the products in the family and individual performance (i.e., distinctiveness) of each product in the family [15]. Both commonality and performance need a set of properly designed metrics for an evaluation purpose. In many cases, such metrics focus on two pillars: commonality and revenue.

The degree of commonality index proposed by Collier [10] was one of the first such indices. It uses information contained in the company's bills of materials to assess commonality of a single end item, a product family, or an entire product line. Jiao and Tseng [16] extended Collier's commonality index and created indices for component part commonality and process commonality. Martin and Ishii [17] also introduced a commonality index similar to Collier's, along with indices for measuring setup costs and product differentiation points, which correlate with many of the indirect costs caused by variety. For measuring the degree of variation within a scale-based product family, the non-commonality index (NCI) was proposed by Simpson et al. [18], which is a normalized measure of the variability of design variable setting across members of the product family. A smaller NCI indicates less variation among design variable settings across the family, and, hence, more commonality within the family. Based on a similar principle to that of the NCI, Tian [19] proposed an exponential coefficient to evaluate the similarity of two design parameter vectors and builds a commonality index based on this similarity.

Another category of metrics is to measure revenue. For example, Li and Azarm [20] adopted the Net Present Value (NPV) in economics to evaluate the benefits of the product family in order to maximize revenue. de Week et al. [21] introduced the sales volume of a product family by comparing it with the benchmark products which have the maximum sales volume. Pullmana et al. [22] took sales volume as an optimization objective and a conjoint analysis was used to assess product characteristics. To balance the benefits of customers and of producers, Jiao et al. [23] propose a concept of shared surplus taking customer preference, customer choice probability, size of the market segments, and cost. However, all these metrics assume that the predominated market is known a priori, and the revenue is normally unrealistic under the competitive circumstances. Taking the competitive products in the market as a benchmark, the metrics of PFCA and commonality are proposed in the paper.

2.2. *Optimization approaches*

Product family positioning essentially entails a type of multi-objective optimization problems [9]. When using multi-objective optimization to determine the best design variable settings for individual products within a product family, there are two basic optimization approaches: two-step approach and single-step approach [24]. The two-step approach divides the optimal process into two steps: the entire alternative product variants are enumerated in the first step, and then promising products are selected to constitute a product portfolio in the second step [9]. Following the two-step approach, Green and Krieger [25][26] introduced several heuristic procedures with the consideration of how to generate a reference set appropriately. In general, the two-step approach is preferred only when the number of alternatives is small and product attributes are simple [9]. The single-step approach uses a single optimized computation for product family planning based on internal and external characteristics directly. An example of the single-step approach was proposed by Fujita et al. [27] who simultaneously optimized the system structure and configuration of a product family and applied their approach to a family of aircraft. In general, the one-step approach is more preferable when the number of alternatives is large, as the intermediate step of enumeration can be eliminated [28].

2.3. *Optimization algorithms*

Product family positioning is classified as a combinatorial optimization problem, in that each company strives for the optimality of its product offerings through various combinations of products [29]. By intuition, finding the near-optimal solution for a finite combinatorial optimization problem could be done by simple enumeration. In practice, however, this technique is often impossible because the number of feasible solutions is often enormous. A number of methods and algorithms have been developed to solve combinatorial optimization problems. Sait and Youssef [30] divided them into two groups: exact algorithms and approximation algorithms. Due to the enumerative nature, exact algorithms are not easy to design with moderate computational effort as can be seen from the complexity theory [31]. Comparing with traditional calculus-based or approximation-based optimization techniques, GAs excel in solving combinatorial optimization problems [32]. However, GAs are in general incapable of fine-tuning for obtaining the global optimum [33]. As a result, various modified GAs have been proposed for particular problems [34]. For example, Jiao et al. [23] used a new coding method and a multi-gene crossover operator. Yang et al. [35] proposed a hybrid algorithm which is a combination of GA and neural network algorithm.

3. Problem Description

This paper aims to deal with the problem of scale-based product family positioning. We will build an optimization model with the goal of finding the optimal product combination among a set of alternative product variants. The optimal product portfolio, considering the tradeoff between PFCA and commonality, constitutes the expected product family.

The parameters of a product family can be divided into three facets: basic requirement parameter (BRP), performance requirement parameter (PRP) and design technology parameter (DTP). BRPs describe a basic function or performance of a product variant, by which the market is divided into several market segments. PRPs reflect characteristics of products that are relevant to customer experience or expectation. To some extent, PRPs can determine the level of customer satisfaction. DTPs refer to parameters which engineers are interested in. For example, when customers want to buy a car, features of style, fuel type and engine power can be considered as BRPs, fuel consumption per kilometer, maximum speed, and price as PRPs, and diameter of axle, gear number of engine as DTPs.

BRPs can be defined as $R=\{r_t \mid 1\le t\le T\}$, where T is the total number of parameters in BRPs. For each r_t ($\forall r_t\in R$), its value is composed of a series of discrete values or ranges, i.e. $r_t^*=\{r_{tl}^* \mid 1\le l\le L_t\}$. For each r_t^*, L_t may be different. According to different value combinations of r_t, the whole market M can be divided into I market segments, where $M=\{m_1,\cdots,m_i\cdots,m_I\}$, $1\le I\le \prod_{t=1}^{T} L_t$ and $\forall m_i\in M$, $m_i=\{r_{1i}^*,\cdots r_{ti}^*,\cdots r_{Ti}^*\}$.

For I market segments, we assume that there are also I corresponding alternative products in a company to be positioned, i.e. $P=\{P_i \mid 1\le i\le I\}$, and P_i ($\forall P_i\in P$) can meet the corresponding market segment m_i ($\forall m_i\in M$). Furthermore, for each P_i ($\forall P_i\in P$), there are a number of (N_i-1) competitive products in the market segment, and N_i may be different for different product P_i.

PRPs can be defined as $G = \{g_z | 1 \le z \le Z\}$, where Z is the total number of variables in PRPs. For N_i products in the same market segment, although the values of BRPs are the same, the values of PRPs may be different because of different technology or economic factors. The value of g_z ($g_z \in G$) is composed of a series of discrete values or ranges, i.e. $g_z^* = \{g_{zk}^* | 1 \le k \le K_z\}$. Among the K_z values, there are always a maximum value $g_z^{*\max}$ and a minimum value $g_z^{*\min}$. For product P_i, its values of g_{zi}^* make a major impact on the product competitive advantage.

On the other hand, a number of S DTPs can be formalized as $D = \{d_s | 1 \le s \le S\}$. For each d_s ($\forall d_s \in D$), its value is composed of a series of discrete values or ranges, i.e. $d_s^* = \{d_{sv}^* | 1 \le v \le V_s\}$. For each d_s^* in product family P, V_s may be different. To improve commonality of product family P, it is necessary to control the value of V_s.

According to the PRPs and DTPs, a decision variable $X = \{x_1, \cdots x_i, \cdots x_I\}$ can be defined to describe which products are selected as the members of product family by a company. x_i ($x_i \in X$) is equal to 1 or 0, where 1 means the product P_i is selected, and vice versa.

4. Metric of PFCA

4.1. *Choice probability*

When a company cannot monopolize a market segment, products of a company will inevitably face competition with products in the same category from other companies. The competition often leads to the fact that a product with satisfied BRPs will not be necessarily purchased by customers. Customer choices also depend on the values of PRPs, which reflect the level of customer satisfaction.

4.1.1. *Measurement of customer satisfaction*

For the variable value g_{zi}^* of the product variant p_i, its satisfaction level u_{zi} ($1 \le z \le Z$, $1 \le i \le I$) can be calculated as follows:

(i) If customer satisfaction is more preferable when the values of PRPs increase (the-more-the-better), u_{zi} is calculated as follows:

$$u_{zi} = \frac{g_{zi}^* - g_z^{*\min}}{g_z^{*\max} - g_z^{*\min}} \tag{1}$$

(ii) If customer satisfaction is more preferable when the values of PRPs decrease (the-smaller-the-better), u_{zi} is calculated as follows:

$$u_{zi} = \frac{g_z^{*\max} - g_{zi}^*}{g_z^{*\max} - g_z^{*\min}} \tag{2}$$

Here, $u_{zi} \in [0,1]$. $u_{zi} = 1$ means that g_{zi}^* is the benchmark among N_i products and possesses a maximum satisfaction level.

For PRPs with Z variables, the customer satisfaction level U_i of product p_i can be evaluated as follows:

$$U_i = \sum_{z=1}^{Z} q_z u_{zi} \tag{3}$$

where q_z is the weight of g_{zi}, and $\sum_{z=1}^{Z} q_z = 1$.

4.2.2. *Measurement of choice probability*

Customer choice probability indicates how likely a customer in market i chooses a product among N_i competing products. According to the multinomial logit model [36], the probability C_i of customers choosing the product variant p_i can be calculated as follows:

$$C_i = \frac{e^{\lambda U_i}}{e^{\lambda U_i} + \sum_{n=1}^{N_i - 1} e^{\lambda U_n}} \tag{4}$$

In Eq.(4), λ is a scaling parameter. When $\lambda \to \infty$, the model is similar to the deterministic model, which means customers are absolutely rational, and the product will be selected according to the values of PRPs. However, the model becomes a uniform distribution as $\lambda \to 0$, which indicates the products will be randomly selected. The value of λ depends on the actual market analysis.

4.2. *Holistic measurement of PFCA*

PFCA evaluates the ability of a product family bringing profits to a company by covering multiple market segments. Higher PFCA means higher probability of obtaining expected profit in a competitive circumstance. PFCA is related to the profit ratio of each variant p_i and market size, in addition to the choice probability of each product. PFCA can be calculated holistically as follows:

$$\begin{aligned} PFCA(X) &= I \cdot \sum_{i=1}^{I} w_i C_i o_i x_i \\ &= I \cdot \sum_{i=1}^{I} w_i \frac{e^{\mu U_i}}{e^{\mu U_i} + \sum_{n=1}^{N_i - 1} e^{\mu U_n}} o_i x_i \end{aligned} \tag{5}$$

In Eq.(5), w_i is the weight of market segment i, and $\sum_{i=1}^{I} w_i = 1$. A larger w_i means that market segment i has more customers, and accordingly, the market size is larger. o_i is the profit ratio of product p_i, and $\sum_{i=1}^{I} o_i = 1$. A larger o_i represents a higher profit. For a company, when assuming the products are mature, each o_i is essentially invariable. x_i ($x_i \in X$) is a decision variable as described in Section 3.

Also, from Eq.(5), it suggests that there are some ways to improve PFCA: (1) Holding more product variants in the product family, and it is possible to cover more market segments; (2) The products should be selected if they satisfy a large number of potential customers at a higher level; (3) The products in the product family should be higher profit ratios, and can bring more profits as a whole.

5. Commonality Index

Higher commonality means more similar technologies will be applied in a product family, and accordingly reduces the product life-cycle cost. For a scale-based product family, commonality reflects the sharing and reusing degree of design technology parameter values among the product variants. The steps of calculating commonality are as follows:

Step1: Analyzing the aggregation degree of design technology parameter value in alternative products. If $\forall d_{sv}^* \in d_s^*$ ($1 \le s \le S, 1 \le v \le V_s$), there are $n_{sv} (1 \le n_{sv} \le I)$ same variants in a company, then the aggregation degree of d_s ($\forall d_s \in D$) can be calculated as follows:

$$A_s = \sum_{v=1}^{V_s} \left(\frac{n_{sv}}{I'}\right)^2 \tag{6}$$

where, $I' = \sum_{i=1}^{I} x_i$ and $\sum_{v=1}^{V_s} n_{sv} = I'$.

Step2: Assessing the weights of all DTPs using pair-wise comparison matrix. To evaluate the importance degree of DTP, grade criteria are shown in Table 1.

To reduce subjectivity and uncertainty, the weight of each design technology parameter can be computed by pair-wise comparison. According to Table 1, a standard pair-wise comparison matrix was established as shown in Table 2. In the matrix, if the relative importance of the *i*-th parameter to the *j*-th parameter is e_{ij}, then the relative importance of the *j*-th parameter to the *i*-th parameter is $e_{ji} = 1/e_{ij}$.

Table 1. Grade criteria of DTPs

Value	Importance degree
1	same
2	Between "same" and "slightly higher"
3	Slightly higher
4	Between "slightly higher" and "obviously higher"
5	Obviously higher
6	Between "obviously higher" and "absolutely higher"
7	Absolutely higher
8	Between "absolutely higher" and "extremely higher"
9	Extremely higher

Table 2. Standard pair-wise comparison matrix

	d_1	…	d_i	…	d_j	…	d_S
d_1	1	…	e_{1i}	…	e_{1j}	…	e_{1S}
…	…	…	…	…	…	…	…
d_i	$1/e_{1i}$	…	1	…	e_{ij}	…	e_{iS}
…	…	…	…	…	…	…	…
d_j	$1/e_{1j}$	…	$1/e_{ij}$	…	1	…	e_{jS}
…	…	…	…	…	…	…	…
d_S	$1/e_{1S}$	…	$1/e_{iS}$	…	$1/e_{jS}$	…	1

Next, the weight value $\alpha_s (1 \le s \le S)$ is determined by normalizing the elements of the eigenvector corresponding to the maximum eigenvalue calculated from the pair-wise comparison matrix [37]. Using the eigenvector assures consistency of the matrix and reduces inconsistency between the comparison values. These parameter weights will be employed during the final calculation of commonality.

Step3: according to the weight of design technology parameter and aggregation degree of its value, the commonality of the product family CI can be calculated as follows:

$$CI = \sum_{s=1}^{S} \alpha_s A_s = \sum_{s=1}^{S} \left(\alpha_s \sum_{v=1}^{V_s} \left(\frac{n_{sv}}{I'} \right)^2 \right) \tag{7}$$

6. Positioning Optimization

6.1. *Optimization model*

In order to tradeoff PFCA and commonality, we combine the Eq.(5) and Eq.(7), and a two-objective optimization model is formulated as Eq.(8). In the optimization model, the first optimization objective function is to determine the maximum value of PFCA, and the second one is to find the maximum commonality of the product family.

$$\max PFCA(X) = \max\left(I \cdot \sum_{i=1}^{I} w_i \frac{e^{\mu U_i}}{e^{\mu U_i} + \sum_{n=1}^{N_i - 1} e^{\mu U_n}} v_i x_i \right)$$

$$\max CI(X) = \max\left(\sum_{s=1}^{S} \left(\alpha_s \sum_{v=1}^{V_s} \left(\frac{n_{sv}}{I^{'}} \right)^2 \right) \right)$$

$$s.t.\ \ X = [x_1, \ldots, x_i, \ldots, x_I]$$

$$I^{'} = \sum_{v=1}^{V_s} n_{sv}$$

$$I^{'} = \sum_{i=1}^{I} x_i$$

$$\sum_{i=1}^{I} x_i \geq 2$$

$$x_i \in \{0,1\}\ (i \leq I) \tag{8}$$

6.2. Optimization algorithm

For the multi-objective optimization problem, it is reasonable to find a Pareto Solution Set (PSS) instead of an optimal solution [38]. In order to do so, a GA is proposed. The basis of GA is about representation of the problem to be solved with a finite-length string called a chromosome. For product family positioning, the length of the string stands for the total number of alternative products in the product family. Each element of the string indexed by "1" or "0", called gene, indicating whether the product is selected or not.

Fig. 1 summarizes the procedures of the optimization solution, as elaborated below:

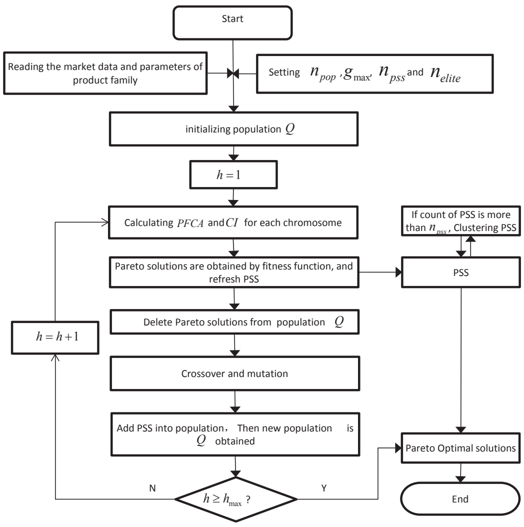

Fig. 1 procedures of optimization solution

Step1: Reading the market and parameters of the product family, such as the distribution of customer requirements, customer choice probability, profitability ratio of each alternative product variant, and so on. At the same time, solution parameters are given, such as the size of population (n_{pop}), maximum iteration loops ($h_{\max}$), the maximum size of PSS (n_{pss}), the maximum number of Pareto solutions (n_{elite}) and so on;

Step2: Generate randomly initial population Q consisted of n_{pop} chromosomes. Set PSS as NULL and iteration loop h is 1;

Step3: Calculate $PFCA$ and CI for each chromosome.

Step4: To find all non-dominated solutions in various GA search directions, a fitness function with a random weight is developed to evaluate these solutions [39]. The weights for $PFCA$ and CI can be calculated as $q_1/(q_1+q_2)$ and $q_2/(q_1+q_2)$, respectively, where q_1 and q_2 are a random numbers in the interval [0,1]. As a result, the fitness degree b_k of the k-th solution (i.e. the k-th chromosome) can be calculated as Eq.(9):

$$b_k = \frac{q_1}{q_1+q_2} PFCA_k + \frac{q_2}{q_1+q_2} CI_k \tag{9}$$

where $PFCA_k$ and CI_k are the respective objective values in the optimization model for the k-th solution.

According to Eq.(9), the number of n_{elite} Pareto solutions can be obtained.

Step5: Delete the Pareto solutions from population Q, and insert these Pareto solutions into PSS. If the count of PSS is more than n_{pss}, apply the clustering method to keep the size of PSS [40].

Step6: Do the crossover and mutation operator for population Q with the probability of n_c and n_m, respectively.

Step7: Add PSS to the population, and a new population Q is generated. $h = h + 1$. If $h \geq h_{\max}$, then end the process. Otherwise, go to step3.

6.3. *Comprehensive optimal selecting*

For a multi-objective optimization problem, there is a contradiction trend between several objectives. The fuzzy set theory is used in this paper to evaluate all the Pareto solutions and the comprehensive optimal solution for all the objectives are then selected [41]. For the two objectives of solution k in PPS, their normalization value μ_{PFCA}^{k} and μ_{CI}^{k} can be defined as Eq. (10) and (11):

$$\mu_{PFCA}^{k} = \frac{F_{PFCA}^{k} - F_{PFCA}^{\min}}{F_{PFCA}^{\max} - F_{PFCA}^{\min}} \tag{10}$$

$$\mu_{CI}^{k}=\frac{F_{CI}^{k}-F_{CI}^{\min}}{F_{CI}^{\max}-F_{CI}^{\min}} \quad (11)$$

in which $F_{PFCA}^{\max}$, $F_{CI}^{\max}$, $F_{PFCA}^{\min}$ and $F_{CI}^{\min}$ are the respective maximum and minimum values for the two objective functions in the optimization model. F_{PFCA}^{k} and F_{CI}^{k} are the respective objective function values for the *k*-th solution in PPS. For the solution *k*, its comprehensive normalization value μ^{k} can be evaluated as Eq. (12):

$$\mu^{k}=\frac{\mu_{PFCA}^{k}+\mu_{CI}^{k}}{\sum_{k=1}^{N_{pss}}(\mu_{PFCA}^{k}+\mu_{CI}^{k})} \quad (12)$$

in which N_{pss} is the count of PSS. The solution which has the maximum μ^{k} is the comprehensive optimal solution. If the solutions in PSS are sorted by μ^{k}, we can get the priorities of the solutions. This will provide a basis for the decision-maker.

7. Case Study

Three-phase asynchronous motor is used as a case to illustrate product family positioning optimization problem. It is widely used in industry because of its simple-structure, easy maintenance, high reliable operation and low cost.

7.1 *Market requirements*

When customers choose a three-phase asynchronous motor, the power and the speed of the motor are often taken into account for decision making. Accordingly, BRPs can be defined as $R=\{\text{Rated Power, Rotation Speed}\}$, and rated power and rotation speed are divided into the following discrete values:

$$\textit{Rated Power}:\ r_1^*=\{3kW, 7.5kW, 15kW, 22kW, 30kW\}$$

$$\textit{Rotation Speed}:\ r_2^*=\{[3000,2800],[1500,1400],[1000,950],[750,700]\}$$

According to the BRPs, the market can be divided into 20 market segments as listed in Table 3. The segments and their market shares are shown in Table 4 and 5.

Table 3. Market segments of three-phase asynchronous motor

Market Segment	Rated Power (kW)	Rotation Speed (r/min)	Market Segment	Rated Power (kW)	Rotation Speed (r/min)
m_1	3	[3000, 2800]	m_{11}	15	[1000, 950]
m_2	3	[1500, 1400]	m_{12}	15	[750, 700]
m_3	3	[1000, 950]	m_{13}	22	[3000, 2800]
m_4	3	[750, 700]	m_{14}	22	[1500, 1400]
m_5	7.5	[3000, 2800]	m_{15}	22	[1000, 950]
m_6	7.5	[1500, 1400]	m_{16}	22	[750, 700]
m_7	7.5	[1000, 950]	m_{17}	30	[3000, 2800]
m_8	7.5	[750, 700]	m_{18}	30	[1500, 1400]
m_9	15	[3000, 2800]	m_{19}	30	[1000, 950]
m_{10}	15	[1500, 1400]	m_{20}	30	[750, 700]

Table 4. Segments and their market shares

Market Segment	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9	m_{10}
Market Share	11%	7%	3%	2%	7%	6%	4%	3%	8%	7%

Table 5. Segments and their market shares (Cont.)

Market Segment	m_{11}	m_{12}	m_{13}	m_{14}	m_{15}	m_{16}	m_{17}	m_{18}	m_{19}	m_{20}
Market Share	4%	4%	7%	5%	3%	2%	7%	5%	3%	2%

7.2. *DTPs and PRPs*

Three-phase asynchronous motors possess dozens of DTPs. In this paper, some of the most representative design parameters are selected, including slot number of each phase d_1, pole number d_2, plane height d_3, rotor slot number d_4, stator inner diameter d_5, stator outer diameter d_6, iron core length d_7 and gap length d_8. Among these parameters, d_1 is a common parameter, and its value is invariable, while others are scalable design parameters. The weights of these DTPs can be computed by pair-wise comparison as listed in Table 6.

Table 6. Weights of DTPs

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8
Weigh	0.14	0.40	0.14	0.06	0.06	0.14	0.03	0.03

Meanwhile, there are five PRPs: rated efficiency g_1, rated power factor g_2, starting current ratio g_3, starting torque ratio g_4, and maximum torque ratio g_5. These performance parameters are proportional to the customer's satisfaction except for g_3 which is inversely proportional to the customer's satisfaction. This is because a larger starting current ratio will result in over-heating, declination of lifespan and shock of the power grid, yet a smaller starting current ratio is a better choice for customers. There are 20 alternative product variants in the product family, the values of the DTPs and PRPs are listed in Table 7.

Table 7. The values of DTPs and PRPs of alternative variants

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	g_1	g_2	g_3	g_4	g_5
P_1	4	2	100	20	94	155	100	0.3	82	0.87	7	2.2	2.2
P_2	4	4	100	38	98	155	135	0.25	82.5	0.81	7	2.2	2.2
P_3	4	6	132	44	148	210	110	0.35	83	0.76	6.5	2.0	2.0
P_4	4	8	132	60	148	210	140	0.35	82	0.72	5.5	2.0	2.0
P_5	4	2	132	20	116	210	125	0.55	86.2	0.88	7	2.2	2.2
P_6	4	4	132	38	136	210	160	0.4	87	0.85	7	2.2	2.2
P_7	4	6	160	58	180	260	145	0.4	86	0.78	6.5	2.0	2.0
P_8	4	8	160	60	180	260	195	0.4	86	0.75	5.5	2.0	2.0
P_9	4	2	160	20	150	260	155	0.65	88.2	0.88	7	2.0	2.2
P_{10}	4	4	160	38	170	260	195	0.5	88.5	0.85	7	2.2	2.2
P_{11}	4	6	180	58	205	290	200	0.45	89.5	0.81	6.5	1.8	2.0
P_{12}	4	8	200	82	230	327	195	0.5	88	0.76	6	1.8	2.0
P_{13}	4	2	180	20	160	290	175	0.8	89	0.89	7	2.0	2.2
P_{14}	4	4	180	44	187	290	220	0.55	91.5	0.86	7	2.0	2.2
P_{15}	4	6	200	58	230	327	220	0.5	90.2	0.83	6.5	1.8	2.0
P_{16}	4	8	225	82	260	368	310	0.5	90	0.78	6	1.8	2.0
P_{17}	4	2	200	20	182	327	180	1	90	0.89	7	2.0	2.2
P_{18}	4	4	200	44	210	327	230	0.65	92.2	0.87	7	2.0	2.2
P_{19}	4	6	225	60	260	368	210	0.5	90.2	0.85	6.5	1.7	2.0
P_{20}	4	8	250	82	285	400	225	0.55	90.5	0.80	6	1.8	2.0

Each alternative variant in the product family will face some competitors in the same market segment. The choice probability of each product reflects the competitive result of this customer group. Take customer group m_1 as an example:

Table 8 lists the PRPs of all products in market segment m_1. In this group, company product P_1 participates in the competition with five other competitors (i.e. $P_{c1} \sim P_{c5}$).

It can be seen from Table 8 that P_{c4} has the best performance, and possess the highest PFCA among all of these products. Accordingly, according to Eq.(1), for customer performance parameter g_1 (rated efficiency) of product P_1, its customer satisfaction level u_{11} can be calculated as follows:

$$u_{11} = \frac{0.82 - 0.80}{0.825 - 0.80} = 0.8 \tag{13}$$

Also, according to Eq. (2), the customer satisfaction level u_{13} of g_3 (starting current ratio) of P_1 is calculated as follows:

$$u_{13} = \frac{7.3-7}{7.3-6.9} = 0.75 \tag{14}$$

Similarly, the customer satisfaction levels of other PRPs are calculated.

Assuming the weights of the PRPs are 0.25, 0.25, 0.2, 0.2 and 0.1, respectively. According to Eq. (3), the customer satisfaction level U_1 is 0.8. Table 9 lists of satisfaction levels of all the products in market segment m_1.

Table 8. PRPs of all products in the market segment m_1

	g_1	g_2	g_3	g_4	g_5
P_1	0.82	0.87	7	2.2	2.2
P_{c1}	0.82	0.86	7.3	2.2	2.2
P_{c2}	0.80	0.84	7.3	2.1	2.0
P_{c3}	0.81	0.85	7.2	2.1	2.1
P_{c4}	0.825	0.87	6.9	2.3	2.2
P_{c5}	0.81	0.86	7	2.1	2.2

Table 9. Customer satisfaction levels of all the products in market segment m_1

	g_1	g_2	g_3	g_4	g_5	U_i
P_1	0.80	1.00	0.75	0.50	1.00	0.80
P_{c1}	0.80	0.66	0.00	0.50	1.00	0.56
P_{c2}	0.00	0.00	0.00	0.00	0.00	0.00
P_{c3}	0.40	0.33	0.25	0.00	0.50	0.28
P_{c4}	1.00	1.00	1.00	1.00	1.00	1
P_{c5}	0.40	0.66	0.75	0..00	1.00	0.52

After obtaining overall customer satisfaction levels of each product in a market segment, customer choice probabilities of the products can be measured. Set $\lambda = 2.5$, then the choice probability of P_1 is calculated as follows:

$$C_1 = \frac{e^{2.5\times U_1}}{e^{2.5\times U_1} + \sum_{n=2}^{6} e^{2.5\times U_n}} = \frac{e^{2.5\times 0.8}}{e^{2.5\times 0.8} + (e^{2.5\times 0.56} + e^{2.5\times 0.00} + e^{2.5\times 0.28} + e^{2.5\times 1} + e^{2.5\times 0.52})} = 24.3\% \tag{15}$$

Similarly, all customer choice probabilities of all products in the market segment m_1 can be obtained as shown in Table 10. Therefore, the customer selection probabilities of 20 alternative product variants in the product family under competitive circumstances can be calculated as shown in Table 11 and 12.

Table 10. customer choice probabilities of all products in the market segment m_1

	P_1	P_{c1}	P_{c2}	P_{c3}	P_{c4}	P_{c5}
Choice probability	24.3%	13.6%	3.3%	6.7%	40.1%	12.0%

Table 11. Customer selection probabilities of 20 alternative product variants

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}
Choice probability	24.3%	5%	35%	30%	25%	5%	30%	5%	30%	25%

Table 12. Customer selection probabilities of 20 alternative product variants (Cont.)

	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	P_{20}
Choice probability	15%	20%	30%	25%	35%	15%	40%	35%	10%	10%

In this case, we assume that the profit of each product in the product family is approximately the same, and the profit ratios (o_i) of these 20 products are all 0.05.

7.3. *Results analysis*

To obtain PSS, GA is applied in this case. The length of chromosome is 20, and the genes in chromosomes are either 1 or 0, which indicates the product is selected or not. Moreover, the size of population (n_{pop}) is set as 200, the maximum iteration loop ($h_{\max}$) is 100, and the maximum size of PSS (n_{pss}) is 50. Crossover probability n_c and mutation probability n_m are 0.8 and 0.01, respectively.

According to Fig. 2, after 90 iteration loops, there are 44 solutions in PSS as shown in Fig. 2. From the Fig. 3, we can see that more solutions have higher PFCA. However, their commonality (CI) is low. According to Eq. (12), after the comprehensive optimal selection, the optimal solutions can be selected in Table 13.

Table 13. Solutions with higher priority

Priority	PFCA	CI	The count of products in solution	Product portfolio in solution
1	0.22574	0.21155	17	P_1 , P_2 , P_3 , P_4 , P_5 , P_6 , P_7 , P_8 , P_9 , P_{10} , P_{11} , P_{12} , P_{13} , P_{14} , P_{15} , P_{17} , P_{18}
2	0.22224	0.22101	16	P_1 , P_3 , P_4 , P_5 , P_6 , P_7 , P_8 , P_9 , P_{10} , P_{11} , P_{12} , P_{13} , P_{14} , P_{15} , P_{17} , P_{18}
3	0.23174	0.19252	19	P_1 , P_2 , P_3 , P_4 , P_5 , P_6 , P_7 , P_8 , P_9 , P_{10} , P_{11} , P_{12} , P_{13} , P_{14} , P_{15} , P_{16} , P_{17} , P_{18} , P_{19}
4	0.22874	0.20067	18	P_1 , P_2 , P_3 , P_4 , P_5 , P_6 , P_7 , P_8 , P_9 , P_{10} , P_{11} , P_{12} , P_{13} , P_{14} , P_{15} , P_{17} , P_{18} , P_{19}
5	0.22424	0.21398	16	P_1 , P_2 , P_3 , P_4 , P_5 , P_6 , P_7 , P_9 , P_{10} , P_{11} , P_{12} , P_{13} , P_{14} , P_{15} , P_{17} , P_{18}

The optimal solution with priority 1 has 17 variants and its portfolio is [P_1 , P_2 , P_3 , P_4 , P_5 , P_6 , P_7 , P_8 , P_9 , P_{10} , P_{11} , P_{12} , P_{13} , P_{14} , P_{15} , P_{17} , P_{18}]. From the Pareto solutions in this case, we can see that CI has a downward trend with the enhancement of PFCA because of the increasing number of product variants. Meanwhile, different combinations within the same number of product variants have different effects on the solutions. Taking the second and the fifth priority combinations as an example, although they both have 16 product variants, the second solution chooses P_8 instead of P_2 , and it possesses a larger CI. The fifth one chooses P_2 instead of P_8, so the solution has a better PFCA.

The comprehensive optimal selection method is an equal weight method. Thus, the priority given in Table 13 is under the premise that PFCA and CI have the same weight. Companies may put much more emphasis on PFCA if a market is in a depressed or oversupply situation, and more products with higher PFCA will allow companies to obtain more orders. However, when the market becomes more prosperous or in a shortage, companies may put more emphasis on CI. Under this circumstance, companies pursue the maximization of internal efficiency. As a result, there will be fewer product variants in the product family by increasing the weight of CI in the optimal selection.

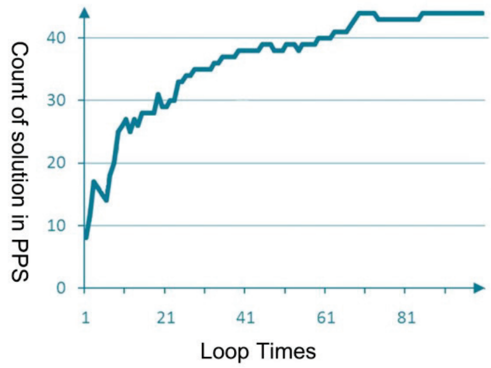

Fig. 2 Iteration process

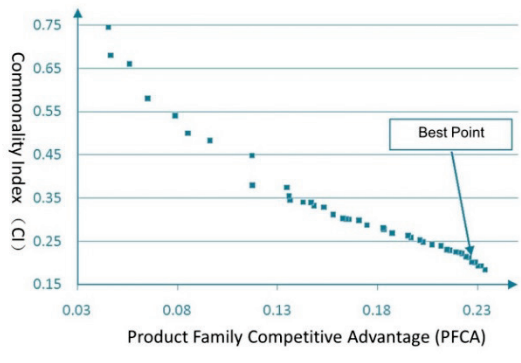

Fig. 3 Pareto optimal solution set

7.4. *Algorithm feature analysis*

(1) In the case study, the possible solution space reaches 1048575 (i.e. $2^{20}-1$), it is difficult to apply exhaustive search techniques or orthogonal arrays to enumerate different combinations of products. The optimization result indicates that the proposed algorithm works efficiently in searching for near-optimal product portfolio solutions.

(2) When applying GA to multi-objective optimization problem, it is important to build a fitness function to find all non-dominated solutions. If the fitness function with constant weight values is constructed, the search direction is fixed, and it is not easy to obtain a variety of non-dominated solutions. In this paper, the fitness function with random weights is developed. And the optimization result reveals that the Pareto frontier for a product family is captured effectively.

8. Conclusions

To stand out in the fierce competitions among current manufacturing industries, companies need to provide a competitive product portfolio for the right market. Positioning method of scale-based product family mainly is essentially a combination optimization of various parameters to tradeoff PFCA and commonality. The metrics of PFCA considering customer choice probability, sales, and profits under a holistic framework is proposed. Meanwhile, commonality is measured according to product DTPs in a product family. Based on the metrics of PFCA and commonality, a multi-objective optimization model balancing external competitive advantages and internal commonality is proposed. For capturing the Pareto solutions for the multi-objective optimization model, a GA cooperating fitness function with random weights is developed. According to the case analysis, the optimization model and algorithm yield quality solutions in a reasonable amount of time. The multinomial logit model is applied in this paper, in which the value of scaling parameter λ depends on market analysis. Future research may incorporate customer discrete-choice models in the decision making process.

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