α-Generalized Semantic Resolution Method in Linguistic Truth-valued Propositional Logic $\mathscr{L}_{V(n\times 2)}$P(X)

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Received 14 October 2012

Accepted 4 July 2013

Abstract

This paper is focused on α-generalized semantic resolution automated reasoning method in linguistic truth-valued lattice-valued propositional logic. Concretely, α-generalized semantic resolution for lattice-valued propositional logic $(\mathscr{L}_n \times \mathscr{L}_2)$P(X) is equivalently transformed into that for lattice-valued propositional logic $\mathscr{L}_n$P(X)$(i \in \{1,2,\cdots,n\})$. A similar conclusion is obtained between the α-generalized semantic resolution for linguistic truth-valued lattice-valued propositional logic $\mathscr{L}_{V(n\times 2)}$P(X) and that for lattice-valued propositional logic $\mathscr{L}_{V(n)}$P(X)$(i \in \{1,2,\cdots,n\})$. Secondly, the generalized semantic resolution for lattice-valued propositional logic $\mathscr{L}_n$P(X) based on a chain-type truth-valued field is investigated and its soundness and weak completeness are given. The Presented work provides some foundations for resolution-based automated reasoning in linguistic truth-valued lattice-valued logic based on lattice implication algebra.

Keywords: Automated reasoning; Resolution principle; Semantic resolution method; Lattice-valued logic; Linguistic truth-valued lattice implication algebras

1. Introduction

Theorem mechanical proving is an important research direction of the study automated reasoning, its aim is to achieve the mechanization of theorem proving, resolution-based automated reasoning is one way of automatic reasoning.

Since its introduction in 1965, automated reasoning based on Robinson's resolution principle [1] has been extensively studied in the context of finding natural an efficient proof systems to support a wide spectrum of computational tasks. They are widely applied to areas such as artificial intelligence, logic programming, problem solving and question answering systems, database theory, and so on.

For improving the efficiency of resolution principle, in 1967, Slagle [2] presented the semantic resolution method, as one of the most important refinements of resolution principle, its main idea is restraining the type and the order of clauses participated in the process of resolution reasoning. Semantic resolution strategy can effectively reduce the redundant clauses and increase the efficiency of reasoning based on resolution principle. Like Robinson's resolution principle, the semantic resolution method is also sound and complete in predicate logic.

With the non-classical logic in the application of information science, computer science and artificial intelligence increasingly important, automated reasoning based on various kinds of non-classical logic has become an active area of research [3,4,5,6,7,8,9].

Lattice-valued logic is an important case of multi-valued logic, can describe phenomenon in real world. In 1993, Xu [14] introduced a new logical algebra structure-lattice implication algebra with incomparable elements by combining lattice and implication algebra, consequently, lattice-valued propositional logic system [10,11,15]and lattice-valued first-order logic system [12] based on lattice implication algebra were proposed, the research work related on lattice implication algebras and lattice-valued logic system based on lattice implication algebra were collected in Ref.[10].

Based on the work mentioned above, α-resolution principle based on lattice-valued propositional logic and α-resolution principle based on lattice-valued first-order logic were given in Ref.[16, 17], which can be used to prove whether a logic formula is false or not in logic systems based on lattice implication algebras. Consequently, Xu et al. [18] presented α-generalized resolution principle for general generalized clausal set in lattice-valued logic system, and moreover its soundness and weak completeness were given. The difference between α-resolution and α-generalized resolution is that the reasoning rule is based on generalized clause and the reasoning rule of generalized resolution principle is based on general generalized clause(some ordinary logical formulae). For this characteristic of α-generalized resolution principle, it make the resolution procedure more natural and intuitive.

In real uncertainty reasoning problem, most information, which are always propositions with truth-values, can be very qualitative in nature, i.e. described in natural language, usually, in a quantitative setting the information is expressed by means of numerical values, However, when we work in a qualitative setting, that is, with vague or imprecise knowledge, this cannot be estimated with an exact numerical value, Then, it may be more realistic to use linguistic truth values instead of numerical values [20,21,22,23]. In 2006, based on symbolic approaches, Xu [24] proposed linguistic truth-valued lattice implication algebra, this algebraic model can be applied for linguistic truth-valued automated reasoning and uncertainty reasoning. In 2007, Xu [25] proved the weak completeness of resolution in a linguistic truth-valued propositional logic which truth-value field is linguistic truth-valued lattice implication algebra. Except for technical means of judging resolution pair, the executive way of Robinson's resolution principle coincides with that of $\alpha-$resolution in lattice-valued logic LP(X), so, applying semantic resolution strategy to the process of $\alpha-$resolution like that in classical logic can increase the efficiency of $\alpha-$resolution in lattice-valued logic. In the present paper, based on the precious works, we establish the generalized semantic resolution method on linguistic truth-valued lattice-valued propositional logic and it can treat the uncertainty information with linguistic valued in real world.

The paper is structured as follows: Section 2 as a preliminary gives an overview of some concepts and results about linguistic truth-valued lattice implication algebra and lattice-valued propositional logic system. Section 3 discusses the equivalence of generalized semantic resolution based on linguistic truth-valued lattice-valued propositional logic $\mathscr{L}_{V(n\times 2)}$P(X) and generalized semantic resolution on lattice-valued propositional logic $\mathscr{L}_{V(n)}$P(X). In Section 4 the generalized semantic resolution on lattice-valued propositional logic $\mathscr{L}_n$P(X) based on a chain-type truth-valued field is investigated and its soundness and weak completeness are given. Section 5 contrives a corresponding resolution reasoning algorithm as a foundation for the implementation purpose. The paper is concluded in Section 6.

2. Preliminaries

In this section, we will give some elementary concepts, the details can be seen in the Ref.[13].

Definition 1. [14] Let $(L,\vee,\wedge,O,I)$ be a bounded lattice with an order-reversing involution $'$, the greatest element I and the smallest element O, and

$$\rightarrow: L\times L \longrightarrow L$$

be a mapping. $\mathscr{L}=(L,\vee,\wedge,',\rightarrow,O,I)$ is called a lattice implication algebra(LIA) if the following conditions hold for any $x,y,z\in L$:

(I_1) $x\rightarrow(y\rightarrow z)=y\rightarrow(x\rightarrow z)$;
(I_2) $x\rightarrow x=I$;
(I_3) $x\rightarrow y=y'\rightarrow x'$;
(I_4) $x\rightarrow y=y\rightarrow x=I$ implies $x=y$;
(I_5) $(x\rightarrow y)\rightarrow y=(y\rightarrow x)\rightarrow x$;
(l_1) $(x\vee y)\rightarrow z=(x\rightarrow z)\wedge(y\rightarrow z)$;
(l_2) $(x\wedge y)\rightarrow z=(x\rightarrow z)\vee(y\rightarrow z)$.

In this paper, we denote $\mathscr{L}$ as a lattice implication algebra $(L,\vee,\wedge,',\rightarrow,O,I)$.

Example 1. [14,13] (Boolean algebra). Let $(L,\vee,\wedge,')$ be a Boolean lattice. For any $x,y\in L$, define $x\rightarrow y=x'\vee y$, then $(L,\vee,\wedge,',\rightarrow)$ is an LIA.

Example 2. [14,13] (Łukasiewicz implication algebra on a finite chain L_n) Let L_n be a finite chain, $L_n=\{a_i|1\leqslant i\leqslant n\}$ and $a_1<a_2<\ldots<a_n$, define for any $a_j,a_k\in L_n$,

$a_j\vee a_k=a_{max\{j,k\}}$,
$a_j\wedge a_k=a_{min\{j,k\}}$,
$(a_j)'=a_{n-j+1}$,
$a_j\rightarrow a_k=a_{min\{n-j+k,n\}}$.

Then $\mathscr{L}_n=(L_n,\vee,\wedge,',\rightarrow,a_1,a_n)$ is an LIA.

Example 3. [14,13] Let $L=\{O,\alpha,\beta,\gamma,\delta,I\}$. The ordering relation on L is given by Fig. 1. For any $x,y\in L$, define $x\vee y=\mathrm{Sup}(x,y), x\wedge y=\mathrm{Inf}(x,y)$, and operations on L are defined in Tab. 1. Then $(L,\vee,\wedge,',\rightarrow,O,I)$ is an LIA.

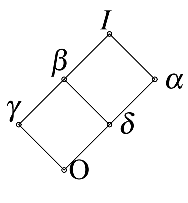

Fig. 1. Hasse Diagram of L.

Table 1. The Operations on L

x	x'	$\rightarrow$	O	α	β	γ	δ	I
O	I	O	I	I	I	I	I	I
α	γ	α	γ	I	β	γ	β	I
β	δ	β	δ	α	I	β	α	I
γ	α	γ	α	α	I	I	α	I
δ	β	δ	β	I	I	β	I	I
I	O	I	O	α	β	γ	δ	I

Example 4. [20,24] Let $L_n=\{d_1,d_2,\cdots,d_n\}$, $d_1<d_2<\cdots<d_n$, $L_2=\{b_1,b_2\}$, $b_1<b_2$, $(L_n,\vee_{L_n},\wedge_{L_n},'_{L_n},\rightarrow_{L_n},d_1,d_n)$ and $(L_2,\vee_{L_2},\wedge_{L_2},'_{L_2},\rightarrow_{L_2},b_1,b_2)$ be two Lukaisewicz implication algebras. For any $(d_i,b_j),(d_k,b_m)\in L_n\times L_2$, define

$(d_i,b_j)\vee(d_k,b_m)=(d_i\vee_{L_n}b_j,d_k\vee_{L_2}b_m)$;
$(d_i,b_j)\wedge(d_k,b_m)=(d_i\wedge_{L_n}b_j,d_k\wedge_{L_2}b_m)$;
$(d_i,b_j)'=(d_i',b_j')$;
$(d_i,b_j)\rightarrow(d_k,b_m)=(d_i\rightarrow_{L_n}b_j,d_k\rightarrow_{L_2}b_m)$,

Then $L_n\times L_2,\vee,\wedge,',\rightarrow,(d_1,b_1),(d_n,b_2)$ is a lattice implication algebra, denoted as $\mathscr{L}_n\times\mathscr{L}_2$.

Example 5. [20,24] Denote MT=$\{f,t\}$, which is called as the set of meta truth values. The lattice implication algebra of defined on the set of meta truth values is called a meta linguistic truth-valued lattice implication algebra, where $f<t$, the operation $'$ is defined as $f'=t$ and $t'=f$, the operation $\rightarrow$ is defined as: $\rightarrow: MT\times MT\longrightarrow MT$, $x\rightarrow y=x'\vee y$.

Example 6. [20,24] Denote $AD_n=\{a_i|1\leqslant i\leqslant n\}$, $a_1<a_2<\cdots<a_n$, and $a_i(i=1,2,\cdots,n)$ be modifier of meta language, the operations on AD_n are defined as, for any $a_j,a_k\in AD_n$,

$a_j\vee a_k=a_{max\{j,k\}}$;
$a_j\wedge a_k=a_{min\{j,k\}}$
$(a_j)'=a_{n-j+1}$;
$a_j\rightarrow a_k=a_{min\{n-j+k,n\}}$.

Then $(AD_n,\vee,\wedge,',\rightarrow,a_1,a_n)$ is a lattice implication algebra, and it is called modifier lattice implication algebra.

Definition 2. [20,24] Let $AD_n=\{a_1,a_2,\cdots,a_n\}$ be a set with n modifiers and $a_1<a_2<\cdots<a_n$, $MT=\{f,t\}$ be a set of meta truth values and $f<t$, denote $L_{V(n\times 2)}=AD_n\times MT$, define a mapping $\sigma: L_{V(n\times 2)}\longrightarrow L_n\times L_2$,

$$\sigma((a_i,mt))=\begin{cases}(d_i',b_1) & \text{when } mt=f,\\ (d_i,b_2) & \text{when } mt=t.\end{cases} \tag{1}$$

Then σ is a bijection mapping, denote its inverse mapping as σ^{-1}, for any $x, y \in L_{V(n\times 2)}$, define

$x \vee y = \sigma^{-1}(\sigma(x) \vee \sigma(y))$,
$x \wedge y = \sigma^{-1}(\sigma(x) \wedge \sigma(y))$,
$x' = \sigma^{-1}((\sigma(x))')$,
$x \to y = \sigma^{-1}(\sigma(x) \to \sigma(y))$,

Then $\mathscr{L}_{V(n\times 2)} = (L_{V(n\times 2)}, \vee, \wedge, ', \to, (a_n, f), (a_n, t))$ is called a linguistic truth-valued lattice implication algebra generated by AD_n and MT, its elements are called linguistic truth-values, and σ is an isomorphic mapping from $\mathscr{L}_{V(n\times 2)}$ to $\mathscr{L}_{n\times 2}$.

Proposition 1. [14,13] *Let $\mathscr{L}$ be a lattice implication algebra. Then for any $x, y, z \in L$, the following conclusions hold:*

(1) $I \to x = x$ and $x \to O = x'$;
(2) $O \to x = I$ and $x \to I = I$;
(3) $x \to y \geqslant x' \vee y$;
(4) $x \leqslant y$ if and only if $x \to y = I$;
(5) If $x \leqslant y$, then $x \to z \geqslant y \to z$ and $z \to x \leqslant z \to y$.

Proposition 2. [13] *All LIAs form a proper class.*

Proposition 3. [10] *For any finite cardinal number $k > 2$, there exists an LIA with the cardinal number k.*

Definition 3. [10,11] Let X be a set of propositional variables, $T = L \cup \{', \to\}$ be a type with ar($'$)=1, ar($\to$)=2 and ar(α)=0 for any $\alpha \in L$. The propositional algebra of the lattice-valued propositional calculus on the set X of propositional variables is the free T algebra on X is denoted by LP(X).

Theorem 4. [10,11] *$LP(X)$ is the minimal set Y which satisfies:*

(1) $X \cup L \subseteq Y$.
(2) if $p, q \in Y$, then $p', p \to q \in Y$.

Definition 4. [10,11] A valuation of LP(X) is a propositional algebra homomorphism $v : LP(X) \to L$.

Specially, when the field with valuation of LP(X) is an $\mathscr{L}_{V(n\times 2)}$, this specific LP(X), i.e. $\mathscr{L}_{V(n\times 2)}$P(X), is a linguistic truth-valued lattice-valued proportional logic system. Similarly, the truth-valued domain of $\mathscr{L}_n$P(X) is a Lukasiewicz implication algebra $\mathscr{L}_n$.

Definition 5. [16] A lattice-valued propositional logical formula f is called an extremely simple form, in short, ESF, if a lattice-valued propositional logical formula f^* obtained by deleting any constant or literal or implication item appearing in f is not equivalent to f.

Definition 6. [16] A lattice-valued propositional logical formula f is called an indecomposable extremely simple form, in short, IESF, if:

(1) f is an ESF containing connective $\to$ and $'$.

(2) for any $g \in LP(X)$, if $g \in \overline{f}$ in $\overline{LP(X)}$, then g is an ESF containing connective $\to$ and $'$ at most, where

$\overline{LP(X)} = (LP(X)/=, \vee, \wedge, ', \to)$ is a lattice implication algebra.

$LP(X)/_{=} = \{\overline{p} | p \in LP(X)\}, \overline{p} = \{q \in LP(X) | q = p\}$.

Definition 7. [16] All the constants, literals and IESFs are called generalized literals. Here, the definition of literal is the same as that in classical logic.

Definition 8. [18] Let $g_1, g_2, \cdots, g_n$ be generalized literals in LP(X). A logical formula Φ is called a general generalized clause if these generalized literals are connected by $\wedge, \vee, \to, '$ and $\leftrightarrow$, denoted by $\Phi(g_1, g_2, \cdots, g_n)$.

Definition 9. [18] A general generalized clause G in LP(X) is called a constant claue if only constants exist in G. Particularly, if for any valuation γ, such that $\gamma(G) = \alpha$, then G is called an α-constant clause. If for any valuation γ, such that $\gamma(G) \leqslant \alpha$, then G is called an α-false constant clause, denoted by $\alpha - \Box$.

Definition 10. [18] (α-Generalized Resolution Principle) Let Φ and Ψ be general generalized clauses in LP(X), g and h generalized literals in Φ and Ψ, respectively. $\alpha \in L$ and $\alpha < I$. If $g \wedge h \leqslant \alpha$, then

$$R_{\alpha-g}(\Phi, \Psi) = \Phi(g = \alpha) \vee \Psi(h = \alpha)$$

is called an α-generalized resolvent of Φ and Ψ, where g and h are called generalized resolution literals.

Definition 11. [18] Let G be generalized clause and g be a generalized literal occurring in G, $A(G)$ and $A(g)$ be the atom sets of G and g respectively, we say G and g are independent if $A(G) \cap A(g) = \varnothing$.

Definition 12. [18] Let g be generalized literal in LP(X), we say g is normal if there exists a valuation γ such that $\gamma(g) = I$.

Definition 13. [18] A general generalized literal clause S is normal if all the IESFs in S are normal.

3. The Equivalence Among Generalized Semantic resolution methods

In this section, we discuss the relation between generalized semantic resolution for linguistic truth-valued lattice-valued propositional logic $\mathscr{L}_{V(n\times 2)}$P(X) and generalized semantic resolution for lattice-valued propositional logic system $\mathscr{L}_{V(n)}$P(X).

Definition 14. Let $(N, E_1, E_2, \cdots, E_q)(q \geqslant 1)$ be general generalized clauses consequence in lattice-valued propositional logic LP(X), v be a valuation in LP(X), $\mathscr{O}$ an order of generalized literals occurring in these clauses. Finite general generalized clauses consequence $(N, E_1, E_2, \cdots, E_q)$ is called α-generalized semantic clash on $\mathscr{O}$ and v(α-generalized $\mathscr{O}v$ clash for short), if the following conditions hold:

1) $v(E_i) \leqslant \alpha$, $1 \leqslant i \leqslant q$;

2) Let $R_1 = N$, for any $i = 1, 2, \cdots, q$, there exists α-generalized resolution formula R_{i+1} of R_i and E_i, the resolution generalized literal in E_i is the leftmost generalized literal in E_i according to $\mathscr{G}$;

3) $v(R_{i+1}) \leqslant \alpha$;

R_{q+1} is called the resolvent of this clash, $E_1, E_2, \cdots, E_q$ are called electrons and N is called the core of this clash..

Definition 15. Suppose S is a set of general generalized clauses in LP(X), $\alpha \in L$, an α-generalized resolution deduction $\omega = \{D_1, D_2, \cdots, D_m\}$ is called an α-generalized semantic resolution deduction from S to the general generalized clause D_m, if

(1) $D_i \in S(i = 1, 2, \cdots, m)$, or

(2) There exists $r_1, r_2 < i$, such that D_i is α-generalized semantic resolution formula of D_{r_1} and D_{r_2}.

If there exists an α-generalized semantic resolution deduction ω from S to an α-constant clause, then ω is called an α-generalized semantic refutation of S.

Theorem 5. [28] *Let g_1, g_2 be generalized literals in lattice propositional logic $(\mathscr{L}_n \times \mathscr{L}_2)P(X)$, v_1 be a valuation in $(\mathscr{L}_n \times \mathscr{L}_2)P(X)$, $\alpha = (d_i, b_2) \in \mathscr{L}_n \times \mathscr{L}_2$, then $g_1 \wedge g_2 \leqslant \alpha$ if and only if g_1, g_2 are interpreted in $\mathscr{L}_nP(X)$ and $g_1 \wedge g_2 \leqslant d_i$.*

Theorem 6. [28] *Let g_1, g_2 be generalized literals in lattice propositional logic $(\mathscr{L}_n \times \mathscr{L}_2)P(X)$, v_1 be a valuation in $(\mathscr{L}_n \times \mathscr{L}_2)P(X)$, $\alpha = (d_i, b_1) \in \mathscr{L}_n \times \mathscr{L}_2$, and $d_i < d_n$, if for any $j = 1, 2, \cdots, m$, $g_j^{\triangle} \leqslant g_j$ when $g_j^{\triangle}, g_j$ are interpreted in $\mathscr{L}_nP(X)$, then $g_1 \wedge g_2 \leqslant \alpha$ if and only if g_1, g_2 are interpreted in $\mathscr{L}_nP(X)$ and $g_1 \wedge g_2 \leqslant d_i$.*

Theorem 7. [28] *Let g be a generalized literal in lattice propositional logic $(\mathscr{L}_n \times \mathscr{L}_2)P(X)$, v_1 be a valuation in $(\mathscr{L}_n \times \mathscr{L}_2)P(X)$, $\alpha = (d_i, b_2) \in \mathscr{L}_n \times \mathscr{L}_2$, then the following conclusions hold:*

(1) $v_1(g) \nleqslant \alpha$ if and only if $v_2(g) > d_i$;

(2) $v_1(g) \leqslant \alpha$ if and only if $v_2(g) \leqslant d_i$, where $v_2 = v_1|\mathscr{L}_nP(X)$.

Theorem 8. [28] *Let g be a generalized literal in lattice propositional logic $(\mathscr{L}_n \times \mathscr{L}_2)P(X)$, v_1 be a valuation in $(\mathscr{L}_n \times \mathscr{L}_2)P(X)$, $\alpha = (d_i, b_1) \in \mathscr{L}_n \times \mathscr{L}_2$, then the following conclusions hold:*

(1) $v_1(g) \geqslant (d_{i+1}, b_1)$ if and only if $v_2(g) > d_i$;

(2) $v_1(g) \leqslant \alpha$ if and only if $v_2(g) \leqslant d_i$, where $v_2 = v_1|\mathscr{L}_nP(X)$.

Theorem 9. *Let $S = C_1 \wedge C_2 \wedge \cdots \wedge C_n$, where $C_1, C_2, \cdots, C_n$ are general generalized clauses in lattice-valued propositional logic $(\mathscr{L}_n \times \mathscr{L}_2)P(X)$, v_1 be a valuation in $(\mathscr{L}_n \times \mathscr{L}_2)P(X)$, $\alpha = (d_i, b_2) \in \mathscr{L}_n \times \mathscr{L}_2$, then the following conclusions are equivalent:*

(1) There exists an α-generalized semantic resolution deduction from S to α-false clause about valuation v_1;

(2) There exists an a_i-generalized semantic resolution deduction from S to d_i-false clause about valuation v_2, where $v_2 = v_1|\mathscr{L}_nP(X)$.

Proof.

(1)$\Rightarrow$(2) Let D be an α-generalized semantic resolution deduction from S to α-false clause about valuation v_1, since $v_2 = v_1|\mathscr{L}_n$P(X), from theorem 7, we have:

(1) If $v_1(g) \leqslant \alpha$, then $v_2(g) \leqslant d_i$;

(2) If $v_1(g) \nleqslant \alpha$, then $v_2(g) > d_i$.

Furthermore, we have any α-resolution pair in D is also d_i-resolution pairs in D^*, by theorem 6, and D^* is an d_i-generalized semantic resolution deductions from S to d_i-false clause about valuation v_2.

(2)⇒(1) we can get the result similarly. □

Theorem 10. *Let $S = C_1 \wedge C_2 \wedge \cdots \wedge C_n$, where $C_1, C_2, \cdots, C_n$ are general generalized clauses in linguistic truth-valued lattice-valued propositional logic $\mathscr{L}_{V(n\times 2)}$P(X), v_1 be a valuation in $\mathscr{L}_{V(n\times 2)}$P(X), $\alpha = (a_i, b_2) \in \mathscr{L}_{V(n\times 2)}$, then the following conclusions are equivalent:*

(1) There exists an α-generalized semantic resolution deduction from S to α-false clause on valuation v_1.

(2) There exists an a_i-generalized semantic resolution deduction from S^* to a_i-false clause on valuation v_2, where $v_2 = v_1|\mathscr{L}_{V(n)}$P(X).

Proof. According to Theorem 9, we can obtain the result easily. □

According to Theorem 9 and Theorem 10, we can find the fact that: (d_i, b_2)-generalized semantic resolution for lattice-valued propositional logic $(\mathscr{L}_n \times \mathscr{L}_2)$P(X) based on lattice implication algebra can be equivalently transformed into d_i-generalized semantic resolution for lattice-valued propositional logic $\mathscr{L}_n$P(X). Similarly, (a_i, t)-generalized semantic resolution for linguistic truth-valued lattice-valued propositional logic $\mathscr{L}_{V(n\times 2)}$P(X) based on lattice implication algebra is also equivalent to a_i-generalized semantic resolution for lattice-valued propositional logic $\mathscr{L}_{V(n)}$P(X).

Theorem 11. *Let $S = C_1 \wedge C_2 \wedge \cdots \wedge C_n$, where $C_1, C_2, \cdots, C_n$ are general generalized clauses in lattice-valued propositional logic $(\mathscr{L}_n \times \mathscr{L}_2)$P(X), v_1 be a valuation in $(\mathscr{L}_n \times \mathscr{L}_2)$P(X), $\alpha = (d_i, b_1) \in \mathscr{L}_n \times \mathscr{L}_2$ and $d_1 < d_n$, if the following conditions hold:*

(1) For any generalized literal g in S, $g^{\triangle} \leqslant g$ if $g^{\triangle}$ and g are interpreted in $\mathscr{L}_{V(n)}$;

(2) For any generalized literal g in S, $v_1(g) \geqslant (d_{i+1}, b_1)$ or $v_1(g) \leqslant \alpha$, then the following conclusions are equivalent:

(i) There exists an α-generalized semantic resolution deduction from S to α-false clause about valuation v_1;

(ii) There exists an d_i-generalized semantic resolution deduction from S to d_i-false clause about valuation v_2, where $v_2 = v_1|\mathscr{L}_n$P(X).

Proof. We can prove it similarly to theorem 9. □

Theorem 12. *Let $S = C_1 \wedge C_2 \wedge \cdots \wedge C_n$, where $C_1, C_2, \cdots, C_n$ are general generalized clauses in linguistic truth-valued lattice-valued propositional logic $\mathscr{L}_{V(n\times 2)}$P(X), v_1 be a valuation in $\mathscr{L}_{V(n\times 2)}$P(X), $\alpha = (a_i, b_1) \in \mathscr{L}_n \times L_2$ and $a_i > a_1$, if the following conditions hold:*

(1) For any generalized literal g in S, $g^{\triangle} \leqslant g$ if $g^{\triangle}$ and g are interpreted in $\mathscr{L}_n$;

(2) For any generalized literal g in S, $v_1(g) \geqslant (a_{i-1}, f)$ or $v_1(g) \leqslant \alpha$, then the following conclusions are equivalent:

(i) There exists an α-generalized semantic resolution deduction from S to α-false clause on valuation v_1;

(ii) There exists an a_{n-i+1}-generalized semantic resolution deduction from S to a_{n-i+1}-false clause on valuation v_2, where $v_2 = v_1|\mathscr{L}_{V(n)}$P(X), and for any propositional variable p(or constant symbol e):

1) If $v_1(p)$(or $v_1(e)$)=(a_j, t), then $v_2(p)$(or $v_2(e)$=a_j, where $a_j \in \mathscr{L}_{V(n)}$;

2) If $v_1(p)$(or $v_1(e)$)=(a_j, f), then $v_2(p)$(or $v_2(e)$=a_{n-j+1}, where $a_j \in \mathscr{L}_{V(n)}$.

Proof. According to Theorem 11, we can obtain the result easily. □

According to Theorem 11 and Theorem 12, (d_i, b_1)-generalized semantic resolution for lattice-valued propositional logic $(\mathscr{L}_n \times \mathscr{L}_2)$P(X) based on lattice implication algebra $\mathscr{L}_n \times \mathscr{L}_2$ can be equivalently transformed into d_i-generalized semantic resolution for lattice-valued propositional logic $\mathscr{L}_n$P(X) under some conditions. Similarly, (a_i, f)-generalized semantic resolution for linguistic truth-valued lattice-valued propositional logic $\mathscr{L}_{V(n\times 2)}$P(X) based on lattice implication algebra is also equivalent to a_{n-i+1}-generalized semantic resolution for lattice-valued propositional logic $\mathscr{L}_{V(n)}$P(X) under some conditions.

Computing out the α-resolution fields of all generalized literals in a general generalized clause set is the premise for applying α-generalized semantic method on the general generalized clause set.

The α-resolvability between two generalized literals not only associates with generalized literals themselves, but also with the valuation field of the logic system. In general, the more complex the structure of generalized literal and valuation field are, the more difficult to calculate its α-resolution field. Even the same generalized literal, its α-resolution field is possible different when it is discussed in different logic systems. Because of the structure of chain-type lattice implication algebra $\mathscr{L}_n$ is more simper than that of the product lattice implication algebra $\mathscr{L}_n \times \mathscr{L}_2$, so, even the same generalized literal, α-resolution field of the generalized literal in lattice-valued propositional logic system $(\mathscr{L}_n \times \mathscr{L}_2)$P(X) is more complex than that in lattice-valued propositional logic system $\mathscr{L}_n$P(X). In this section, the main issue is the equivalence between generalized semantic resolution for a complex lattice-valued propositional logic system and generalized semantic resolution for a relative simple lattice-valued propositional logic system. i. e., the complex problem can be equivalently transformed into simple problem and the difficult problem can be equivalently transformed into easier problem.

4. Generalized semantic resolution method in $\mathscr{L}_n$P(X)

In this section, we discuss the generalized semantic resolution method for lattice-valued propositional logic system $\mathscr{L}_n$P(X) and give the soundness theorem and weak completeness theorem of the resolution deduction.

Theorem 13. *(Soundness of α-generalized $\mathscr{O}v$ deduction) Let S be the set of general generalized clauses in lattice-valued propositional logic $\mathscr{L}_n$P(X), v be a valuation of $\mathscr{L}_n$P(X), $\mathscr{O}$ be an order of generalized literals occurring in these clauses, if there exists an α-generalized $\mathscr{O}v$ deduction from S to α-clause, then $S \leqslant \alpha$.*

Proof. Followed by the soundness theorem of α-generalized resolution principle [18] in LP(X). □

Theorem 14. *(Completeness of α-generalized $\mathscr{O}v$ deduction) Let S be the set of general generalized clauses in lattice-valued propositional logic $\mathscr{L}_n$P(X), v be an valuation in $\mathscr{L}_n$P(X), $\mathscr{O}$ be an order of non-constant generalized literals occurring in these clauses, $\alpha \in L$, if $S \leqslant \alpha$ and the following conditions hold:*

(1) If S^* do not contains generalized literal g, then g and S^* are independent each other, where S^* is the set of general generalized clause and $S^* \subseteq S$;

(2) The truth-value of the rightmost generalized literal g regarding to $\mathscr{O}$ is α-false under valuation v and g is normal. If there exists a generalized literal $h \in S$ such that $g \wedge h \leqslant \alpha$, then g and h have the same order.

Then there exists an α-generalized $\mathscr{O}v$ deduction from S to α-false clause.

Proof. Let M be the set of non-constant generalized literals occurring in S, we prove it by induction on $|M|$.

1. If $|M| = 1$, then there exists a generalized literal p, such that $p = S \leqslant \alpha$, therefore, there exists an α-generalized $\mathscr{O}v$ clash $(p,p) = p(p = \alpha) \vee p(p = \alpha) = \alpha$. So, there exists an α-generalized $\mathscr{O}v$ deduction from S to α-false generalized clause. The conclusion holds.

2. IF $|M| = 2$, say $M = \{p,q\}$. If $S = p \vee q$, then generalized clause $S = p \vee q \leqslant \alpha$ by $S \leqslant \alpha$, i.e., generalized clause $p \vee q$ is α-false clause, thus, there exists an α-generalized $\mathscr{O}v$ clash $(p \vee q, p \vee q) = (p \vee q)((p \vee q) = \alpha) \vee (p \vee q)((p \vee q) = \alpha) = \alpha$. So, there exists an α-generalized $\mathscr{O}v$ deduction from S to α-false clause. If $S = p \wedge q \leqslant \alpha$, then there exists an α-generalized Gv clash $(p,q) = p(p = \alpha) \vee q(q = \alpha) = \alpha$ or $(p \vee q, p \vee q) = (p \wedge q)((p \wedge q) = \alpha) \vee (p \wedge q)((p \wedge q) = \alpha) = \alpha$. So, there exists an α-generalized $\mathscr{O}v$ deduction from S to α-false clause.

3. Suppose the result holds for $|M| = n (n \geqslant 3)$.

4. Now we need to prove the result for $|M| = n+1$.

4.1. If there exists a general generalized clause ϕ_u in S, such that ϕ_u contains one non-constant generalized literal only and $v(\phi_u) \leqslant \alpha$. Since g is normal, there exists a valuation v_0, such that $v_0(g) = I$.

4.1.1) If there exists no generalized literal $h \in S$, such that $g \wedge h \leqslant \alpha$, let

$$S_1 = \{\phi^* | \phi^* \in S, \phi^* = \phi(g = I)\}$$

Since S_1 is α-unsatisfiable, and S contains generalized literals g but S_1 do not contains it, therefore, the number of generalized literals in S_1 is less than n. By the hypothesis of induction, there exists an α-generalized $\mathscr{O}v$ deduction D_1 from S_1 to α-false clause, and v satisfies: for any generalized literal $x \in S$, if $x \in S_1$, then $v(x) = v_1(x)$, i.e., v is an expansion of v_1. For each α-generalized $\mathscr{O}v_1$ clash $(N_1, E_{11}, \cdots, E_{i2}, \cdots, E_{q1})$, a). If $E_{i1} = E_i(g = I)$, where $E_i \in S$, then we changing E_{i1} into its corresponding clause in S; b). If $N_1 = N(g = I)$, where $N \in S$, then we changing N_1 into its corresponding clause in S, in this way, D_1 can be expanded to an α-generalized $\mathscr{O}v$ deduction D_{11} from S to α-false clause.

4.1.2) If there exists a generalized literal $h \in S$, such that $g \wedge h \leqslant \alpha$, let

$$S_1 = \{\phi^* | \phi^* \in S, \phi^* = \phi(g = I, h = v_0(h))\}$$

Since S_1 is α-unsatisfiable, and S contains generalized literals g, h but S_1 do not contains them, therefore, the number of α-resolution pairs in S_1 is less than n, by the hypothesis of induction, there exists an α-generalized $\mathscr{O}v_1$ deduction D_1 from S_1 to α-false clause, and v satisfies: for any generalized literal $x \in S$, if $x \in S_1$, then $v(x) = v_1(x)$, i.e., v is an expansion of v_1. For each α-generalized $\mathscr{O}v_1$ clash $(N_1, E_{11}, \cdots, E_{i2}, \cdots, E_{q1})$, a). If $E_{i1} = E_i(g = I)$, where $E_i \in S$, then we changing E_{i1} into its corresponding clause in S; b). If $N_1 = N(g = I)$, where $N \in S$, then we changing N_1 into its corresponding clause in S, c). If $E_{i1} = E_i(h = v_0(h))$, then we add an α-generalized Gv clash $(\phi_u, E_i) = E_i(h = \alpha) \vee \phi_u(g = \alpha)$ on E_{i1}; d). If $N_1 = N(h = v_0(h))$, then we replace α-generalized $\mathscr{O}v_1$ clash $(N_1, E_{11}, \cdots, E_{i2}, \cdots, E_{q1})$ with $(N, \phi_u, E_1, \cdots, E_i, \cdots, E_q)$, e). If $E_{i1} = E_i(g = I, h = v_0(h))$, then we add an α-generalized $\mathscr{O}v$ clash $(\phi_u, E_i) = E_i(h = \alpha) \vee \phi_u(g = \alpha)$ on E_{i1} and change constant I occurring in of E_{i1} into g; f). If $N_1 = N(g = I, h = v_0(h))$, then we replace α-generalized $\mathscr{O}v_1$ clash $(N_1, E_{11}, \cdots, E_{i2}, \cdots, E_{q1})$ with $(N, \phi_u, E_1, \cdots, E_i, \cdots, E_q)$ and change constant I of N_1 into g, in this way D_1 can be expanded to an α-generalized $\mathscr{O}v$ deduction D_{11} from S to α-false clause.

4.2. If there exists no unit general generalized clause in S, such that the truth value of this generalized clause is less than or equal to α under valuation v, we take the rightmost generalized literal g regarding to the order $\mathscr{G}$, since g is a normal generalized literal, there exists a valuation v_0, such that $v_o(g) = I$.

4.2.1. If there exists no generalized literal $h \in S$, such that $g \wedge h \leqslant \alpha$, let

$$S_2 = \{\phi^* | \phi^* \in S, \phi^* = \phi(g = I)\}$$

Then S_2 is α-unsatisfiable, and S contain generalized literal g but S_2 do not, therefore, the number of α-resolution pairs in S_2 is less than n, by the hypothesis of induction, there exists an α-generalized $\mathscr{O}v_2$ deduction D_2 from S_2 to α-false clause, and v satisfies: for any generalized literal $x \in S$, if $x \in S_2$, then $v(x) = v_2(x)$, i.e., v is an expansion of v_2. We replace ϕ^* with ϕ, since g is the rightmost generalized literal according to $\mathscr{O}$. So, α-generalized $\mathscr{O}v_2$ clash in D_2 can be amended to α-generalized $\mathscr{O}v$ clash, in this way, we get an α-generalized resolution deduction D_{21} from S to α-false clause or a unit generalized clause ϕ_u that contains g only or a unit generalized clause ϕ_u that contains h only or a unit generalized clause ϕ_u that contains g and h.

If D_{21} is an α-generalized $\mathscr{O}v$ deduction from S to α-false clause, the conclusion holds; Otherwise, if D_{21} is an α-generalized $\mathscr{O}v$ deduction from S to a unit generalized clause ϕ_u that contains g only, by the structure and properties of general generalized clause, we have: each general generalized clauses in D_{21} is less than or equal to its counterpart clause in D_2, hence, D_{21} is also an α-generalized $\mathscr{O}v$ deduction from S to α-false clause.

4.2.2. If there exists a generalized literal $h \in S$, such that $g \wedge h \leqslant \alpha$, let

$$S_2 = \{\phi^* | \phi^* \in S, \phi^* = \phi(h = I, g = v_0(g))\}$$

Then S_2 is α-unsatisfiable, and S contain generalized literal g, h but S_2 do not, therefore, the number of α-resolution pairs in S_2 is less than n, by the hypothesis of induction, there exists an α-generalized $\mathscr{O}v_2$ deduction D_2 from S_2 to α-false clause, and v

satisfies: for any generalized literal $x \in S$, if $x \in S_2$, then $v(x) = v_2(x)$, i.e., v is an expansion of v_2.

We replace ϕ^* with ϕ, since g and h are the right-most generalized literals according to $\mathscr{O}$. So, α-generalized $\mathscr{O}v_2$ clash in D_2 can be amended to α-generalized $\mathscr{O}v$ clash, thus, we get an α-generalized resolution deduction D_{21} from S to α-false clause or a unit generalized clause ϕ_u that contains g only or a unit generalized clause ϕ_u that contains h only or a unit generalized clause ϕ_u that contains g and h. If D_{21} is an α-generalized $\mathscr{O}v$ deduction from S to α-false clause, the conclusion holds; Otherwise, if D_{21} is an α-generalized $\mathscr{O}v$ deduction from S to a unit generalized clause ϕ_u that contains h only or a unit generalized clause ϕ_u that contains g and h, by the structure and properties of general generalized clause, we have: each general generalized clauses in D_{21} is less than or equal to its counterpart clause in D_2, hence, D_{21} is also an α-generalized Gv deduction from S to α-false clause; Otherwise, if D_{21} is an α-generalized $\mathscr{O}v$ deduction from S to a unit generalized clause ϕ_u that contains g only, then we consider clause set $S \cup \{\phi_u\}$, this clause set is α-false and $\phi_u(g = \alpha) \leqslant \alpha$, so there exists a unit general generalized clause which truth value is less than or equal to α under valuation v, by the proof of case 4.1, we get an α-generalized $\mathscr{O}v$ deduction D_{22} from $S \cup \{g\}$ to α-clause. Connecting D_{21} and D_{22}, we get an α-generalized $\mathscr{O}v$ deduction D from S to α-false clause.

This completes the proof. □

Example 7. Let L_7 be chain-type lattice implication algebra with 7 elements, $L_7 = \{a_i | 1 \leqslant i \leqslant 7\}$, and $O = a_1 < a_2 < \cdots < a_7 = I$. Suppose that $S = \{(x \to y) \wedge (w \vee u), (x \to y)' \vee u', (z \to w)'\}$, where $(x \to y) \wedge (w \vee u), (x \to y)' \vee u', (z \to w)'$ are general generalized clauses in $\mathscr{L}_7P(X)$, $\alpha = a_5$, then $S \leqslant \alpha$. Subsequently, we prove the α-unsatisfiability of S by α-generalized semantic resolution method, let v be a valuation in $\mathscr{L}_7P(X)$ of S and $\mathscr{O}$ be an order of generalized literals occurring in S, and v satisfies: $v(x \to y) = a_6$, $v(z) = O$, $v(w) = a_5$, $v(u) = a_7$, the order $\mathscr{O}$ satisfies: $u' > (x \to y)', w > (x \to y)', x \to y > (x \to y)'$. Then $S_1 = \{(x \to y)' \vee u', (z \to w)'\}$, $S_2 = \{(x \to y) \wedge (w \vee u)\}$, where S_1 is the set of clauses which truth value is less than or equal to α under v, S_2 is the set of clauses which truth value is more than α under v. Then

1. $(x \to y)' \vee u'$
2. $(z \to w)'$
3. $(x \to y) \wedge (w \vee u)$

Applying α-generalized semantic resolution method on S_1 and S_2, we can get two new α-generalized resolution formulae:

4. $\alpha \vee [(x \to y) \wedge (\alpha \vee u)]$ by(2)(3)
5. $\alpha \vee (x \to y)' \vee [(x \to y) \wedge (w \vee \alpha)]$ by (1)(3)

We input clauses (4), (5) into S_2, S_1 respectively, thus, imposing α-generalized semantic resolution method on S_1 and S_2, we can get one new generalized resolution formula only:

6. $\alpha \vee (x \to y)'$ by (1)(4)

We input clauses (6) into S_1, and imposing α-generalized semantic resolution method on S_1 and S_2, we can get two new generalized resolution formulae:

7. α by (3)(6)
8. α by (4)(6)

Connecting these α-generalized resolution formulae above, we get an α-generalized semantic resolution deduction ω from S to α-false clause, that is $\{(x \to y)' \vee u', (z \to w)', (x \to y) \wedge (w \vee u), \alpha \vee [(x \to y) \wedge (\alpha \vee u)], \alpha \vee (x \to y)' \vee [(x \to y) \wedge (w \vee \alpha)], \alpha \vee (x \to y)', \alpha\}$.

There rise 4 formulae if we use α-generalized semantic resolution method.

5. An algorithm for α-generalized semantic resolution in linguistic truth-valued lattice-valued proportional logic

From the soundness and weak completeness of α-generalized semantic resolution, $S \leqslant \alpha$ if and only if there exists an α-generalized semantic resolution deduction from S to $\alpha - \square$, so we can contrive a corresponding resolution reasoning algorithm as a foundation for the implementation purpose.

Algorithm

Let $S = \{\phi_i | i = 1, 2, \cdots, n\}$ be general generalized clause set in $\mathscr{L}_n$P(X), denote H_i the set of generalized literals occurring in $\phi_i (i = 1, 2, \cdots, n)$.

Step 0. Given a valuation v in $\mathscr{L}_n$P(X) for S, and give a order $\mathscr{O}$ for all generalized literals occurring in S. Let $M = \{$ generalized literal C occurring in $S \mid C$ be α-false under valuation v $\}$, $N = \{$ generalized literal C occurring in $S \mid C$ be α-satisfiable under valuation $v\}$;

Step 1. Set $j = 1$;

Step 2. Suppose that $A_0 = \varnothing, B_0 = N$;

Step 3. Set $i = 0$;

Step 4. If $\alpha - \Box \in A_i$, then the algorithm terminate, S is α-unsatisfiable; Otherwise, go to next step;

Step 5. If $B_i = \varnothing$, then go to step 8: Otherwise go to next step;

Step 6. Let $W_{i+1} = \{\alpha$-generalized resolvent of C_1 and $C_2 | C_1 \in M, C_2 \in M$ or $C_2 \in B_i$, the generalized resolution literals in C_1 be the leftmost literals in C_1 according to the order $\mathscr{O}\}$, suppose that A_{i+1}=$\{$ generalized literal C occurring in $W_{i+1} | C$ be α-false under valuation $v\}$ and B_{i+1}=$\{$ generalized literal C occurring in W_{i+1} $|C$ be α-satisfiable under valuation $v\}$;

Step 7. Set $i = i + 1$, go to step 4;

Step 8. Let $T = A_0 \cup A_1 \cup \cdots \cup A_i, M = M \cup T$;

Step 9. $j = j + 1$;

Step 10. Let $R = \{\alpha$-generalized resolvent of C_1 and $C_2 | C_1 \in T$ and $C_2 \in N$, the generalized resolution literals in C_1 be the leftmost literals in C_1 according to the order $\mathscr{O}\}$ and $A_0 = \{$ generalized literal C occurring in $R | C$ be α-false under valuation $v\}$, $B_0 = \{$ generalized literal C occurring in $R | C$ be α-satisfiable under valuation $v\}$;

Step 11. Go to step 3.

Theorem 15. *(Soundness of algorithm) Let S be general generalized clause set in $\mathscr{L}_n P(X)$, $\alpha \in L$, carry the α-generalized semantic resolution algorithm on S, if the algorithm terminate in step 4, then S is α-unsatisfiable.*

Proof. If the algorithm terminate in step 4, namely there exists an $\alpha - \Box$ in the α-generalized semantic resolution process, then it follows that S is α-unsatisfiable from the soundness of α-generalized semantic resolution deduction. □

Theorem 16. *(Completeness of algorithm) Let S be general generalized clause set in $\mathscr{L}_n P(X)$, $\alpha \in L$, carrying the α-generalized semantic resolution algorithm on S, if S is α-unsatisfiable, then the algorithm terminate in step 4.*

Proof. (1) If there exists a general generalized clause ϕ in S, which truth value is less than or equal to α, then these general generalized clause must occur in M, according to circular variable i, the generalized clause ϕ can be decided by two father generalized clause from M and B_0(namely N), therefore the resolution formula belongs to W_1 and moreover it also belongs to A_1, thus $\alpha - \Box \in A_1$, then the algorithm can terminate in step 4.

(2) If there exists a general generalized clause ϕ in S, when the algorithm is carried out according to circular variables, because of the finiteness of generalized literals occurring in S, so only finite generalized resolution formulae can be produced in α-generalized semantic resolution process. The algorithm, hence, can terminate. □

6. Conclusions

In real world, some uncertainty phenomena more suitable be described by natural language than number values, linguistic truth-valued lattice-valued logic is one class of important non-classical logic which valuation fields is a set of linguistic values. The course of human behavior of thinking resolving some real problems can be treated as a proof of soft theorems (some conclusions with uncertainty), in which a lot of natural language reasoning got involved.

Automated reasoning based on resolution principle is an important and efficient method among many automated reasoning methods. In the present paper, our main aim is that the α-generalized semantic resolution reasoning for linguistic truth-valued propositional logic, the method can be used to check if a clausal set is false in a certain linguistic truth-valued level. Concretely, the equivalence of α-generalized semantic resolution based on linguistic truth-valued lattice-valued propositional logic and d_i-generalized semantic resolution on lattice-valued propositional logic $\mathscr{L}_n$P(X) was probed firstly; Secondly, the a_i-generalized semantic resolution on lattice-valued propositional logic $\mathscr{L}_n$P(X) based on a chain-type truth-valued field was investigated and its soundness and weak completeness were given.

We intend to continue this research with the uncertainty and imprecise information, and we have also started to study the automated reasoning theory and method in linguistic truth-valued propositional logic. Future research will be focused on the generalized semantic resolution method on concrete clausal set and the applications of α-generalized semantic resolution method.

Acknowledgments

This work is supported by National Science Foundation of China (Grant No.61175055)Sichuan Key Technology Research and Development Program (Grant No.2011FZ0051), Radio Administration Bureau of MIIT of China(Grant No.[2011]146), China Institution of Communications(Grant No.[2011]051) and Science and Technology Fund Project of Guizhou Province(Grant No.LKB[2012]02).

References

1. J.P. Robinson, " A machine-oriented logic based on the resolution principle", *J. ACM.*, **12**, 23-41(1965).
2. J.R.Slagle, " Automatic theorem proving with renamable and semantic resolution", *J. ACM.*, **14**, 687-697(1967).
3. D. Anatoli, N. Robert, V. Andrei, " Stratified Resolution", *Journal of Symbolic Computation*, **36**, 79-99(2003).
4. C. Areces, M. de Rijke, H. de Nivelle, " Resolution in Modal, Description and Hybrid Logic", *Journal of Logic and Computation*, **11**, 717-736(2001).
5. M. Baaz, C. G. Fermvller, " Resolution-Based Theorem Proving for Many Valued Logics", *J. Symbolic Comput.*, **19**, 353-391(1995).
6. D. Dubois, H. Prade, " Resolution principle in possibilistic logic", *J. Approx. Reason.*, **4**, 1-21(1990).
7. R. Hahnle, " Automated Deduction in Multiple-valued Logics ", *Oxford University Press*, 1994.
8. R. C. T. Lee, " Fuzzy logic and the resolution principle ", *J. ACM,*, **19**, 109-119(1972).
9. C. G. Morgan, " Resolution for many-valued logics ", *Logique et Analyses* , **19**, 11-339(1976).
10. K.Y. Qin, Y. Xu, " Lattice-valued propositional logic(II)", *J. Southwest Jiaotong Univ.*, **2**, 22-27(1994)(in English).
11. Y. Xu, K.Y. Qin, " Lattice-valued propositional logic(I)", *J. Southwest Jiaotong Univ.*, **1**, 123-128(1993)(in English).
12. Y. Xu, K.Y. Qin, Z.M. Song, " On syntax of first-order lattice-valued logic system FM", *Chinese Sci. Bull.*, **42**, 1052-1055(1997).
13. Y. Xu, D. Ruan, K.Y. Qin, J. Liu, " Lattice-Valued Logic: An Alternative Approach to Treat Fuzziness and incomparability ", *springer-verlag*, 2003.
14. Y. Xu, " Lattice implication algebra ", *J. Southwest Jiaotong Univ.*, **1**, 20-27(1993)(in Chinese).
15. K.Y. Qin, Y. Xu, " Lattice-valued propositional logic(II)", *J. Southwest Jiaotong Univ.*, **2**, 22-27 (1994) (in English).
16. Y. Xu, D. Ruan, E. E. Kerre, J. Liu, " α-Resolution principle based on lattice-valued propositional logic LP(X)", *Information Science* , **130**, 195-223(2000).
17. Y. Xu, D. Ruan, E. E. Kerre, J. Liu, " α-Resolution principle based on first-order lattice-valued logic LF(X)", *Information Sciences* , **132**, 221-239(2001).
18. Y. Xu, W. T. Xu, X. M. Zhong, X. X. He, " α-Generalized resolution principle based on lattice-valued propositional logic LP(X)", *The 9th International FLINS Conference on Foundations and Applications of Computational Intelligence* , **14**, August 2-4, 2010, Chengdu(Emei), China, 66-71.
19. X. H. Wang, X. H. Liu, " Generalized Resolution ", *J. ACM.*, **2**, 81-92(1982).
20. Z. Pei, D. Ruan, J. Liu, Y. Xu, " Linguistic Values-based Intelligent Information Processing: Theory, Methods and Applications ", *Atlantic Press* , 2009.
21. L. A. Zadeh, " Fuzzy logic = computing with words ", *IEEE Transactions on Fuzzy Systems* , **4**, 103-111(1996).
22. L. A. Zadeh, " The concept of a linguistic variable and its application to approximate reasoning(I)", *Information Sciences* , **8**, 199-249(1975).
23. L. A. Zadeh, J. Kacprzyk, " Computing with words in information/intelligent system: Foundations ", *Berlin: Springer-Verlag*, 1999.
24. Y. Xu, S. W. Chen, J. Ma, " Linguistic truth-valued lattice implication algebra and its properties ", *IMACS Multi-conference on Computational Engineering in System Application* , 1413-1418(2006).
25. Y. Xu, S. W. Chen, J. Liu, D. Ruan, " Weak Completeness of Resolution in a Linguistic Truth-Valued propositional logic ", *proc.IFSA2007: Theoretical Advance and Applications of Fuzzy Logic and Soft computing* , June 18-21, 2007, Cancun, Mexico, 358-366 (2007).
26. X. B. Li, " The study of resolution automated reasoning for linguistic truth-valued lattice-valued logic ", *Southwest Jiaotong University Dissertation*, Chengdu, China, 2008.
27. W. T. Xu, " α-Generalized linear resolution method for linguistic truth-valued lattice-valued logic system

based on lattice implication algebras ", *Southwest Jiaotong University Dissertation* , Chengdu, China, 2011.
28. X. M. Zhong, Y. Xu, J. Liu, D. Ruan, S. W. Chen, " General form of α-resolution based on linguistic truth-valued lattice-valued logic ", *Soft Computing* , received.
29. X. M. Zhong, J. Liu, S. W. Chen, Y. Xu, " α-Quasi-lock semantic resolution method for linguistic truth-valued lattice-valued propositional logic $\mathscr{L}_{V(n\times 2)}$P(X)", *Proceedings of the 2011 International Conference on Intelligent Systems and Knowledge Engineering (ISKE2011)*, Shanghai, China, Dec.15-17, 2011, 159-169.
30. J. F. Zhang, Y. Xu, et al., " α-resolution fields of generalized literals of lattice-valued logic ", *Proceedings of the international Conf. on Fuzzy Logic and Intelligent Technologies in Nuclear Science(FLINS2010*, **4**, Chengdu, China, August 2-4 2010, 99-104.
31. J. F. Zhang, Y. Xu, " α-Semantic resolution method on lattice-valued propositional logic LP(X)", *Journal of Liaoning Technical University(Natural Science)*, **29**, 767-770(2010)(in Chinese).
32. J. F. Zhang, Y. Xu, X. X. He, " Lattice-valued Semantic resolution reasoning method ", *Computer Science* , **38**, 201-203(2011)(in Chinese).
33. J. F. Zhang, Y. Xu, " α-Semantic resolution method in lattice-valued logic LP(X)", *Proceedings of the 8th International Conference on Fuzzy Systems and Knowledge Discovery*, July 26-28, 2011, Shanghai, China, 402-406.
34. L. Zou, J. L. Li, K. J. Xu, Y. Xu, " A Kind of Resolution Method of Linguistic Truth-valued Propositional Logic Based on LIA ", *Proc. 4th International Conference on Fuzzy Systems and Knowledge discovery* , August, 2007, Haikou, China, 32-36.