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$IFI-$ideals of lattice implication algebras

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Abstract

The notion of $IFI-$ideal is introduced in lattice implication algebras. Firstly, the equivalent conditions of $IF-$ideals and $IFI-$ideals are given in lattice implication algebras. Then the proposition of $IFI-$ideal is investigated in lattice implication algebras. Next, the relations between $IFI-$ideal and $IF-$ideal, between $IFI-$ideal and $IFI-$filter, between $IFI-$ideal and fuzzy impilcative ideals, between $IFI-$ideal and implicative ideals are discussed in lattice implication algebras. Moreover, the extension theorem of $IFI-$ideals is obtained, and $\Psi(L)$ which is composed of all $IFI-$ideals constitutes a closure system. Finally, we prove that $\forall\alpha \in [0,1], A = (\mu_{0,\alpha}, \overline{\mu_{0,\alpha}})$ is an $IFI-$ideal of lattice implication algebra L if and only if L is a lattice H implication algebra.

Keywords: lattice implication algebra; ideals; $IF-$ideals; $IFI-$ideals.

1. Introduction

There are some certain information and uncertain information in the real world. As we know, we can use classical logic to deal with certain information and some non-classical logic to deal with fuzzy information and uncertain information, for example lattice logic, fuzzy logic, etc. Many-valued logic has always been a crucial direction in non-classical logic [2]. Incomparability which can be encountered in our life is an important one among all kinds of uncertainties. In order to research the many-valued logical system whose propositional value is given in a lattice, Xu Yang proposed the concept of lattice implication algebras by combining algebraic lattice and implication algebra[16]. Since then, several researchers have investigated this logic algebra, and many beautiful results have been obtained [5,6,8,9,13,14,15,18,19,20]. For example, in [4], Jun introduced the notion of $LI-$ ideals in lattice implication algebras and investigated some of its properties. Zhao defined the notions of implicative ideals and fuzzy implicative ideals in lattice implication algebras, and investigated their some properties[17]. In 2003, Liu introduced the concepts of $ILI-$ideals and maximal $LI-$ideals in lattice implication algebras and investigated some of their properties[7]. In 2006, Zhu proposed the concept of the primary ideals in lattice implication algebras and investigated the related properties[18]. After that, Zhu proposed the notions of ideal's radical in lattice implication algebras and discussed the relation between the ideal's radical

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and $LI-$ideals[19]. Xu have collected details of lattice implication algebras and lattice-value logic based on lattice implication algebras in [16].

In 1983, K. T. Atanassov introduced the notion of the intuitionistic fuzzy sets, which was a great extension of fuzzy sets [1]. Since then, many researchers have investigated the intuitionistic fuzzy sets. In [10], Pei applied the intuitionistic fuzzy set to lattice implication algebras, and introduced the notion of the intuitionistic fuzzy filter in lattice implication algebras. In 2009, Xu defined the notion of the intuitionistic fuzzy implicative filter in lattice implication algebras and investigated its related properties[12]. Zhu proposed the notion of the intuitionistic fuzzy ideal (briefly, $IF-$ideal) in lattice implication algebras, which was the dual algebraic structure of the intuitionistic fuzzy filter [21].

Now, it is natural to investigate similar type of generalizations of the existing fuzzy subsystems with other algebraic structure. In this paper, with this objective in view, we introduce the notion of the intuitionistic fuzzy implicative ideal(briefly, $IFI-$ideal) in lattice implication algebras. Then we give the properties of the $IFI-$ideals in lattice implication algebras. Next, we discuss the relations between $IFI-$ideal and $IF-$ideal, between $IFI-$ideal and $IFI-$filter, between $IFI-$ideal and fuzzy impilcative ideals, between $IFI-$ideal and implicative ideals in lattice implication algebras. Then the extension theorem of $IFI-$ideals is obtained. That all $IFI-$ideals are closed under the operation $\cap$ is proved. Finally, we prove that an $IFI-$ideal is equivalent to an $IF-$ideal in lattice H implication algebras and give the equivalent conditions in lattice H implication algebras. It will be important to provide theoretical foundation to design intelligent information processing systems.

2. Preliminaries

By a lattice implication algebra [16]we mean a bounded lattice $(L,\vee,\wedge,0,1)$ with order-reversing involution $'$, in which 1 and 0 are the greatest and the smallest element of L respectively, and a binary operation $\rightarrow$ satisfying the follow axioms:

(I_1) $x\rightarrow(y\rightarrow z)=y\rightarrow(x\rightarrow z)$;
(I_2) $x\rightarrow x=1$;
(I_3) $x\rightarrow y=y'\rightarrow x'$;
(I_4) $if\ x\rightarrow y=y\rightarrow x=1$, then $x=y$;
(I_5) $(x\rightarrow y)\rightarrow y=(y\rightarrow x)\rightarrow x$;
(L_1) $(x\vee y)\rightarrow z=(x\rightarrow z)\wedge(y\rightarrow z)$;
(L_2) $(x\wedge y)\rightarrow z=(x\rightarrow z)\vee(y\rightarrow z)$;
for all $x,y,z\in L$.

If a lattice implication algebra L satisfies: for any $x,y,z\in L, x\vee y\vee((x\wedge y)\rightarrow z)=1$, then we call it a lattice H implication algebra.

A mapping $f:L_1\rightarrow L_2$ from lattice implication algebras L_1 to L_2 is called a lattice implication homomorphism if it satisfies: for any $x,y\in L_1$,

$$\begin{aligned} f(x\rightarrow y) &= f(x)\rightarrow f(y),\\ f(x\vee y) &= f(x)\vee f(y),\\ f(x\wedge y) &= f(x)\wedge f(y),\\ f(x') &= (f(x))'. \end{aligned}$$

A one-to-one and onto lattice implication homomorphism is called a lattice implication isomorphism.

In the following, unless otherwise stated, L always represents any given lattice implication algebra. $\forall x,y,z\in L$, the following holds[16]:
$(1) 0\rightarrow 1=1, 1\rightarrow x=x, x\rightarrow 1=1$;
$(2) x\rightarrow y=1\Longleftrightarrow x\leqslant y$;
$(3) y\leqslant x\rightarrow y$;
$(4) x\rightarrow y\leqslant(y\rightarrow z)\rightarrow(x\rightarrow z)$,
$x\rightarrow y\leqslant(z\rightarrow x)\rightarrow(z\rightarrow y)$;
(5) if $x\leqslant y$, then $y\rightarrow z\leqslant x\rightarrow z$ and $z\rightarrow x\leqslant z\rightarrow y$;
$(6) x\vee y=(x\rightarrow y)\rightarrow y$;
$(7) x\wedge y=((y\rightarrow x)\rightarrow y')'=((x\rightarrow y)\rightarrow x')$;
$(8)(x\rightarrow y)\rightarrow(x\rightarrow z)=x\wedge y\rightarrow z$.

Definition 1. [17]. Let A be a non-empty subset of L. If A satisfies $\forall x,y,z\in L$,
$(I1) 0\in A$;
$(I2)((x\rightarrow y)'\rightarrow z)'\in A$ and $(y\rightarrow z)'\in A$ imply $(x\rightarrow z)'\in A$;
then A is said to be an implicative ideal of L.

Definition 2. [16]. Let A_0 be a non-empty fuzzy subset of L. If A_0 satisfies $\forall x,y,z\in L$,
$(FI1) A_0(x)\leqslant A_0(0)$;
$(FI2) A_0(x\rightarrow z)'\geqslant\min\{A_0((x\rightarrow y)'\rightarrow z)', A_0(y\rightarrow$

$z)'\}$;
then A_0 is said to be a fuzzy implicative ideal of L.

Definition 3. [21]. Let $A=(\mu_A,\nu_A)$ be an intuitionistic fuzzy set of L. If A satisfies $\forall x,y\in L$,
$(IF1)\mu_A(0)\geqslant\mu_A(x),\nu_A(0)\leqslant\nu_A(x)$;
$(IF2)\mu_A(x)\geqslant\min\{\mu_A(x\to y)',\mu_A(y)\}$;
$(IF3)\nu_A(x)\leqslant\max\{\nu_A(x\to y)',\nu_A(y)\}$;
then A is said to be an intuitionistic fuzzy ideal (briefly, $IF-$ideal) of L.

Proposition 1. [16]. *Let L be a lattice implication algebra, then for any $x\in L$, $x\to 0=x'$.*

For more details of lattice implication algebras, we refer readers to [16].

3. Intuitionistic Fuzzy Implicative Ideals

Firstly, the notion of the intuitionistic fuzzy implicative ideals (briefly, $IFI-$ideals) are introduced in lattice implication algebras.

Definition 4. Let $A=(\mu_A,\nu_A)$ be an intuitionistic fuzzy set on L. If A satisfies $\forall x,y,z\in L$,
$(IF1)\mu_A(0)\geqslant\mu_A(x),\nu_A(0)\leqslant\nu_A(x)$;
$(IFI2)\mu_A(x\to z)'\geqslant\min\{\mu_A(y\to z)',\mu_A((x\to y)'\to z)'\}$;
$(IFI3)\nu_A(x\to z)'\leqslant\max\{\nu_A(y\to z)',\nu_A((x\to y)'\to z)'\}$,
then A is said to be an intuitionistic fuzzy implicative ideal(briefly, $IFI-$ideal) of L.

The following example shows that the $IFI-$ideal exists in lattice implication algebras.

Example 1. Let $L=\{0,a,b,1\}$ be a set with Cayley tables as follows:

$\to$	0	a	b	1
0	1	1	1	1
a	b	1	b	1
b	a	a	1	1
1	0	a	b	1

x	x'
0	1
a	b
b	a
1	0

Define $\vee$ and $\wedge$ operations on L as follows:

$$x\vee y = (x\to y)\to y$$
$$x\wedge y = ((x'\to y')\to y')'$$

for all $x,y\in L$, then $(L,\vee,\wedge,\to,')$ is a lattice implication algebra [16].

Define a function $\mu_A: L\longrightarrow[0,1]$ on L by

$$\mu_A(x)=\begin{cases}1, & x=0,a\\ s, & x=x\end{cases}$$

and a function $\nu_A: L\longrightarrow[0,1]$ on L by

$$\nu_A(x)=\begin{cases}0, & x=0,a\\ t, & x=x\end{cases}$$

which satisfies $0\leqslant s,t\leqslant 1$ and $s+t\leqslant 1$, then $A=(\mu_A,\nu_A)$ can be verified to be an $IFI-$ideal of L.

Theorem 2. [21]. *Let $A=(\mu_A,\nu_A)$ be an $IF-$ideal (intuitionistic fuzzy ideal) on L. $\forall x,y\in L$, if $x\leqslant y$, then $\mu_A(x)\geqslant\mu_A(y),\nu_A(x)\leqslant\nu_A(y)$.*

Next, an $IFI-$ideal is proved to be an $IF-$ideal in lattice implication algebras.

Theorem 3. *If $A=(\mu_A,\nu_A)$ is an $IFI-$ideal on L, then $A=(\mu_A,\nu_A)$ is an $IF-$ideal on L.*

Proof. Suppose $A=(\mu_A,\nu_A)$ is an $IFI-$ideal of L, (IF1) holds by Definition 4. And replace z by 0 in Definition 4 (IFI2) and (IFI3), we have

$$(IF2)\mu_A(x)\geqslant\min\{\mu_A(x\to y)',\mu_A(y)\};$$
$$(IF3)\nu_A(x)\leqslant\max\{\nu_A(x\to y)',\nu_A(y)\}$$

by proposition 1. Then $A=(\mu_A,\nu_A)$ is an $IF-$ideal of L by Definition 3.

Moreover, the equivalent conditions of $IF-$ideal are given in lattice implication algebras.

Proposition 4. *Let L be a lattice implication algebra, then for all $x,y,z\in L$, the following hold:*

$$(1)(x\to z)'\to y=(x\to y)'\to z;$$
$$(2)(x\to y)'\to y=x\to y.$$

Proof. $(1)\forall x,y,z\in L$,

$$\begin{aligned}&((x\to z)'\to y)\to((x\to y)'\to z)\\ =\ &(y'\to(x\to z))\to(z'\to(x\to y))\\ =\ &(x\to(y'\to z))\to(x\to(z'\to y))\\ =\ &(x\to(z'\to y))\to(x\to(z'\to y))\\ =\ &1,\end{aligned}$$

then we have$((x \to z)' \to y) \leqslant ((x \to y)' \to z)$. And

$$\begin{aligned} & ((x \to y)' \to z) \to ((x \to z)' \to y) \\ = \ & (z' \to (x \to y)) \to (y' \to (x \to z)) \\ = \ & (x \to (z' \to y)) \to (x \to (y' \to z)) \\ = \ & (x \to (y' \to z)) \to (x \to (y' \to z)) \\ = \ & 1, \end{aligned}$$

so we have $((x \to y)' \to z) \leqslant ((x \to z)' \to y)$.
Combing above, $(x \to z)' \to y = (x \to y)' \to z$.

(2)$\forall x, y \in L$,

$$\begin{aligned} & ((x \to y)' \to y) \to (x \to y) \\ = \ & (y' \to (x \to y)) \to (x \to y) \\ = \ & y' \vee (x \to y) \\ \geqslant \ & y' \vee y \\ = \ & 1, \end{aligned}$$

then we have$((x \to y)' \to y) \leqslant (x \to y)$.
On the other hand,

$$\begin{aligned} & (x \to y) \to ((x \to y)' \to y) \\ = \ & (x \to y) \to (y' \to (x \to y)) \\ = \ & y' \to ((x \to y) \to (x \to y)) \\ = \ & 1, \end{aligned}$$

then we have $(x \to y) \leqslant ((x \to y)' \to y)$.
Hence $(x \to y)' \to y = x \to y$.

Theorem 5. *Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy set on L. Then $A = (\mu_A, \nu_A)$ is an $IF-$ideal on L if and only if $A = (\mu_A, \nu_A)$ satisfies (IF1) and $\forall x, y, z \in L$,*
$(IFI4)\mu_A(x \to y)' \geqslant \min\{\mu_A(z), \mu_A(((x \to y)' \to y)' \to z)'\}$;
$(IFI5)\nu_A(x \to y)' \leqslant \max\{\nu_A(z), \nu_A(((x \to y)' \to y)' \to z)'\}$.

Proof. "$\Leftarrow$" we need only to verify $A = (\mu_A, \nu_A)$ satisfies (IF2) and (IF3). Replace y by 0 in (IFI4) and (IFI5), then we have

$$(IF2)\mu_A(x) \geqslant \min\{\mu_A(x \to z)', \mu_A(z)\};$$

$$(IF3)\nu_A(x) \leqslant \max\{\nu_A(x \to z)', \nu_A(z)\}.$$

Thus $A = (\mu_A, \nu_A)$ is an $IF-$ideal of L by Definition 3.

"$\Rightarrow$" Suppose A is an $IF-$ideal on L, then we have $\forall x, y, z \in L$,
$\mu_A((x \to y)' \to y)' \geqslant \min\{\mu_A(z), \mu_A(((x \to y)' \to y)' \to z)'\}$;
$\nu_A((x \to y)' \to y)' \leqslant \max\{\nu_A(z), \nu_A(((x \to y)' \to y)' \to z)'\}$.
Because

$$\mu_A((x \to y)' \to y)' = \mu_A(x \to y)'$$

and

$$\nu_A((x \to y)' \to y)' = \nu_A(x \to y)'$$

by Proposition 8, we have

$$(IFI4)\mu_A(x \to y)' \geqslant \min\{\mu_A(z), \mu_A(((x \to y)' \to y)' \to z)'\};$$

$$(IFI5)\nu_A(x \to y)' \leqslant \max\{\nu_A(z), \nu_A(((x \to y)' \to y)' \to z)'\}.$$

Corollary 6. *If $A = (\mu_A, \nu_A)$ is an $IFI-$ideal on L, then the following holds:*

$$(IFI4)\mu_A(x \to y)' \geqslant \min\{\mu_A(z), \mu_A(((x \to y)' \to y)' \to z)'\};$$

$$(IFI5)\nu_A(x \to y)' \leqslant \max\{\nu_A(z), \nu_A(((x \to y)' \to y)' \to z)'\}.$$

Proof. It can be obtained by Theorem 3 and Theorem 5.

The equivalent definition of $IFI-$ideal is given in the following by Definition 4 and Proposition 4(1).

Theorem 7. *Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy set on L. A is an $IFI-$ideal of L if and only if $\forall x, y, z \in L$,*
$(IF1)\mu_A(0) \geqslant \mu_A(x), \nu_A(0) \leqslant \nu_A(x)$;
$(EIFI2)\mu_A(x \to z)' \geqslant \min\{\mu_A(y \to z)', \mu_A((x \to z)' \to y)'\}$;
$(EIFI3)\nu_A(x \to z)' \leqslant \max\{\nu_A(y \to z)', \nu_A((x \to z)' \to y)'\}$.

The Propositions of $IFI-$ideals are investigated in the following.

Proposition 8. *Let $A = (\mu_A, \nu_A)$ be an $IFI-$ideal on L, then the following holds:*
$(1)\mu_A(x \to z)' \geqslant \min\{\mu_A(y), \mu_A((x \to y)' \to z)'\}$;
$(2)\nu_A(x \to z)' \leqslant \max\{\nu_A(y), \nu_A((x \to y)' \to z)'\}$.

Proof. Because

$$\begin{aligned} & (y \to z)' \to y \\ = \ & y' \to (y \to z) \\ = \ & y' \to (z' \to y') \\ = \ & 1, \end{aligned}$$

we have $(y \to z)' \leqslant y$. And $\mu_A(y \to z)' \geqslant \mu_A(y)$, $\nu_A(y \to z)' \leqslant \nu_A(y)$ by Theorem 2.

Suppose$A = (\mu_A, \nu_A)$ be an $IFI-$ideal on L, we have
$(IFI2)\mu_A(x \to z)' \geqslant \min\{\mu_A(y \to z)', \mu_A((x \to y)' \to z)'\}$;
$(IFI3)\nu_A(x \to z)' \leqslant \max\{\nu_A(y \to z)', \nu_A((x \to y)' \to z)'\}$
by Definition 4. Thus

$$(1)\mu_A(x \to z)' \geqslant \min\{\mu_A(y), \mu_A((x \to y)' \to z)'\};$$

$$(2)\nu_A(x \to z)' \leqslant \max\{\nu_A(y), \nu_A((x \to y)' \to z)'\}.$$

The relation between $IFI-$ideal and $IFI-$filter of lattice implication algebras is as follows.

Definition 5. [12]. Let $J = (\mu_J, \nu_J)$ be an intuitionistic fuzzy set on L. If J satisfies $\forall x, y, z \in L$,
$(IF1')\mu_J(1) \geqslant \mu_J(x), \nu_J(1) \leqslant \nu_J(x)$;
$(IFI2')\mu_J(x \to z) \geqslant \min\{\mu_J(x \to y), \mu_J(x \to (y \to z))\}$;
$(IFI3')\nu_J(x \to z) \leqslant \max\{\nu_J(x \to y), \nu_J(x \to (y \to z))\}$,
then J is said to be an intuitionistic fuzzy implicative filter(briefly, $IFI-$filter) of L.

Theorem 9. *$A = (\mu_A, \nu_A)$ is an $IFI-$ideal on L if and only if $A' = (\mu'_A, \nu'_A)$ is an $IFI-$filter on L, where $\forall x \in L$, $\mu'_A(x) = \mu_A(x'), \nu'_A(x) = \nu_A(x')$.*

Proof. " $\Rightarrow$ " Suppose $A = (\mu_A, \nu_A)$ is an $IFI-$ideal on L, then $\forall x, y, z \in L$,

$$\mu'_A(1) = \mu_A(0) \geqslant \mu_A(x') = \mu'_A(x)$$

$$\nu'_A(1) = \nu_A(0) \leqslant \nu_A(x') = \nu'_A(x)$$

and

$$\begin{aligned} & \mu'_A(x \to z) \\ = \ & \mu_A(x \to z)' = \mu_A(z' \to x')' \\ \geqslant \ & \min\{\mu_A(y' \to x')', \mu_A((z' \to y')' \to x')'\} \\ = \ & \min\{\mu'_A(y' \to x'), \mu'_A(x \to (z' \to y'))\} \\ = \ & \min\{\mu'_A(x \to y), \mu'_A(x \to (y \to z))\} \end{aligned}$$

$$\begin{aligned} & \nu'_A(x \to z) \\ = \ & \nu_A(x \to z)' = \nu_A(z' \to x')' \\ \leqslant \ & \max\{\nu_A(y' \to x')', \nu_A((z' \to y')' \to x')'\} \\ = \ & \max\{\nu'_A(y' \to x'), \nu'_A(x \to (z' \to y'))\} \\ = \ & \max\{\nu'_A(x \to y), \nu'_A(x \to (y \to z))\}. \end{aligned}$$

By Definition 5, we have $A' = (\mu'_A, \nu'_A)$ is an $IFI-$filter on L.

" $\Leftarrow$ " It can be obtained analogously.

Next, the relation between $IFI-$ideals and fuzzy implicative ideals, between $IFI-$ideals and implicative ideals are discussed in lattice implication algebras.

Theorem 10. *$A = (\mu_A, \nu_A)$ is an $IFI-$ideal on L if and only if μ_A and $\overline{\nu_A}$ are fuzzy implicative ideals on L, where $\overline{\nu_A} = 1 - \nu_A$.*

Proof. " $\Rightarrow$ " Suppose $A = (\mu_A, \nu_A)$ is an $IFI-$ideal on L, then μ_A can be easily verified to be a fuzzy implicative ideal on L by Definition 2. Because $\forall x, y, z \in L$,

$$(1) \quad \overline{\nu_A}(0) = 1 - \nu_A(0) \geqslant 1 - \nu_A(x) = \overline{\nu_A}(x),$$

$$\begin{aligned} (2) \quad & \overline{\nu_A}(x \to z)' = 1 - \nu_A(x \to z)' \\ \geqslant \ & 1 - \max\{\nu_A(y \to z)', \nu_A((x \to y)' \to z)'\} \\ = \ & \min\{1 - \nu_A(y \to z)', 1 - \nu_A((x \to y)' \to z)'\} \\ = \ & \min\{\overline{\nu_A}(y \to z)', \overline{\nu_A}((x \to y)' \to z)'\}, \end{aligned}$$

then $\overline{\nu_A}$ is a fuzzy implicative ideal on L by Definition 2.

" $\Leftarrow$ " If μ_A and $\overline{\nu_A}$ are fuzzy implicative ideals on L, then $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy set on L. In the following, we prove $A = (\mu_A, \nu_A)$ satisfies the Definition 4.

Because $\forall x, y, z \in L$,

$$\begin{array}{lrl} (IF1) & & \mu_A(0) \geqslant \mu_A(x), \\ & & \nu_A(0) \leqslant \nu_A(x) \;\; by \;\; \overline{\nu_A}(0) \geqslant \overline{\nu_A}(x); \\ (IFI2) & & \mu_A(x \to z)' \\ & \geqslant & \min\{\mu_A(y \to z)', \mu_A((x \to y)' \to z)'\}; \end{array}$$

and

$$\begin{array}{rl} & 1 - \nu_A(x \to z)' = \overline{\nu_A}(x \to z)' \\ \geqslant & \min\{\overline{\nu_A}(y \to z)', \overline{\nu_A}((x \to y)' \to z)'\} \\ = & \min\{1 - \nu_A(y \to z)', 1 - \nu_A((x \to y)' \to z)'\} \\ = & 1 - \max\{\nu_A(y \to z)', \nu_A((x \to y)' \to z)'\}, \end{array}$$

then we have
$(IFI3)$ $\nu_A(x \to z)' \leqslant \max\{\nu_A(y \to z)', \nu_A((x \to y)' \to z)'\}$.
By Definition 4, we have $A = (\mu_A, \nu_A)$ is an $IFI-$ideal on L.

Corollary 11. *$A = (\mu_A, \overline{\mu_A})$ is an $IFI-$ideal on L if and only if μ_A is a fuzzy implicative ideal on L.*

Corollary 12. *$A = (\mu_A, \overline{\nu_A})$ is an $IFI-$ideal on L if and only if $A_1 = (\mu_A, \overline{\mu_A})$ and $A_2 = (\overline{\nu_A}, \nu_A)$ are $IFI-$ideals on L.*

Proof. It can be obtained by Theorem 10 and Corollary 11.

Theorem 13. [17]. *Let A be a fuzzy set on L. Then A is a fuzzy implicative ideal of L if and only if $\forall \lambda \in [0,1]$, $A_\lambda = \{x \mid x \in L, A(x) \geqslant \lambda\}$ is an implicative ideal of L.*

Corollary 14. *$A = (\mu_A, \nu_A)$ is an $IFI-$ideal on L if and only if $(\mu_A)_\lambda$ and $(\overline{\nu_A})_\lambda$ are implicative ideals on L.*

Proof. It can be get by Theorem 10 and Theorem 13.

Theorem 15. *A is an implicative ideal of L if and only if $\forall \alpha \in [0,1]$, $F_{A,\alpha} = (\mu_{A,\alpha}, \overline{\mu_{A,\alpha}})$ is an $IFI-$ideal on L, where*

$$\mu_{A,\alpha}(x) = \begin{cases} \alpha, & x \in A \\ 0, & x \notin A \end{cases}$$

and $\overline{\mu_{A,\alpha}} = 1 - \mu_{A,\alpha}$.

Proof. " $\Rightarrow$ " Suppose A is an implicative ideal of L, then $0 \in A$. So we have $\forall x \in L$,

$$\begin{array}{rcl} \mu_{A,\alpha}(0) & = & \alpha \geqslant \mu_{A,\alpha}(x), \\ \overline{\mu_{A,\alpha}}(0) & = & 1 - \mu_{A,\alpha}(0) \leqslant 1 - \mu_{A,\alpha}(x) = \overline{\mu_{A,\alpha}}(x). \end{array}$$

In the following, firstly, we prove $\mu_{A,\alpha}$ satisfies (FI2).

Assume $\mu_{A,\alpha}$ is not satisfied (FI2), that is
$\exists x, y, z \in L, \mu_{A,\alpha}(x \to z)' < \min\{\mu_{A,\alpha}(y \to z)', \mu_{A,\alpha}((x \to y)' \to z)'\}$.

By the definition of $\mu_{A,\alpha}$, we have

$$\begin{array}{rcl} \mu_{A,\alpha}(x \to z)' & = & 0, \\ \mu_{A,\alpha}(y \to z)' & = & \alpha, \\ \mu_{A,\alpha}((x \to y)' \to z)' & = & \alpha, \end{array}$$

that is $(y \to z)' \in A, ((x \to y)' \to z)' \in A$, but $(x \to z)' \notin A$.
This contradict that A is an implicative ideal of L by Definition 1. So (FI2) holds, and $\mu_{A,\alpha}$ is an fuzzy implicative ideal of L by Definition 2.

By Corollary 11, we have $F_{A,\alpha} = (\mu_{A,\alpha}, \overline{\mu_{A,\alpha}})$ is an $IFI-$ideal on L.

" $\Leftarrow$ " It is similar to the above proof.

Theorem 16. *$\forall \alpha \in [0,1], A_{0,\alpha} = (\mu_{0,\alpha}, \overline{\mu_{0,\alpha}})$ is an $IF-$ideal of L.*

Proof. By the definition of $A_{0,\alpha}$, we have

$$\mu_{0,\alpha}(x) = \begin{cases} \alpha, & x = 0 \\ 0, & x \neq 0 \end{cases}$$

Obviously $\mu_{0,\alpha}(0) \geqslant \mu_{0,\alpha}(x)$, and
$\overline{\mu_{0,\alpha}}(0) = 1 - \mu_{0,\alpha}(0) \leqslant 1 - \mu_{0,\alpha}(x) = \overline{\mu_{0,\alpha}}(x)$.
Assume $A_{0,\alpha}$ does not satisfy the Definition 3 (IF2), that is
$\exists x, y \in L, \mu_{0,\alpha}(x) < min\{\mu_{0,\alpha}(x \to y)', \mu_{0,\alpha}(y)\}$.
According to the definition of $A_{0,\alpha}$, we have $\mu_{0,\alpha}(x) = 0$ and $\mu_{0,\alpha}(x \to y)' = \alpha, \mu_{0,\alpha}(y) = \alpha$, that is $(x \to y)' = 0, y = 0$, but $x \neq 0$. Because $(x \to y)' = 0$, we have $x \leqslant y$, and we obtain $x = 0$ by $y = 0$. This contradicts $x \neq 0$. Hence $A_{0,\alpha}$ satisfies the Definition 3 (IF2).
$A_{0,\alpha}$ that satisfies the Definition 3 (IF3) is similar to the proof of (IF2). Thus $A_{0,\alpha}$ is an $IF-$ideal of L, ending the proof.

Now, we give the extension theorem of $IFI-$ideals in the following.

Theorem 17. *Let $A=(\mu_A,\nu_A)$ be an $IF-$ideal of L. Then the following are equivalent: $\forall x,y,z,u\in L$,*
(1) A is an $IFI-$ideal of L.
(2)

$$\mu_A(x\to y)'\geqslant \mu_A((x\to y)'\to x')'$$
$$\nu_A(x\to y)'\leqslant \nu_A((x\to y)'\to x')'$$

(3)

$$\mu_A((y\to z)\to(x\to z))'\geqslant \mu_A((x\to y)'\to z)'$$
$$\nu_A((y\to z)\to(x\to z))'\leqslant \nu_A((x\to y)'\to z)'$$

(4)

$$\begin{aligned}&\mu_A((y\to z)\to(x\to z))'\\ \geqslant\ &\min\{\mu_A((((x\to y)'\to z)'\to u)',\mu_A(u)\};\\ &\nu_A((y\to z)\to(x\to z))'\\ \leqslant\ &\max\{\nu_A((((x\to y)'\to z)'\to u)',\nu_A(u)\}.\end{aligned}$$

Proof. "(1) $\Rightarrow$ (2)" Suppose A is an $IFI-$ideal of L, then by Definition 4,

$$\begin{aligned}&\mu_A(x\to y)'\\ \geqslant\ &\min\{\mu_A(x'\to x')',\mu_A((y'\to x')'\to x')'\}\\ =\ &\min\{\mu_A(0),\mu_A((x\to y)'\to x')'\}\\ =\ &\mu_A((x\to y)'\to x')',\\ &\nu_A(x\to y)'\\ \leqslant\ &\max\{\nu_A(x'\to x')',\nu_A((y'\to x')'\to x')'\}\\ =\ &\max\{\nu_A(0),\nu_A((x\to y)'\to x')'\}\\ =\ &\nu_A((x\to y)'\to x')'.\end{aligned}$$

"(2) $\Rightarrow$ (3)"

$$\begin{aligned}&\mu_A((y\to z)\to(x\to z))'\\ =\ &\mu_A((z'\to y')\to(z'\to x'))'\\ =\ &\mu_A(z'\to((z'\to y')\to x'))'\\ \geqslant\ &\mu_A((z'\to((z'\to y')\to x'))'\to z)'\\ =\ &\mu_A(z'\to((z'\to y')\to(z'\to x')))'\\ =\ &\mu_A(z'\to(z'\wedge y'\to x'))'\\ \geqslant\ &\mu_A(z'\to(y'\to x'))'\\ =\ &\mu_A((x\to y)'\to z)',\end{aligned}$$

$\nu_A((y\to z)\to(x\to z))'\leqslant\nu_A((x\to y)'\to z)'$ can be proved by analogy.

"(3) $\Rightarrow$ (4)" Because A is an $IF-$ideal of L, we have

$$\begin{aligned}&\mu_A((x\to y)'\to z)'\\ \geqslant\ &\min\{\mu_A((((x\to y)'\to z)'\to u)',\mu_A(u)\}\end{aligned}$$

and

$$\begin{aligned}&\nu_A((x\to y)'\to z)'\\ \leqslant\ &\max\{\nu_A((((x\to y)'\to z)'\to u)',\nu_A(u)\}.\end{aligned}$$

By (3), we have

$$\begin{aligned}&\mu_A((y\to z)\to(x\to z))'\\ \geqslant\ &\min\{\mu_A((((x\to y)'\to z)'\to u)',\mu_A(u)\}\end{aligned}$$

and

$$\begin{aligned}&\nu_A((y\to z)\to(x\to z))'\\ \leqslant\ &\max\{\nu_A((((x\to y)'\to z)'\to u)',\nu_A(u)\}.\end{aligned}$$

"(4) $\Rightarrow$ (1)" Because

$$\begin{aligned}&((x\to z)'\to z)'\to(y\to z)'\\ =\ &(y\to z)\to((x\to z)'\to z)\\ =\ &(z'\to y')\to(z'\to(x\to z))\\ =\ &z'\wedge y'\to(x\to z)\\ \geqslant\ &y'\to(x\to z)\\ =\ &(x\to z)'\to y\\ =\ &(x\to y)'\to z,\end{aligned}$$

we have

$$\mu_A(((x\to z)'\to z)'\to(y\to z)')'\geqslant\mu_A((x\to y)'\to z)',$$

and

$$\nu_A(((x\to z)'\to z)'\to(y\to z)')'\leqslant\nu_A((x\to y)'\to z)'$$

by Theorem 2. By(4), we have

$$\begin{aligned}&\mu_A(x\to z)'=\mu_A((z\to z)\to(x\to z))'\\ \geqslant\ &\min\{\mu_A(((x\to z)'\to z)'\to(y\to z)')',\mu_A(y\to z)'\}\\ \geqslant\ &\min\{\mu_A((x\to y)'\to z)',\mu_A(y\to z)'\},\\ &\nu_A(x\to z)'=\nu_A((z\to z)\to(x\to z))'\\ \leqslant\ &\max\{\nu_A(((x\to z)'\to z)'\to(y\to z)')',\nu_A(y\to z)'\}\\ \leqslant\ &\max\{\nu_A((x\to y)'\to z)',\nu_A(y\to z)'\}.\end{aligned}$$

We have A is an $IFI-$ideal of L by Definition 4, ending the proof.

Let $A=(\mu_A,\nu_A)$ and $B=(\mu_B,\nu_B)$ be two $IF-$ideals of L, define operators of $IF-$ideals on L as follows:

$$\begin{aligned} A\cap B &= (\mu_{A\cap B},\nu_{A\cap B})=(\mu_A\wedge\mu_B,\nu_A\vee\nu_B)\\ A\cup B &= (\mu_{A\cup B},\nu_{A\cup B})=(\mu_A\vee\mu_B,\nu_A\wedge\nu_B)\\ A\subseteq B &\Leftrightarrow \mu_A(x)\leqslant\mu_B(x)\ and\ \nu_A(x)\geqslant\mu_B(x),\forall x\in L.\end{aligned}$$

Theorem 18. *Let $A=(\mu_A,\nu_A)$ and $B=(\mu_B,\nu_B)$ be two $IF-$ideals of L and $A\subseteq B$. If A is an $IFI-$ideal of L, then B is an $IFI-$ideal of L.*

Proof. By $B=(\mu_B,\nu_B)$ is an $IF-$ideal of L, we have
$\mu_B(x\to y)'\geqslant min\{\mu_B((x\to y)'\to x')',\mu_B((x\to y)'\to((x\to y)'\to x')')'\}$.
Because

$$\begin{aligned} &\mu_B((x\to y)'\to((x\to y)'\to x')')'\\ \geqslant\ &\mu_A((x\to y)'\to((x\to y)'\to x')')'\\ =\ &\mu_A(((x\to y)'\to x')\to(x\to y))'\\ =\ &\mu_A(x\to(((x\to y)'\to x')\to y))'\\ \geqslant\ &\mu_A((x\to(((x\to y)'\to x')\to y))'\to x')'\\ =\ &\mu_A(x\to(x\to(((x\to y)'\to x')\to y)))'\\ =\ &\mu_A(((x\to y)'\to x')\to(x\to(x\to y)))'\\ =\ &\mu_A((x\to(x\to y))\to(x\to(x\to y)))'\\ =\ &\mu_A(0)=\mu_B(0),\end{aligned}$$

we have

$$\mu_B(x\to y)'\geqslant\mu_B((x\to y)'\to x')',$$

and $\nu_B(x\to y)'\leqslant\nu_B((x\to y)'\to x')'$ can be proved by analogy. Then by Theorem 17(2), B is an $IFI-$ideal of L.

Theorem 19. *Let $A=(\mu_A,\nu_A)$ and $B=(\mu_B,\nu_B)$ be two $IFI-$ideals of L, then $A\cap B$ is an $IFI-$ideals of L.*

Proof. Obviously, $A\cap B$ is an intuitionistic fuzzy set of L. $\forall x,y,z\in L$,

$$\begin{aligned} \mu_{A\cap B}(0) &= \mu_A(0)\wedge\mu_B(0)\\ &\geqslant \mu_A(x)\wedge\mu_B(x)\\ &= \mu_{A\cap B}(x),\\ \nu_{A\cap B}(0) &= \nu_A(0)\vee\nu_B(0)\\ &\leqslant \nu_A(x)\vee\nu_B(x)\\ &= \nu_{A\cap B}(x).\end{aligned}$$

$$\begin{aligned} &\mu_{A\cap B}(x\to z)'=\mu_A(x\to z)'\wedge\mu_B(x\to z)'\\ \geqslant\ &\min\{\mu_A(y\to z)',\mu_A((x\to y)'\to z)'\}\wedge\\ &\min\{\mu_B(y\to z)',\mu_B((x\to y)'\to z)'\}\\ \geqslant\ &\min\{\mu_A(y\to z)'\wedge\mu_B(y\to z)',\\ &\mu_A((x\to y)'\to z)'\wedge\mu_B((x\to y)'\to z)'\}\\ =\ &\min\{\mu_{A\cap B}(y\to z)',\mu_{A\cap B}((x\to y)'\to z)'\}\end{aligned}$$

and

$$\begin{aligned} &\nu_{A\cap B}(x\to z)'=\nu_A(x\to z)'\wedge\nu_B(x\to z)'\\ \leqslant\ &\max\{\nu_A(y\to z)',\nu_A((x\to y)'\to z)'\}\vee\\ &\max\{\nu_B(y\to z)',\nu_B((x\to y)'\to z)'\}\\ \leqslant\ &\max\{\nu_A(y\to z)'\vee\nu_B(y\to z)',\\ &\nu_A((x\to y)'\to z)'\vee\nu_B((x\to y)'\to z)'\}\\ =\ &\max\{\nu_{A\cap B}(y\to z)',\nu_{A\cap B}((x\to y)'\to z)'\},\end{aligned}$$

then by Definition 4, $A\cap B$ is an $IFI-$ideals of L.

Definition 6. [11]. Let L be a complete lattice, $\wp\subseteq L$. If $\wp$ is closed under intersection, then $\wp$ is called a closure system of L.

Remark 1. Let L be a lattice implication algebra, then L is a complete lattice.

Theorem 20. *Let $\Psi(L)$ be a set of all $IFI-$ideals on L, $\digamma(L)$ be a set of all intuitionistic fuzzy sets on L. Then $\Psi(L)$ constitutes a closure system of $\digamma(L)$.*

Proof. Because $\Psi(L)$ is closed under $\cap$ by Theorem 19, the conclusion is obtained.

Remark 2. $\Psi(L)$ may be not closed under $\cup$. It is shown by the following example.

Example 2. In example 1, redefine a function $\mu_B: L\longrightarrow[0,1]$ on L by

$$\mu_B(x)=\begin{cases}1, & x=0,b\\ s, & x=x\end{cases}$$

and a function $\nu_B : L \longrightarrow [0,1]$ on L by

$$\nu_B(x) = \begin{cases} 0, & x = 0, b \\ t, & x = x \end{cases}$$

which satisfies $0 \leqslant s, t \leqslant 1$ and $s + t \leqslant 1$, then $B = (\mu_B, \nu_B)$ can be verified to be an $IFI-$ideal of L. we have $A \cup B = (\mu_{A\cup B}, \nu_{A\cup B}) = (\mu_A \vee \mu_b, \nu_A \wedge \nu_B)$ is an intuitionistic set of L, and

$$\mu_{A\cup B}(x) = \begin{cases} 1, & x = 0, a, b \\ s, & x = 1 \end{cases}$$

$$\nu_{A\cup B}(x) = \begin{cases} 0, & x = 0, a, b \\ t, & x = 1 \end{cases}$$

But $\mu_{A\cup B}(1 \to 0)' = \mu_{A\cup B}(1) = s$, and $\mu_{A\cup B}(a \to 0)' = \mu_{A\cup B}(a) = 1$, $\mu_{A\cup B}((1 \to a)' \to 0)' = \mu_{A\cup B}(a) = 1$, that is

$$\mu_{A\cup B}(1 \to 0)' < \min\{\mu_{A\cup B}(a \to 0)', \mu_{A\cup B}((1 \to a)' \to 0)'\},$$

this contradicts the Definition 4 (IFI2), we have $A \cup B$ is not an $IFI-$ideal of L.

In the following, we discuss the $IFI-$ideal in lattice H implication algebras.

Proposition 21. [17]. *In a lattice H implication algebra L, $\forall x, y, z \in L$, $(x \to y)' \to z = (x \to z)' \to (y \to z)'$.*

Theorem 22. *Let L be a lattice H implication algebra. If $A = (\mu_A, \nu_A)$ is an $IF-$ideal of L, then A is an $IFI-$ideal of L.*

Proof. By Proposition 21, we have $\forall x, y, z \in L$,

$$\mu_A((x \to y)' \to z)' = \mu_A((x \to z)' \to (y \to z)')',$$

$$\nu_A((x \to y)' \to z)' = \nu_A((x \to z)' \to (y \to z)')'.$$

and by $A = (\mu_A, \nu_A)$ is an $IF-$ideal of L, we have

$$\begin{aligned} & \mu_A(x \to z)' \\ \geqslant\ & \min\{\mu_A(y \to z)', \mu_A((x \to z)' \to (y \to z)')'\} \\ =\ & \min\{\mu_A(y \to z)', \mu_A((x \to y)' \to z)'\}, \\ & \nu_A(x \to z)' \\ \leqslant\ & \max\{\nu_A(y \to z)', \nu_A((x \to z)' \to (y \to z)')'\} \\ =\ & \max\{\nu_A(y \to z)', \nu_A((x \to y)' \to z)'\}. \end{aligned}$$

So we have A is an $IFI-$ideal of L by Definition 4.

Corollary 23. *Let L be a lattice H implication algebra. $A = (\mu_A, \nu_A)$ is an $IF-$ideal of L if and only if A is an $IFI-$ideal of L.*

Proof. It can be obtained by Theorem 3 and Theorem 22.

Theorem 24. [17]. *L is a lattice H implication algebra if and only if $A = \{0\}$ is an implicative ideal of L.*

Theorem 25. *L is a lattice H implication algebra if and only if $\forall \alpha \in [0,1], A = (\mu_{0,\alpha}, \overline{\mu_{0,\alpha}})$ is an $IFI-$ideal of L.*

Proof. "$\Rightarrow$" It can be obtained by Theorem 16 and Theorem 22.
"$\Leftarrow$" It can be obtained by Theorem 15 and Theorem 24.

Theorem 26. *Let L_1 and L_2 be two lattice implication algebras, the mapping $f : L_1 \to L_2$ is a lattice implication homomorphism.*
(1) if $B = (\mu_B, \nu_B)$ is an $IFI-$ideal of L_2, then $f^{-1}(B)$ is an $IFI-$ideal of L_1.
(2) if $A = (\mu_A, \nu_A)$ is an $IFI-$ideal of L_1 and f is an isomorphism, then $f(A)$ is an $IFI-$ideal of L_2.

Proof.(1) since f is a lattice implication homomorphism, we have $f(0) = 0$.
$\forall x, y, z \in L_1$,

$$\begin{aligned} & f^{-1}(\mu_B)(x) = \mu_B(f(x)) \\ \leqslant\ & \mu_B(0) = \mu_B(f(0)) = f^{-1}(\mu_B)(0); \\ & f^{-1}(\nu_B)(x) = \nu_B(f(x)) \\ \geqslant\ & \nu_B(0) = \nu_B(f(0)) = f^{-1}(\nu_B)(0); \end{aligned}$$

$$\begin{aligned} & f^{-1}(\mu_B)(x \to z)' = \mu_B(f(x \to z)') \\ =\ & \mu_B(f(x) \to f(z))' \\ \geqslant\ & \min\{\mu_B(f(y) \to f(z))', \mu_B((f(x) \to f(y))' \to f(z))'\} \\ =\ & \min\{\mu_B(f(y \to z)'), \mu_B(f((x \to y)' \to z)')\} \\ =\ & \min\{f^{-1}(\mu_B)(y \to z)', f^{-1}(\mu_B)((x \to y)' \to z)'\}, \end{aligned}$$

and $f^{-1}(\nu_B)(x \to z)' \leqslant \max\{f^{-1}(\nu_B)(y \to z)', f^{-1}(\nu_B)((x \to y)' \to z)'\}$ can be proved analogously. By Definition 4, we have $f^{-1}(B)$ is an $IFI-$ideal of L_1.

(2) Considering the mapping $f^{-1} : L_2 \to L_1$ and the conclusion is obtained by the result of (1).

4. Conclusion

The intuitionistic fuzzy set has become a considerable formal tool to deal with fuzzy information in the real world. In this paper, we apply the intuitionistic fuzzy sets to the implicative ideals of lattice implication algebras and propose the notion of $IFI-$ideals in lattice implication algebras. Then we investigate the equivalent conditions of $IF-$ideals and $IFI-$ideals and discuss the relations of various $IFI-$ideals. Finally, we prove that $\forall\alpha \in [0,1], A = (\mu_{0,\alpha}, \overline{\mu_{0,\alpha}})$ is an $IFI-$ideal of lattice implication algebra L if and only if L is a lattice H implication algebra. We hope the above work would supply a foundation for further application of the intuitionistic fuzzy sets in lattice implication algebras.

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