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A Mechanism to Improve the Interpretability of Linguistic Fuzzy Systems with Adaptive Defuzzification based on the use of a Multi-objective Evolutionary Algorithm

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Abstract

This paper proposes a mechanism that helps improve the interpretability of linguistic fuzzy ruled based systems with common adaptive defuzzification methods. Adaptive defuzzification significantly improves the system accuracy, but introduces weights associated with each rule of the rule base, decreasing the system interpretability. The suggested mechanism is based on three goals: 1) reducing the number of total rules considering that rule weight close to zero can be removed; 2) reducing the rules with weights coupled because rules with weights close to one do not need the weight, and 3) reducing rules triggered jointly, all of them by using several metrics and a proposed interpretability index. This is performed using a multi-objective evolutionary algorithm, obtaining a set of solutions with different trade-offs between accuracy and interpretability. In addition, it is important to note that adaptive defuzzification and therefore the proposal developed in this work can be used together with other methodologies to improve system interpretability and accuracy, so it can be viewed as an interesting component.

Keywords: Linguistic fuzzy modelling, interpretability-accuracy trade-off, multi-objective genetic algorithms, adaptive defuzzification methods.

1. Introduction

Fuzzy logic introduced by Zadeh[1] is well-known by its suitability for linguistic concept modelling and its use in system identification. The semantic expressivity of fuzzy logic, using linguistic variables[2] and linguistic rules[3], is quite close to expert natural language. Therefore, the use of fuzzy logic in system modelling favours the interpretability of the final model, at least from the structural transparency viewpoint. Fuzzy modelling[4], i.e., system modelling with fuzzy rule base systems (FRBSs), is an important and active research line within the fuzzy logic community.

Accuracy and interpretability are in general terms contradictory goals: It is usually assumed that the more complex the FRBS is, the lower its interpretability, which means that complexity is implicitly considered to be related with a lack of interpretability. The ideal objective of fuzzy modelling would meet both criteria to a great extent, but the aforesaid opposition between both targets has shifted the problem to finding a balance between them [5-13].

To summarize, two main trends are found regarding the improvement of the accuracy–interpretability trade-off in the context of fuzzy systems[5]:

- Linguistic fuzzy modelling (LFM), focused on the interpretability and then trying to improve the system accuracy[6].
- Precise fuzzy modelling (PFM), which focuses on the accuracy and then attempts to achieve better system interpretability[7].

A promising way is the LFM approach since it allows us to deal with the modelling of systems by building a linguistic model as starting point, which is clearly interpretable by humans. Concentrating then on LFM, historically at the researchers have usually initially focused on improving the accuracy of the models obtained without paying special attention to

interpretability[6,14-20]. Nowadays, the interest of the researchers in interpretability has grown; they deal with improving the accuracy, trying to avoid decreases in interpretability of the system, and in general terms, looking for a good balance[5,21-29] between both features.

Considering the interpretability and due to its subjective nature and the large amount of factors involved, the choice of appropriate measures is still an open problem[21,28,30-34]. In the specialist literature, there are proposal of different measures[27,28,30-34] and techniques[8-10,22,23,35-37] for obtaining more interpretable linguistic fuzzy models. There are also different proposal for taxonomies[30-33] to help better understand better the interpretability matter.

On the other hand, the adaptive inference system and especially the adaptive defuzzification methods have shown to be two important elements that could easily improve the accuracy of the system[14-16]. They are not the most relevant way to improve the accuracy, but they are well-matched with most other methodologies to do so. However, adaptive defuzzification methods introduce a loss of interpretability[15,30-33], with implications for the overall meaning[15,32].

Multi-objective evolutionary algorithms[38,39] (MOEAs), have been shown to be a particularly interesting instrument to deal with the trade-off between accuracy and interpretability, due to the multi-objective nature of the problem [21-25,35,40]. They present in a single step a set of solutions with different equilibrium between the two contradictory features, letting the engineers select the most satisfactory for their purposes in each situation, which could also change, and it is not necessary to re-run the algorithm to seek another more appropriate solution.

Consequently, due to the interest of adaptive defuzzification methods from the point of view of accuracy, and bearing in mind their aforementioned drawback regarding interpretability issues, it was considered interesting to work on their accuracy-interpretability trade-off. To do so, this paper proposes to introduce a mechanism to improve the interpretability when using adaptive defuzzification method that reduces the number of global rules of the FRBS, decreases the number of rules with an associated weight and also reduces the number of rules triggered simultaneously. A MOEA is used to obtain a set of linguistic fuzzy models more interpretable than usual with adaptive defuzzification and still accurate by using both objectives related with interpretability and accuracy through several measures and a specifically designed interpretability index.

The paper is structured as follows: in section 2 we review the state of the art of the use of MOEA in interpretable linguistic FRBS modelling, and a description of adaptive defuzzification methods, their components, effects and recent works related on their interpretability. Section 3 begins with the description of the foundations of the mechanism proposed to improve the interpretability, and later shows two multi-objective models. In Section 4, two experimental studies are carried out to show the usefulness of the two models on thirteen real world problems. Finally, section 5 presents the conclusions of the studies performed.

2. Preliminaries

This section helps situate the presented work in the framework of the FRBS, and particularly with the previous works on the main streams found in the literature about two points: the use of MOEA in the trade-off between accuracy and interpretability, and the adaptive defuzzification methods and their issues in interpretability. We have not included preliminaries on interpretability here because they can be found in Refs. 30, 31 and 33, so we encourage the reader to review them if needed.

2.1. *Accuracy-interpretability trade-off and MOEAs*

As commented in the Introduction section, since accuracy and interpretability are conflicting goals, the use of evolutionary multi-objective[38, 39] strategies to find some points of equilibrium between the two features has become very popular in the linguistic FRBSs modelling field[21]. This subsection is devoted to a review of some important works from the literature on MOEAs in the FRBS regarding the trade-off between interpretability and accuracy, following a line based on concepts together with historical evolution.

Ishibuchi has carried out some research on the application of MOEAs to the linguistic FRBS design, applied to classification problems. His earlier works[9] were devoted to using simple first-generation MOEAs to perform a rule selection on an initial set of classification rules involving “don’t care” conditions and considering two different objectives: classification accuracy and number of rules. The two-objective

formulation in Ref. 9 was extended to three objectives in Ref. 41 by introducing the total number of antecedent conditions (i.e., the total rule length) as an additional complexity measure. A more effective MOEA, i.e., the multi-objective genetic local search (MOGLS) was used for three-objective genetic fuzzy rule selection in Ref. 42. An additional interesting study on the use of several multi-objective strategies in classification problems was discussed in Ref. 24.

Another contribution in this framework previously commented, and also applied to classification problems, is on Ref. 43, where Cordon et al. use a classical MOEA, (MOGA), to perform feature selection and fuzzy set granularity learning jointly with only two objectives.

On the other hand, there are only a few works in the framework of fuzzy modelling for regression problems. Next, we highlight some of the most significant papers on regression and MOEAs.

Ishibuchi in Ref. 44 show how a simple MOEA can be, applied to a three-objective optimization problem to obtain Mamdani FRBSs for regression problems.

Another interesting study is described in Ref. 29 where the authors present an adaptation of the efficient (2+2) PAES for the identification of Mamdani FRBSs for regression problems by considering the minimization of two objectives (the system error and number of variables involved in the antecedents of the rules).

Later, some papers considered the use of MOEAs for learning or tuning of membership functions for regression problems. Thus, in Ref. 22 the authors used MOEAs in order to improve the fuzzy model accuracy while keeping its interpretability regarding both membership functions and fuzzy rules. Furthermore, the authors of Ref. 23 used MOEAS and a process for rule reduction jointly with a novel approach consisting of considering a new linguistic rule representation model based on the linguistic 2-tuples representation to perform a genetic lateral tuning of membership functions[35]. Moreover, in Ref. 27, another MOEA has been adopted to perform context adaptation (adaptation of the membership functions by using scaling functions) as a post-processing algorithm applied to an initial knowledge base.

Recently, Ref. 25 has used a MOEA for granularity, membership function and rule learning. Furthermore, Ref. 40 improves the fuzzy model accuracy preserving the interpretability promoting the cooperation between fuzzy operators and rule base, also based on multi-objective strategies.

Notice that usually only two objectives are considered in the majority of the works. One of these usually concerns the accuracy which can easily be defined by checking how similar the outputs of the model and the real system are, for instance using the mean squared error. Nevertheless, some problems arise to characterize the second objective, interpretability, which is still an open problem.

2.2. *Adaptive defuzzification methods*

This subsection concisely revises the fundamentals of the main adaptive defuzzification methods used on fuzzy modelling.

Following the studies developed in Ref. 15, most adaptive defuzzification methods follow the expression (1),

$$y_0 = \frac{\sum_i^N f(h_i) \cdot V_i}{\sum_i^N f(h_i)}, \tag{1}$$

where h_i is the matching degree between the input variables and the rule antecedent fuzzy sets, so $f(h_i)$ is a functional of matching degree, and V_i represents a characteristic value of the fuzzy set inferred from rule R_i, the Maximum Value (MV) or the Centre of Gravity (CG). As shown, it is an expression of defuzzification method acting in Mode B, i.e. it defuzzifies first the individual contribution of each rule and then the final result is computed utilizing a weighted sum.

The functional term can take different arrangements, but the most frequent is the product: $f(h_i) = h_i \cdot \alpha_i$, where α_i corresponds to a parameter for each rule R_i, i = 1 to N, as well as the Centre of Gravity (CG) as characteristic value, due to its computational efficiency, linearity and similar results to other options for the functional term[15]. The final expression of the adaptive defuzzification method used in this paper is shown in (2).

$$y_0 = \frac{\sum_i^N h_i \cdot \alpha_i \,.\, CG_i}{\sum_i^N h_i \,.\, \alpha_i}, \tag{2}$$

The effect of α_i zone is as follows:

$\alpha_i \cdot h_i$, $\alpha \in [1,\infty)$:empowerment of h_i, (3)

$\alpha_i \cdot h_i$, $\alpha \in [0,1]$: penalty of h_i.

In practice, the interval for α_i used is usually restricted to [0,1] in order to act only as a penalty in different levels, obtaining similar results.

The product functional term, which employs a different parameter for each rule, has an effect similar to weighted rules[45]. The following is an example of a set of weighted rules, where the weights are α_i :

R_1 : If X_{11} is A_{11} and … and X_{1m} is A_{1m} then Y is B_1 with α_1
R_2 : If X_{21} is A_{21} and … and X_{2m} is A_{2m} then Y is B_2 with α_2
…
R_n : If X_{n1} is A_{n1} and … and X_{nm} is A_{nm} then Y is B_n with α_n

The α_i values associated with rule R_i acquires the meaning of how significant or important that rule is for the inference process.

The result of the use of rule weights in system modelling is a significant improvement of accuracy[15]. The rule weight adaptation is carried out frequently by using an evolutionary algorithm. The process of adaptation also produces a rule subset with better cooperation among the rules composing it[15], due to rules with high level of cooperation among them get high values of weights and on the contrary, the rest of the rules obtain values close to zero.

From the point of view of the influence of adaptive defuzzification on the interpretability of fuzzy systems, this is an issue occasionally discussed. On one hand, Nauck and Kruse[46] consider that rule weights could be equivalently replaced by modifications in the membership functions in the antecedents or consequents part, and they consider this a severe negative aspect in the semantic interpretability of the FRBSs.

Nevertheless, on the other hand, Ishibuchi and later Nakashima analysed the importance of weights viewed as *certainty grades*[47]. They showed that compact FRBSs can be designed *without adjusting membership functions* and can be partially replaced by the adjustment of the aforementioned *certainty grades*. They consider that the comprehensibility of the model viewing weights as *certainty grades* is not deteriorated significantly. In this way, the rule weight can be interpreted as the strength of each rule. They show that the larger the rule weight is, the large the decision area of each rule is. This approach allows a better approximation of interpretability working with rule weights because it retains the linguistic meaning of the data base.

3. A new Proposal to Improve the Interpretability of Adaptive Defuzzification

This section provides a detailed description of the mechanism to achieve interpretability improvements for FRBSs with adaptive defuzzification methods.

We begin by describing the mechanism, its metrics and interpretability indexes to set up a basic model and later, in subsection 3.2, an improved adaptive model also based on the same principles.

3.1. *Description of the proposed mechanism*

In this subsection, we shall describe the elements of the mechanism to improve the interpretability and several metrics to measure it when an adaptive defuzzification method is used in a linguistic FRBS.

The necessary reduction of rules with weight from the adaptive defuzzification[30-33,45], the importance of rule selection[8-12,42,44,48] and the interest in the reduction of rules triggered together[49], all in order to reduce the system complexity and favour the compactness of the rule base, are the lines followed in our proposal.

3.1.1. Mechanism and metrics

We first describe the two threshold mechanisms, and then the reduction of rules with weight and rules triggered together.

A. Mechanism of global rule selection based on a low threshold

As mentioned in subsection 2.2, adaptive defuzzification methods are equivalent to using rules with weights. Therefore, rules with weights close to *zero* are those with the lowest influence on the final output. Hence, we propose avoiding the use of rules with the lowest weight values, removing those rules during the learning process of the weights. In this way, the effect of their contribution, even though it is small, is taken into account for the rest of the system.

This idea is applied establishing a threshold, T_L (*threshold low*) that defines the cut-off level to remove rules. In order to measure its effect, we propose to use the usual metric:

- Number of final rules, ($\#R_F$)

Its expression is:

$$\text{Minimize } \#R_F = \#R - \#(\text{rules with weights under } T_L) \quad (4)$$

with #R as the number of rules of the rule base (initially), before starting the defuzzification parameter learning process, and *#(rules with weights under T_L)* the number of rules whose weight is close to *zero*.

The rule selection improves the system interpretability, because the *high-level interpretability*[31] and *complexity of the system at rule base level*[33] are then improved. The *description interpretability*, a concept introduced in Ref. 32 regarding the system structure readability, is enhanced.

Rule selection can also enhance the system accuracy[8-11,23,34] because it improves the cooperation between the rules of the rule base.

B. Mechanism to reduce the number of rules with weights based on a high threshold

Weight values close to *one* belong to the most important rules for the FRBS. For this reason, we consider that these rules could be used without weights, and so we propose removing theses values during the learning process of the defuzzification parameters by using a new second threshold, T_H (*threshold high*). The simply metric proposed to connect with this concept is the following:

- Number of rules with weight, ($\#R_W$)

It measures the number of rules with a weight associated of the rule base. Its expression is as follows:

$$Minimize\ \#R_W = \#R - \#(rules\ with\ weights\ above\ T_H) \quad (5)$$

with #R being the initial number of rules of the rule base, and *#(rules with weights above T_H)* the number of rules whose weight is close to *one.*

This technique reduces the complexity, lessens the impact on the *semantic interpretability* of the adaptive defuzzification[46,33] and improves the *explanation interpretability*[32] which is related with the systems' comprehension of the model.

C. Mechanism to reduce the average number of rules triggered together:

Despite the fact that FRBSs obtain the output by combining the contribution of several rules triggered together with different levels, the system is more interpretable the lower the number of them triggered at the same time is [49].

To take this fact into account, we shall use the following metric:

- Average number of rules triggered by each example, (AvR_{TG})

Its expression is the following:

$$Minimize\quad AvR_{TG} = \frac{\sum_{j}^{M} R^{j}{}_{TG}}{P} \quad (6)$$

where P is the number of examples and $\#R^j{}_{TG}$ is the number of rules triggered by the example j.

Thus, reducing AvR_{TG}, the *rule base level*[31] and the *semantic interpreatability*[33] are enhanced, as well as the *explanation interpretability*[32].

3.1.2. An interpretability index

In the present work, we propose the aggregation of two metrics, $\#R_W$ and AvR_{TG}, in a global index based on the arithmetic mean of them, which is denoted as $R_W\_AvR_{TG}$.

To calculate the final expression of the new index (7), we first normalize both metrics to the range of 0 to 1. Then, the value of $R_W\_AvR_{TG}$ ranges between 0 (the highest level of interpretability) and 1 (the lowest).

$$Minimize\quad R_W\_AvR_{TG} = \frac{\#R_W + AvR_{TG}}{2} \quad (7)$$

The interpretability index defined is based on two metrics which perform in the semantic[33] and explanation[32] interpretability as was commented in subsection 3.1.1 when they were introduced. The minimization of this index recovers the interpretability lost due to by the effect of the adaptive defuzzification, while enhancing that of the global system, which could have a margin of improvement due to the learning effect or optimization of the knowledge base.

3.2. *Evolutionary multi-objective models proposed*

This subsection describes the two evolutionary models proposed that use a multi-objective algorithm with the mechanism to improve the interpretability for adaptive defuzzification systems described in the previous subsection.

3.2.1. A first model: Fixed Thresholds

The first model presented learns the parameters of the adaptive defuzzification, using the interpretability improvement mechanism described above with fixed

thresholds and optimizing several objectives based on the metrics and index previously cited.

The MOEA

In this paper, an evolutionary model based on the popular NSGA-II[50] was used. In the following, we state its main components and parameters.

Objectives

In a previous work[51], we used the minimization of the $\#R_F$ and $R_W\_AvR_{TG}$ as an objective, but in the current study we only use the index $R_W\_AvR_{TG}$. This is due to the fact that $\#R_F$ will be also minimized when minimizing the interpretability index, because it includes the R_W and uses the T_L mechanism to reduce the number of rules (section 3.1.1), so when minimizing the R_W through the interpretability index, the $\#R_F$ will be also minimized.

Therefore, we will use the following two objectives:

- Interpretability maximization: Minimizing the index of interpretability, $R_W\_AvR_{TG}$.
- Accuracy maximization: Minimizing the classical Mean Square Error (MSE), defined in expression (8),

$$MSE\ (S) = \frac{\frac{1}{2}\sum_{k=1}^{P}(y_k - S\ (x_k))^2}{P} \qquad (8)$$

where *S* denotes the fuzzy model using the adaptive defuzzification method shown in expression (2), and the minimum t-norm as conjunction and inference operators. This measure uses a set of system evaluation data formed by P pairs of numerical data $Z_k = (x_k, y_k)$, k=1,..,P, with x_k being the values of the input variables, and y_k being the corresponding values of the associated output variables.

Coding scheme and initial population

We use a real coding scheme, where *n* is the number of parameters α_i (of the adaptive defuzzification, one for each rule of the rule base R_i), whose values are within the interval [0,1].

$$C = (\alpha_1, \ldots, \alpha_n) \mid \alpha_i \in \{0, 1\}$$

The initial population is set up as follows: An individual of the initial population has all the genes initially set to '1' in order to begin the evolutionary process with all the rules without weight. The remaining individuals of the initial population are set randomly.

NSGA-II based multi-objective genetic algorithm

NSGA-II[50] is one of the most widely known and used second generation MOEA in the literature for solving multi-objective problems. It generates the offspring population from the current population through selection, crossover and mutation. It builds the next generation from the current population and the offspring until it reaches the stop condition, which in this work is based on the number of evaluations. The NSGA-II algorithm has two features that make it one of the main and most important MOEA: One is the assignment of fitness based on the Pareto ranking and crowding operator and the other is the procedure for updating each generation through elitism.

In this work, we have employed a MOEA based on the original NSGA-II with some adaptations. The proposed algorithm makes use of the NSGA-II selection mechanism, but in order to improve the search ability of the algorithm for our application, the searching effort is concentrated in a more interesting and reduced zone of the Pareto, the density of the obtained solutions being higher in this zone.

To do so, in each stage of the algorithm, we force the population to have a number of individuals dominated by error as a basis for the selection of the next generation, and make this number smaller as the algorithm progresses by reducing the number of required dominated solutions from 80% to 0% progressively with the generations, or what is the same, increasing the number of non-dominated solutions allowed from 20% to 100% gradually. Then, the solutions non-dominated are sorted from the best to the worst (considering accuracy as criterion).

Important issues related with implementation

This subsection raises two questions related with the implementation.

The first is related with the completeness[52] of the rule bases obtained with this methodology; to ensure this property when removing rules from the rule base, we do not delete a rule if that rule is the only one to cover a specific example, since the effect on the FRBS is negative[53, 54].

The second issue is related with the implementation of the thresholds and the search ability of the

mechanism: during the evolutionary process, the value of the chromosome could involve removing a rule, or eliminating the weight of a rule, but the original value is maintained within the chromosome in order to avoid affecting the crossover operator and evolutionary process.

3.2.2. A Second Model: Adaptive Thresholds

In this subsection, we propose a new approach based on the use of self-adapted thresholds instead of preset ones, in the same process of defuzzification parameters learning. In this way, the multi-objective learning process will obtain a set of FRBSs with different trade-off between accuracy and interpretability by using not only the defuzzification parameters but also different thresholds.

The adaptive thresholds mechanism

We propose to adapt the available range for the adaptive defuzzification parameters using a single parameter β_T for the whole system, whose value is the width and is centred in the middle of the interval, as is shown in Figure 1. The use of a single parameter to carry out the adaptation of both thresholds, T_H and T_L, jointly as proposed versus the use of two parameters representing the thresholds is less flexible, but reduces the search space of the evolutionary algorithm.

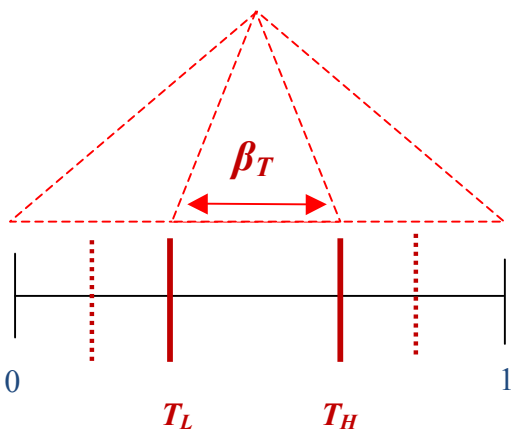

Fig. 1. Description of the parameter to adapt the thresholds

The MOEA

The new MOEAs have the same characteristics as the first proposal described. The difference is that this approach uses a double real coding scheme: the first part belongs to the parameters of the adaptive defuzzification (C_D) and the second part is used for the parameter of the adaptive thresholds (C_T). Then, the resulting chromosome is $C = C_D + C_T$.

The new C_T part is also real coded, and is composed of a single gene: $C_T = \beta_T \mid \beta_T \in [0.30, 1]$. This interval lets the thresholds be between T_L=0 and T_H=1, and T_L=0.35 and T_H=0.65. It has been selected empirically performing different tests.

4. Experimental Study

To analyse the practical behaviour of the proposed methodology, an experimental study was carried out, divided into two parts. The first focused on the first model and the second on the adaptive thresholds model, with thirteen problems of varying complexities (different numbers of variables and available data). Table 1 summarizes the main characteristics of the thirteen datasets selected from the KEEL project[55] webpage (http://www.keel.es) where they can be downloaded.

Table 1 Data sets considered for the experimental study

Datasets	Name	Variables	Patterns
Plastic Strength	PLA	3	1650
Quake	QUA	4	2178
Electrical Maintenance	ELE	5	1056
AutoMPG6	AU6	6	392
AutoMPG8	AU8	8	392
Anacalt	ANA	8	4052
Abalone	ABA	9	4177
Concrete	CON	9	1030
Stock prices	STP	10	950
Ankara Weather	WAN	10	1609
Izmir Weather	WIZ	10	1461
Mortgage	MOR	16	1409
Treasure	TRE	16	1409

4.1. *Common set-up of experiments*

Two algorithms were used to obtain the initial set of candidate linguistic rules: the well-known ad-hoc data-driven algorithm of Wang and Mendel [56] (WM) and the first stage of MOGUL-IRL[52], which we name FS_MOGUL. No other methods from the literature were used, due to the problems they present when using a high number of variable datasets like the ones used in this work (see the lower zone of Table 1). In this paper, the linguistic partitions consist of five linguistic terms in

the case of datasets with less than nine variables and three linguistic terms in the remaining ones (which helps to obtain a more reasonable and compact number of rules in the main datasets).

We adopted a 5-fold cross-validation model, i.e. 5 random partitions of the data each with 20% of the patterns of the data set, and used four folds for training and one for testing. For each of the data partitions, the learning methods were run 6 times using different seeds for the random number generator. For each data set, we therefore consider the average results of 30 runs.

The average results of the initial reference FRBSs obtained with WM and FS_MOGUL are shown in Tables 2 and 3.

Table 2 Initial Results using WM rule base

Name	#R	MSE_{TRA}	MSE_{TST}
PLA	14.80	3.434	3.557
QUA	53.60	0.0258	0.0267
ELE	65	56135	56359
AU6	116.00	4.338	6.819
AU8	70.60	12.71	13.73
ANA	72.40	0.187	0.189
ABA	68.20	8.407	8.424
CON	135.40	91.176	94.190
STP	122.80	9.074	9.042
WAN	156.00	16.063	16.403
WIZ	104.80	6.945	7.139
MOR	77.60	0.985	0.973
TRE	75.00	1.636	1.632

Table 3 Initial Results using the first stage of FS_MOGUL rule base

Name	#R	MSE_{TRA}	MSE_{TST}
PLA	75.40	5.246	5.262
QUA	227.60	0.0633	0.0648
ELE	88.80	129400	133564
AU6	170.60	8.565	13.154
AU8	78.80	21.47	22.07
ANA	211.20	0.195	0.200
ABA	50.20	24.790	24.720
CON	124.40	163.157	164.638
STP	45.60	16.745	16.906
WAN	33.40	56.898	58.880
WIZ	52.40	38.414	41.442
MOR	31.40	2.002	1.995
TRE	33.00	2.667	2.685

In the case of multi-objectives approaches, we adopt the use of the three representative points of the accuracy-interpretability objectives, as in Ref. 28 and 57: the most interpretable (MAX INT), the median (MEDIAN INT/ACC) and the most accurate in training (MAX ACC) points. This methodology was used for two-objective problems in Ref. 57. Then, in Ref. 28, it was extended to problems with more than two objectives by projecting the obtained Pareto fronts in the planes generated by considering pairs of objectives. In this way, the non-dominated solutions can be analyzed by considering the aforesaid interesting points for each pair of objectives (each projected plane).

Therefore, for each representative point, we computed the mean values over the 30 trials of the MSEs on the training and test sets (MSE_{TRA} and MSE_{TST}), the $\#R_F$ (although it is not an objective of the MOEA) and $R_W\_AvR_{TG}$ index.

For the single-objective based approaches, we compute the same mean values over the 30 solutions obtained for each dataset. These three points are representative positions on the MSE - $R_W\_AvR_{TG}$ plane, and were only considered in order to perform a statistical analysis. Anyway, the final user could select the most appropriate solution from the final Pareto front, by also looking for a trade-off between MSE, R_F and $R_W\_AvR_{TG}$ depending on their own preferences.

In some cases, particularly with the second model, it was necessary to use another measure to capture the total MOEA performance[38,58]. We selected the Two Set Coverage[58] (CS) ratio as a tool to compare the Pareto fronts of different multi-objective approaches. CS considers X', X'' ⊆ X' as two sets of phenotype decision vectors and a' and a'' are two points belong to sets X' and X'', respectively. CS is defined as the mapping of the order pair (X', X'') to the interval [0, 1] per equation (9).

$$CS(X',X'') = \frac{|\{a''\in X'';\exists a'\in X': a'\geq a''\}|}{|X''|}, \qquad (9)$$

If all points in X' dominate or are equal to all points in X'', then by definition CS=1. CS=0 implies the opposite. In general CS (X',X'') and CS(X'',X'), both have to be considered due to set intersection not being empty. The advantage of this metric is that it is easy to calculate and provides a relative comparison between MOEAS.

To assess whether there are significant differences among the results, we adopted statistical analysis[59-62] and in particular non-parametric tests, according to the recommendations made in Ref. 61. In particular, for pair-wise comparison we use the Wilcoxon signed-rank test[63,64], and for multiple comparison we shall employ

different approaches, including Friedman's test[65], to detect statistical differences among a group of results, and the Finner post-hoc test[66] to observe the difference in performance between the methods and the retention or rejection of the hypothesis with the level of significance fixed. To perform the tests, we used a level of confidence $\alpha = 0.05$. In particular, these tests are based on computing the differences on sample means (typically, mean test errors obtained by a pair of different algorithms on different datasets). In the classification framework, these differences are well defined since the errors are in the same domain. In the regression framework, to make the differences comparable, we adopt the normalized difference proposed in Ref. 57 and 28, namely DIFF and defined as:

$$DIFF = \frac{Mean\,(other) - Mean\,(reference)}{Mean\,(other)} \qquad (10)$$

where Mean(x) represents the MSE or $\#R_F$ (it is not necessary in the case of $R_W\_AvR_{TG}$) obtained by the x algorithm. This difference expresses the improvement percentage of the reference algorithm on the other one.

4.2. *First part of the experimental study*

In the first part of the experimental study we analyse the practical behaviour of the first model.

A. Particular experimental set-up

The FRBSs considered for this first part of the experimental study are summarized in Table 4, i.e. we are proposing the multi-objective approach with the interpretability improvement MO-AD_I, but we also use two single-objective approaches: SO-AD which is the standard adaptive defuzzification[15] accuracy oriented, and SO-AD_I which is the same, but adding the mechanisms proposed to improve the interpretability.

Table 4 Methods considered for comparison

FRBSs	Description
	Single-Objective FRBSs
SO-AD	Adaptive Defuzzification[15]
SO-AD_I	Adaptive Defuzzification with the mechanism to improve interpretability
	Multi-Objective FRBSs
MO-AD_I	Adaptive Defuzzification with the mechanism to improve interpretability

Two different things will be studied in this first part of the experimental study: the influence of the values of the different thresholds for the interpretability improvement mechanism introduced first (subsection B), and then we will compare the single and the multi-objective approaches (subsection C).

In the case of the studied adaptive defuzzification approaches, the values of the input parameters considered by the single-objective (SO-AD and SO-AD_I) methods are: population size of 61, 200000 evaluations, 0.6 as crossover probability and 0.2 as mutation probability per chromosome. In the case of the MOEAs, (MO-AD_I), they are: population size of 200 individuals, external population size of 200 individuals, 200000 evaluations, and 0.2 as mutation probability.

B. Analysis of the influence of different values for the thresholds

This section shows the results and analyses the proposed algorithm (MO-AD_I) to evaluate the behaviour of the different thresholds values. Therefore, we analyse the use of the multi-objective approach with different thresholds in order to study the influence of these values on the interpretability and accuracy criteria. To do this, we have chosen three fixed thresholds configurations:

- T_L=0.1 and T_H=0.9
- T_L=0.2 and T_H=0.8
- T_L=0.3 and T_H=0.7

Then, the multi-objective approaches were denoted by MO-$AD_{I(0.1\text{-}0.9)}$, MO-$AD_{I(0.2\text{-}0.8)}$, MO-$AD_{I(0.3\text{-}0.7)}$, respectively.

Tables 5 and 6 show the three representative points for the different configurations (different thresholds, using the rule bases of WM and FS_MOGUL).

All the configurations have to be compared in order to determine which of them should be preferred. Since we will compare together more than two algorithms, on this occasion we use non-parametric tests for multiple comparisons. Particular details of the study can be consulted in Appendix A, where in order to perform a multiple comparison, it was necessary to check whether any of the results obtained by the algorithms presented any inequality. In the event of finding some, we can know by using a post-hoc test which algorithms partners' average results were dissimilar.

- Examining the results of the statistical analysis join to Tables 5 and 6, we can conclude the following points:

Table 5. Results obtained by MO-AD_I for different configurations of thresholds using the WM rule base

		MAX $_{INT}$						MEDIAN ($_{INT / ACC}$)						MAX $_{ACC}$					
Dataset	MO-AD_I T_L - T_H	MSE_{tra}	MSE_{tst}	$\#R_F$	$R_W\_AvR_{TG}$	($\#R_W$	AvR_{TG})	MSE_{tra}	MSE_{tst}	$\#R_F$	$R_W\_AvR_{TG}$	($\#R_W$	AvR_{TG})	MSE_{tra}	MSE_{tst}	$\#R_F$	$R_W\_AvR_{TG}$	($\#R_W$	AvR_{TG})
PLA	0.1 - 0.9	6.947800	6.940028	6.1	0.128	(0.00	1.02)	2.073691	2.091053	10.6	0.385	(2.23	2.48)	1.699226	**1.759935**	14.0	0.771	(10.80	3.25)
	0.2 - 0.8	7.042636	7.042633	6.0	0.125	(0.00	1.00)	2.237865	2.245128	10.6	0.346	(1.37	2.40)	1.769444	1.875019	13.9	0.675	(8.50	3.11)
	0.3 - 0.7	7.042636	7.042633	6.0	0.125	(0.00	1.00)	2.543587	2.545364	10.4	0.309	(0.60	2.31)	1.830941	1.900387	12.4	**0.551**	(6.13	2.75)
QUA	0.1 - 0.9	0.023570	0.024276	22.7	0.179	(4.93	2.12)	0.020914	0.022153	27.2	0.287	(10.60	3.01)	0.020675	**0.021944**	32.6	0.448	(22.67	3.79)
	0.2 - 0.8	0.024555	0.025273	19.9	0.138	(1.23	2.02)	0.021178	0.022430	25.6	0.237	(6.67	2.80)	0.020782	0.022043	32.1	0.416	(20.73	3.57)
	0.3 - 0.7	0.026361	0.027337	17.7	0.114	(0.13	1.80)	0.021700	0.022975	24.7	0.193	(3.13	2.62)	0.020897	0.022139	31.4	**0.380**	(17.37	3.49)
ELE	0.1 - 0.9	57867	60608	35.8	0.167	(7.37	3.10)	35428	39094	38.6	0.285	(16.30	4.48)	32518	**35633**	45.2	0.469	(33.27	5.96)
	0.2 - 0.8	83303	84897	27.2	0.082	(1.10	2.06)	44049	47997	34.0	0.159	(6.30	3.11)	32947	35747	45.3	0.440	(29.23	6.02)
	0.3 - 0.7	143152	150408	20.0	0.063	(0.03	1.77)	64459	70994	27.6	0.086	(1.17	2.16)	33433	35677	45.4	**0.421**	(25.97	6.19)
AU6	0.1 - 0.9	3.487613	6.568578	84.6	0.214	(6.60	6.07)	2.746415	6.482094	73.4	0.293	(28.57	5.57)	2.568845	6.457802	73.6	0.387	(48.77	5.77)
	0.2 - 0.8	3.715838	7.137473	61.2	0.143	(2.77	4.29)	2.787005	6.521554	64.7	0.216	(15.17	4.94)	2.623889	6.440329	72.0	0.348	(41.27	5.57)
	0.3 - 0.7	5.551319	9.068401	40.9	0.086	(0.27	2.78)	3.326684	7.252765	52.2	0.132	(3.43	3.84)	2.687347	**6.430410**	72.5	**0.325**	(35.53	5.61)
AU8	0.1 - 0.9	11.281517	13.152617	32.1	0.132	(3.00	3.93)	9.102294	11.28545	33.9	0.201	(10.80	4.45)	8.897037	**10.759520**	36.2	0.303	(23.00	4.99)
	0.2 - 0.8	16.321254	18.062810	19.0	0.070	(0.37	2.39)	9.601045	11.69286	26.3	0.124	(3.80	3.47)	8.991249	10.937447	35.1	0.276	(19.97	4.80)
	0.3 - 0.7	21.088986	23.062541	14.8	0.054	(0.00	1.93)	10.672853	12.84438	21.4	0.090	(1.13	2.91)	9.127924	11.186454	35.9	**0.255**	(17.23	4.73)
ANA	0.1 - 0.9	0.647718	0.646577	63.7	0.190	(1.77	1.43)	0.107990	0.109300	67.1	0.267	(3.67	1.94)	0.007488	**0.009588**	67.5	0.659	(59.50	1.98)
	0.2 - 0.8	0.871531	0.871570	54.9	0.151	(0.27	1.19)	0.337729	0.336884	60.4	0.216	(1.27	1.65)	0.007523	0.009669	65.6	0.585	(48.97	1.97)
	0.3 - 0.7	0.955815	0.960777	45.6	0.137	(0.03	1.09)	0.438822	0.440663	50.6	0.195	(0.70	1.52)	0.007551	0.009691	64.8	**0.531**	(41.37	1.96)
ABA	0.1 - 0.9	7.613683	7.712487	15.2	0.083	(2.00	2.58)	4.836747	4.858605	19.8	0.142	(6.23	3.66)	4.753375	**4.789237**	26.5	0.225	(13.47	4.79)
	0.2 - 0.8	12.693606	12.611176	11.2	0.050	(0.20	1.85)	5.533293	5.538769	15.3	0.090	(2.17	2.81)	4.790064	4.817361	25.9	0.213	(12.17	4.73)
	0.3 - 0.7	14.830294	14.893969	10.3	0.044	(0.00	1.66)	6.432483	6.432756	13.4	0.066	(0.37	2.42)	4.821530	4.845469	23.1	**0.184**	(9.37	4.39)
CON	0.1 - 0.9	70.753300	76.129910	83.8	0.131	(6.30	10.97)	55.1029	61.1530	71.5	0.169	(20.23	9.65)	50.3219	**57.5966**	59.9	0.219	(37.07	8.39)
	0.2 - 0.8	98.572969	104.99026	41.1	0.056	(1.83	5.03)	55.8998	64.5417	42.2	0.093	(9.70	5.87)	51.4730	58.4786	56.0	**0.190**	(30.87	7.70)
	0.3 - 0.7	128.97494	134.53928	25.9	0.030	(0.03	3.06)	73.2893	83.2969	33.2	0.049	(1.60	4.37)	52.9665	59.9898	60.0	0.198	(31.23	8.43)
STP	0.1 - 0.9	3.708981	3.794866	25.7	0.061	(3.20	5.04)	2.129880	2.227016	29.8	0.092	(7.83	6.25)	2.052056	**2.154450**	36.7	0.139	(15.43	7.94)
	0.2 - 0.8	4.652642	4.674927	17.0	0.037	(0.60	3.57)	2.321608	2.455662	23.8	0.064	(3.83	5.06)	2.115639	2.209151	34.1	0.116	(11.53	7.15)
	0.3 - 0.7	8.334997	8.401997	10.5	0.024	(0.07	2.44)	3.113456	3.240279	15.9	0.039	(1.13	3.58)	2.191832	2.272137	33.8	**0.106**	(9.67	6.91)
WAN	0.1 - 0.9	6.463753	7.172746	38.8	0.115	(12.30	13.24)	6.033333	6.780082	43.1	0.146	(18.50	15.22)	5.983361	**6.674879**	48.6	0.190	(27.90	17.52)
	0.2 - 0.8	9.346113	10.098808	31.0	0.060	(1.20	9.81)	6.214014	6.936514	38.5	0.092	(5.40	13.06)	6.040380	6.706163	46.7	0.159	(19.83	16.73)
	0.3 - 0.7	16.437768	17.227865	19.2	0.029	(0.03	5.03)	7.376043	8.159211	27.6	0.052	(0.67	8.66)	6.130478	6.859088	46.0	**0.140**	(14.50	16.28)
WIZ	0.1 - 0.9	3.846971	4.359892	30.5	0.147	(7.90	10.00)	2.718277	3.149809	36.1	0.211	(14.73	12.86)	2.629827	**3.068601**	43.2	0.297	(27.57	15.13)
	0.2 - 0.8	7.513500	8.062620	24.5	0.079	(0.87	6.87)	3.009577	3.614640	32.7	0.146	(5.47	11.02)	2.675106	3.230414	43.8	0.279	(23.43	15.32)
	0.3 - 0.7	24.760387	25.488658	14.8	0.034	(0.03	3.11)	5.940635	6.631707	19.3	0.062	(0.73	5.35)	2.734843	3.302554	45.1	**0.265**	(20.30	15.39)
MOR	0.1 - 0.9	1.590566	1.564395	9.2	0.042	(0.77	2.97)	0.201851	0.219410	12.9	0.079	(2.83	4.83)	0.153782	**0.165901**	16.5	0.144	(9.57	6.50)
	0.2 - 0.8	3.173316	3.214704	6.5	0.024	(0.13	1.81)	0.321694	0.342161	10.6	0.053	(1.37	3.54)	0.159248	0.171457	17.5	0.142	(8.90	6.69)
	0.3 - 0.7	3.180415	3.323322	5.6	0.021	(0.00	1.64)	0.551852	0.574422	8.5	0.039	(0.43	2.86)	0.163368	0.175955	18.2	**0.137**	(7.90	6.86)
TRE	0.1 - 0.9	1.720608	1.786207	10.3	0.045	(0.67	2.96)	0.248659	0.280729	14.2	0.086	(3.10	4.75)	0.202813	**0.232601**	20.3	0.177	(12.23	7.00)
	0.2 - 0.8	3.553366	3.630207	6.5	0.028	(0.00	2.07)	0.557518	0.555809	10.7	0.056	(1.00	3.61)	0.208263	0.237011	19.0	0.154	(9.67	6.58)
	0.3 - 0.7	4.611414	4.668308	5.5	0.025	(0.00	1.80)	1.005509	1.023034	7.9	0.039	(0.23	2.72)	0.212469	0.238197	18.9	**0.144**	(7.93	6.68)

Table 6. Results obtained by MO-AD$_I$ for different configurations of thresholds using the FS_MOGUL rule base

		MAX $_{INT}$					MEDIAN ($_{INT\,/\,ACC}$)					MAX $_{ACC}$				
Dataset	MO-AD$_I$ / T_L - T_H	MSE$_{tra}$	MSE$_{tst}$	#R$_F$	R$_W$_AvR$_{TG}$	(#R$_W$ AvR$_{TG}$)	MSE$_{tra}$	MSE$_{tst}$	#R$_F$	R$_W$_AvR$_{TG}$	(#R$_W$ AvR$_{TG}$)	MSE$_{tra}$	MSE$_{tst}$	#R$_F$	R$_W$_AvR$_{TG}$	(#R$_W$ AvR$_{TG}$)
PLA	0.1 - 0.9	1.574572	1.623150	21.9	0.130	(4.80 4.68)	1.171741	1.207446	29.3	0.216	(11.57 6.61)	1.163733	**1.201293**	40.5	0.379	(28.03 9.16)
	0.2 - 0.8	3.278771	3.305375	11.5	0.051	(0.23 2.37)	1.246104	1.273259	20.3	0.111	(2.83 4.40)	1.164596	1.201882	36.9	0.298	(19.17 8.12)
	0.3 - 0.7	5.290166	5.346323	7.0	0.029	(0.00 1.38)	1.754391	1.810790	13.2	0.063	(0.10 2.97)	1.166274	1.204826	33.5	**0.251**	(14.87 7.24)
QUA	0.1 - 0.9	0.021684	0.022334	199.7	0.292	(13.63 26.19)	0.017291	0.018149	177.8	0.385	(71.50 22.83)	0.017111	**0.018167**	170.8	0.553	(141.50 24.26)
	0.2 - 0.8	0.017468	0.018472	132.1	0.208	(20.67 16.32)	0.017125	0.018199	123.1	0.260	(43.27 16.55)	0.017071	0.018207	124.1	0.320	(65.73 17.58)
	0.3 - 0.7	0.017608	0.018610	86.4	0.129	(13.13 10.08)	0.017135	0.018257	98.0	0.174	(22.40 12.52)	0.017088	0.018183	117.2	**0.268**	(48.00 16.29)
ELE	0.1 - 0.9	64241	67141	56.0	0.129	(4.23 4.10)	42156	45526	49.5	0.228	(17.10 5.09)	39495	**42072**	52.0	0.352	(33.80 6.28)
	0.2 - 0.8	84698	87379	35.9	0.068	(1.40 2.35)	46395	50173	40.8	0.126	(6.07 3.57)	39976	42689	49.6	0.297	(26.70 5.68)
	0.3 - 0.7	108723	113721	23.2	0.048	(0.07 1.85)	61099	65626	29.1	0.078	(2.07 2.56)	40766	43063	48.5	**0.258**	(22.10 5.18)
AU6	0.1 - 0.9	3.821884	6.764193	143.4	0.197	(4.50 10.80)	3.296278	6.516126	129.9	0.266	(32.60 9.97)	2.941543	**6.263927**	111.0	0.389	(79.53 9.18)
	0.2 - 0.8	3.454157	7.474670	79.4	0.133	(10.20 6.07)	3.009759	6.955385	81.0	0.173	(21.03 6.51)	2.925950	6.544321	86.8	0.235	(38.40 7.15)
	0.3 - 0.7	4.202431	7.836509	58.7	0.083	(3.43 4.31)	3.124690	7.149861	69.9	0.126	(10.77 5.52)	2.976153	6.502033	88.3	**0.226**	(34.93 7.27)
AU8	0.1 - 0.9	10.610777	11.814385	24.6	0.120	(3.97 3.30)	7.526326	9.129575	27.7	0.185	(11.33 3.95)	7.386325	**9.000788**	33.3	0.283	(22.67 4.81)
	0.2 - 0.8	16.296509	18.007791	18.0	0.069	(0.63 2.28)	7.898196	9.633468	23.8	0.122	(4.43 3.28)	7.435334	9.047838	31.6	0.250	(18.63 4.57)
	0.3 - 0.7	19.910480	20.495670	14.7	0.056	(0.00 2.00)	9.872984	11.83105	19.3	0.081	(0.37 2.76)	7.505406	9.082736	32.1	**0.238**	(17.17 4.52)
ANA	0.1 - 0.9	0.044437	0.047909	174.4	0.188	(12.07 4.67)	0.013463	0.015529	154.4	0.250	(44.00 4.24)	0.007158	0.009335	133.0	0.358	(91.53 4.12)
	0.2 - 0.8	0.038210	0.040613	129.3	0.141	(12.93 3.22)	0.009456	0.011776	114.0	0.191	(27.63 3.68)	0.007033	**0.009237**	112.5	0.252	(50.90 3.84)
	0.3 - 0.7	0.126768	0.126480	85.7	0.094	(8.33 2.16)	0.015763	0.018568	92.5	0.125	(10.17 2.93)	0.007180	0.009461	108.3	**0.216**	(34.20 3.92)
ABA	0.1 - 0.9	10.034018	9.980401	15.6	0.098	(1.10 2.33)	6.252071	6.261997	20.2	0.180	(5.57 3.34)	6.198179	**6.219666**	24.9	0.311	(15.07 4.32)
	0.2 - 0.8	13.518071	13.474387	10.1	0.064	(0.03 1.70)	6.450398	6.484010	15.2	0.115	(1.40 2.69)	6.200991	6.221171	24.6	0.258	(10.83 4.02)
	0.3 - 0.7	13.471095	13.490648	9.1	0.058	(0.00 1.56)	6.933849	6.956984	13.8	0.099	(0.70 2.45)	6.204145	6.222632	22.9	**0.220**	(7.90 3.77)
CON	0.1 - 0.9	67.672232	71.789422	86.8	0.143	(5.13 12.02)	59.010937	65.03955	66.0	0.201	(25.23 9.74)	56.380503	**62.489223**	62.6	0.257	(39.37 9.66)
	0.2 - 0.8	80.296355	88.380934	40.9	0.071	(3.53 5.52)	58.333450	65.72600	45.4	0.113	(10.77 6.87)	56.679133	62.796797	55.0	0.195	(27.00 8.43)
	0.3 - 0.7	116.57328	123.10188	27.7	0.036	(0.30 3.45)	66.961798	73.82797	36.8	0.066	(2.67 5.44)	57.450287	63.305839	58.3	**0.189**	(24.53 8.79)
STP	0.1 - 0.9	5.405339	5.474713	13.3	0.096	(1.33 3.21)	4.259753	4.362481	18.6	0.167	(4.63 4.54)	4.159594	**4.255350**	25.0	0.287	(11.73 6.19)
	0.2 - 0.8	7.052710	7.174048	9.1	0.063	(0.13 2.41)	4.338720	4.473762	16.2	0.136	(3.17 3.98)	4.179792	4.288358	25.0	0.268	(9.90 6.25)
	0.3 - 0.7	8.748136	8.816542	7.9	0.058	(0.00 2.25)	4.455629	4.560821	14.7	0.125	(2.67 3.75)	4.194782	4.309624	24.9	**0.252**	(8.50 6.22)
WAN	0.1 - 0.9	26.310868	26.725976	9.6	0.088	(0.53 2.86)	14.522565	14.77936	13.3	0.193	(4.40 4.46)	13.774738	**14.009191**	16.1	0.361	(12.27 6.19)
	0.2 - 0.8	29.878818	30.684648	7.9	0.069	(0.00 2.43)	15.051468	15.54537	12.1	0.150	(2.43 4.00)	13.832013	14.079000	15.6	0.331	(10.67 5.96)
	0.3 - 0.7	32.988571	34.115293	7.9	0.065	(0.00 2.26)	15.540758	16.02159	11.5	0.122	(0.93 3.81)	13.948231	14.207306	15.2	**0.302**	(9.47 5.61)
WIZ	0.1 - 0.9	12.545764	13.543774	15.7	0.107	(2.57 4.18)	3.881428	4.467590	19.7	0.193	(7.50 6.11)	3.413844	**3.856717**	25.0	0.351	(19.80 8.12)
	0.2 - 0.8	34.049969	35.694754	11.6	0.059	(0.13 2.93)	4.965794	5.631655	16.9	0.121	(2.30 5.01)	3.463373	3.912941	24.4	0.314	(16.77 7.72)
	0.3 - 0.7	34.337238	36.299840	10.5	0.054	(0.00 2.74)	6.706427	7.391861	14.5	0.091	(0.87 4.18)	3.547122	4.014734	24.5	**0.291**	(14.73 7.61)
MOR	0.1 - 0.9	3.056375	3.052106	6.0	0.060	(0.03 1.89)	0.423308	0.439410	8.6	0.134	(2.57 2.93)	0.267971	**0.275199**	12.8	0.305	(9.03 5.06)
	0.2 - 0.8	3.325074	3.280224	5.6	0.055	(0.00 1.74)	0.502724	0.495026	7.8	0.118	(1.90 2.76)	0.275008	0.281564	13.0	0.302	(8.77 5.12)
	0.3 - 0.7	3.689980	3.747015	5.8	0.050	(0.00 1.58)	0.548342	0.553402	7.8	0.109	(1.50 2.69)	0.283597	0.290467	13.0	**0.293**	(8.13 5.15)
TRE	0.1 - 0.9	4.887197	5.066371	5.8	0.054	(0.17 1.80)	0.806103	0.792245	8.7	0.104	(0.87 3.17)	0.448043	0.436540	10.4	0.229	(7.20 4.17)
	0.2 - 0.8	5.307431	5.513163	5.4	0.048	(0.00 1.67)	0.931746	0.905101	8.4	0.092	(0.43 2.96)	0.450184	**0.436108**	10.3	0.218	(6.60 4.10)
	0.3 - 0.7	4.886871	4.991007	5.7	0.046	(0.00 1.62)	0.991394	0.975439	8.0	0.084	(0.23 2.81)	0.455389	0.437021	10.3	**0.211**	(6.17 4.10)

1. The best interpretability is obtained with the thresholds T_L=0.3 and T_H=0.7 (interpretability index and metrics, are better with them) while the best accuracy is obtained with the thresholds T_L=0.1 and T_H=0.9. We think that this is because the more aggressive setup (T_L=0.3 and T_H=0.7) lets the learning algorithm attack the interpretability index more strongly, reducing rules with weight and rules triggered together converging to zones with solutions more interpretable quickly. In contrast, the more conservative setup (T_L=0.1 and T_H=0.9) forces the learning algorithm to search in a less interpretable zone, favouring easier convergence with the most accurate solutions. This is a coherent result because a T_H of 0.7 considerably reduces the number of rules with weight, and a T_L of 0.3 also reduces the number of rules powerfully, but there is a reduction in the range available for the rule weights from 0.3 to 0.7, which is thinner than the range from 0.1 to 0.9. Wider ranges of rule weights allow greater accuracy.
2. Results obtained in the most accurate point (MAX ACC) by the three thresholds are similar regarding accuracy. As commented before, they are better with T_L=0.1 and T_H=0.9 but when using the other two thresholds setups, the accuracy is not strongly harmed while the interpretability is enhanced more substantially, e.g. the number of rules with weight is about the half of the total number of rules.

Finally, Figure 2 shows the comparison between the Pareto fronts obtained with different thresholds, on this occasion from FRBSs obtained using the WM rule base, and particularly for the MOR and STO problems, as an example. Results are extensive to the rest of the datasets and FRBSs that use FS_MOGUL rule bases. We plotted the three points, the MAX INT, the MEDIAN (INT/ACC) and the MAX ACC for each MOEA. The approximations of the Pareto fronts achieved by T_L=0.1 and T_H=0.9 are in general more accurate than the Pareto fronts obtained by T_L=0.3 and T_H=0.7 which are more interpretable, as was expected. Moreover, it can be observed that the Pareto fronts obtained by MO-$AD_{I(0.3-0.7)}$ are wider than those obtained with MO-$AD_{I(0.1-0.9)}$. It is coherent with the second main previous result obtained studying the tables: the best accuracies are reached by the MO-$AD_{I(0.1-0.9)}$ models, but MO-$AD_{I(0.3-0.7)}$ are close to them in accuracy (MAX ACC), and also reach the greatest level in accuracy (MAX INT).

C. Comparing the proposals against single-objective approaches

This subsection analyses the performance of our multi-objective proposal against the single-objective approach.

We selected the set-up T_L=0.1 and T_H=0.9 because, as shown before, it has achieved the best results in accuracy, and the single-objective approaches also use a single-objective based on accuracy.

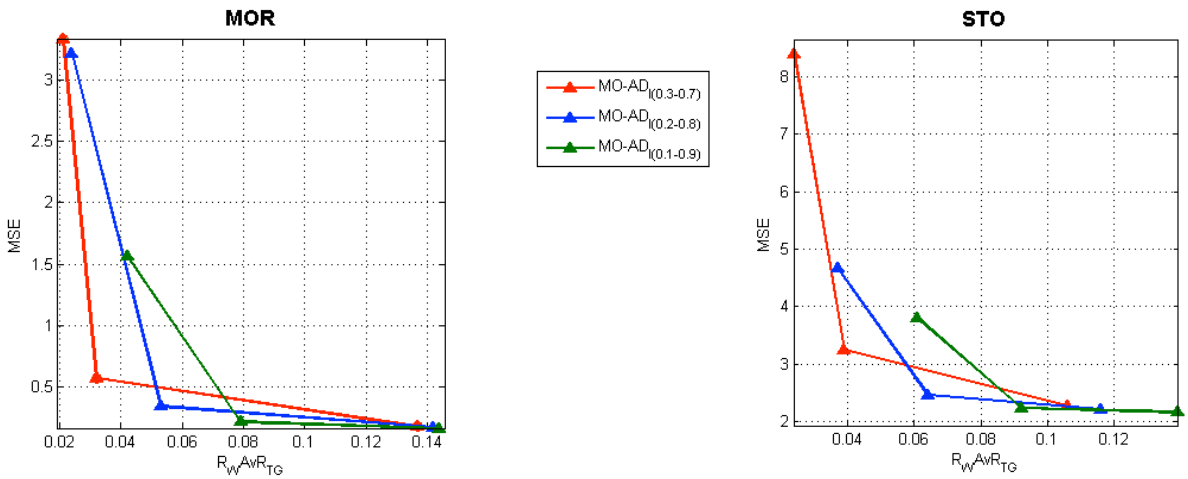

Fig. 2. Average Pareto fronts obtained for STO and MOR problems in the accurate-interpretability plane for WM.

Clearly it would make no sense to consider the interpretability index in the single-objective accuracy oriented, because this index affects the interpretability and the objective employed is accuracy only (assuming that accuracy oriented approaches have the worst values for R_W_AvR_{TG}). However, with the aim of comparing the most accurate point in multi-objective approach with the single-objective approach, we also introduced these indexes in the single-objective approach (SO_AD_I) in order to see how the accuracy of the index was affected.

Table 7 shows the result of the single-objective approaches, where SO-AD is the single-objective with adaptive defuzzification and SO-$AD_{I(0.1\text{-}0.9)}$ is the single-objective approach with the mechanism to improve the interpretability, together with the results of the multi-objective approach MO-$AD_{I(0.1\text{-}0.9)}$ for the most accurate point from Table 5 and 6.

To compare the multi-objective approach against the single-objective ones, we performed a statistical analysis in the different measures to check if there are

Table 7 Results obtained by standard single-objectives models accuracy oriented (SO-AD) vs single-objectives models with the interpretability improvement mechanism (SO-$AD_{I(0.1\text{-}0.9)}$),and multi-objective models proposal (MO-$AD_{I(0.1\text{-}0.9)}$), with both, WM and FS_MOGUL rule bases

		WM						FS_MOGUL					
Dataset	FRBSs	MSE_{tra}	MSE_{tst}	$\#R_F$	R_W_AvR_{TG}	($\#R_W$	AvR_T)	MSE_{tra}	MSE_{tst}	$\#R_F$	R_W_AvR_{TG}	($\#R_W$	AvR_T)
PLA	SO-AD	1.694533	1.749531	14.80	0.913	(14.80	3.30)	1.274698	1.295137	75.40	0.904	(75.40	19.25)
	SO-AD_I	1.705751	1.765311	14.33	0.813	(11.97	3.27)	1.252672	1.269589	58.97	0.645	(52.83	14.02)
	MO-AD_I	1.699226	**1.759935**	14.0	0.771	(10.80	3.25)	1.163733	**1.201293**	40.5	0.379	(28.03	9.16)
QUA	SO-AD	0.021303	0.022412	53.60	0.892	(53.60	6.27)	0.017394	0.018187	227.60	0.829	(227.60	32.89)
	SO-AD_I	0.021163	0.022351	42.40	0.633	(38.17	4.43)	0.017366	0.018170	205.97	0.720	(197.87	28.48)
	MO-AD_I	0.020675	**0.021944**	32.6	0.448	(22.67	3.79)	0.017111	**0.018167**	170.8	0.553	(141.50	24.26)
ELE	SO-AD	36455	38917	65.00	0.880	(65.00	10.66)	43517	47035	88.80	0.759	(88.80	10.04)
	SO-AD_I	36546	39295	58.17	0.738	(54.03	9.02)	43444	46938	80.43	0.662	(74.63	9.37)
	MO-AD_I	32518	**35633**	45.2	0.469	(33.27	5.96)	39495	**42072**	52.0	0.352	(33.80	6.28)
AU6	SO-AD	2.917724	6.002718	116.00	0.777	(116.00	9.10)	3.252887	6.565788	170.60	0.731	(170.60	13.54)
	SO-AD_I	2.914147	6.034773	104.67	0.673	(99.17	8.06)	3.245635	6.629009	154.97	0.640	(146.47	12.36)
	MO-AD_I	2.568845	6.457802	73.6	0.387	(48.77	5.77)	2.941543	**6.263927**	111.0	0.389	(79.53	9.18)
AU8	SO-AD	9.222880	10.51600	70.60	0.771	(70.60	9.65)	7.804103	8.910248	78.80	0.812	(78.80	11.05)
	SO-AD_I	9.204716	10.72178	59.90	0.621	(56.07	7.97)	7.796410	9.112414	66.13	0.648	(61.93	9.04)
	MO-AD_I	8.897037	**10.759520**	36.2	0.303	(23.00	4.99)	7.386325	**9.000788**	33.3	0.283	(22.67	4.81)
ANA	SO-AD	0.008360	0.010359	72.40	0.752	(72.40	2.01)	0.023771	0.026515	211.20	0.740	(211.20	7.00)
	SO-AD_I	0.010606	0.012647	70.30	0.698	(64.83	2.00)	0.020551	0.023209	189.73	0.617	(183.13	5.34)
	MO-AD_I	0.007488	**0.009588**	67.5	0.659	(59.50	1.98)	0.007158	0.009335	133.0	0.358	(91.53	4.12)
ABA	SO-AD	5.101607	5.078695	68.20	0.912	(68.20	15.64)	6.374271	6.385199	50.20	0.854	(50.20	9.40)
	SO-AD_I	5.077808	5.059224	56.57	0.707	(51.90	12.40)	6.235069	6.249418	40.30	0.614	(36.87	6.59)
	MO-AD_I	4.753375	**4.789237**	26.5	0.225	(13.47	4.79)	6.198179	**6.219666**	24.9	0.311	(15.07	4.32)
CON	SO-AD	58.142348	62.98518	135.40	0.692	(135.40	19.63)	61.601930	65.901353	124.40	0.697	(124.40	19.30)
	SO-AD_I	57.845984	62.69151	118.10	0.569	(110.30	16.45)	61.289333	65.672337	108.17	0.568	(100.80	15.99)
	MO-AD_I	50.3219	**57.5966**	59.9	0.219	(37.07	8.39)	56.380503	**62.489223**	62.6	0.257	(39.37	9.66)
STP	SO-AD	3.784825	3.810319	122.80	0.849	(122.80	36.25)	4.371739	4.459575	45.60	0.870	(45.60	14.49)
	SO-AD_I	3.556481	3.571853	98.40	0.643	(91.97	27.91)	4.302168	4.399084	34.57	0.567	(30.03	9.32)
	MO-AD_I	2.052056	**2.154450**	36.7	0.139	(15.43	7.94)	4.159594	**4.255350**	25.0	0.287	(11.73	6.19)
WAN	SO-AD	7.775341	8.005151	156.00	0.845	(156.00	60.23)	14.300552	14.573703	33.40	0.864	(33.40	13.09)
	SO-AD_I	7.626420	7.883073	136.73	0.713	(129.27	52.19)	14.194849	14.484166	25.43	0.622	(23.00	10.00)
	MO-AD_I	5.983361	**6.674879**	48.6	0.190	(27.90	17.52)	13.774738	**14.009191**	16.1	0.361	(12.27	6.19)
WIZ	SO-AD	3.331755	3.574799	104.80	0.883	(104.80	35.21)	4.034584	4.544982	52.40	0.863	(52.40	18.26)
	SO-AD_I	3.316957	3.583501	92.80	0.750	(87.43	30.59)	3.943292	4.418400	42.47	0.659	(39.60	14.19)
	MO-AD_I	2.629827	**3.068601**	43.2	0.297	(27.57	15.13)	3.413844	**3.856717**	25.0	0.351	(19.80	8.12)
MOR	SO-AD	0.329428	0.328112	77.60	0.868	(77.60	29.29)	0.361349	0.364585	31.40	0.857	(31.40	11.25)
	SO-AD_I	0.263595	0.273204	58.50	0.633	(54.50	22.42)	0.346304	0.347567	21.20	0.557	(19.43	7.80)
	MO-AD_I	0.153782	**0.165901**	16.5	0.144	(9.57	6.50)	0.267971	**0.275199**	12.8	0.305	(9.03	5.06)
TRE	SO-AD	0.428898	0.432897	75.00	0.884	(75.00	28.12)	0.595556	0.587049	33.00	0.827	(33.00	11.39)
	SO-AD_I	0.374909	0.378188	57.00	0.653	(53.30	21.73)	0.578306	0.573042	23.00	0.553	(20.63	8.36)
	MO-AD_I	0.202813	**0.232601**	20.3	0.177	(12.23	7.00)	0.448043	0.436540	10.4	0.229	(7.20	4.17)

significant differences. This study can be consulted in Appendix B.

Thus, viewing Table 7 and the statistical analysis of Appendix B, we can conclude:

- The multi-objective proposal, for the most accurate point, statistically overcomes the accuracy of the single-objective accuracy oriented approaches. This is an important result, because the use of the mechanism proposed to improve the interpretability of adaptive defuzzification methods together with the MOEA is improving not only their interpretability but also their accuracy.
- As was expected, the interpretability measures of the multi-objective approach for the most accurate point are better than the ones for the single-objective accuracy oriented approaches.
- Taking into account only the single-objective approaches, SO-AD$_{I(0.1\text{-}0.9)}$ outperforms SO-AD. Thus, the mechanism to improve the interpretability also proposed for the single-objective methods, improves their accuracy when using T_L=0.1 and T_H=0.9

Figure 3 shows the representative points for MO-AD$_{I(0.1\text{-}0.9)}$, SO-AD and SO-AD$_{I(0.1\text{-}0.9)}$ for TRE dataset obtained using WM rule bases. It shows the approximations of the Pareto fronts achieved by MO-AD$_{I(0.1\text{-}0.9)}$ and the relative position of the results obtained with the single-objective methods, SO-AD and SO-AD$_{I(0.1\text{-}0.9)}$. Similar results are obtained for all datasets and FS-MOGUL rule bases.

Figure 4 shows a representative example on WIZ dataset in order to reveal that the #R_F is moving in the same way as the R_W_AvR_{TG} index in the approximated Pareto fronts provided by MO-AD$_{I(0.1\text{-}0.9)}$. We plot the solutions from MO-AD$_{I(0.1\text{-}0.9)}$ two-dimensionally, and plot the projections of these solutions on the accuracy-#R_F and accuracy-R_W_AvR_{TG} planes jointly with the single-objective SO-AD and SO-AD$_{I(0.1\text{-}0.9)}$. In order to retain all the information, in this figure the dominated solutions obtained from the projections have not been removed. Some researchers have also used these kinds of projections for graphic representation when three objectives are optimized together[28, 57].

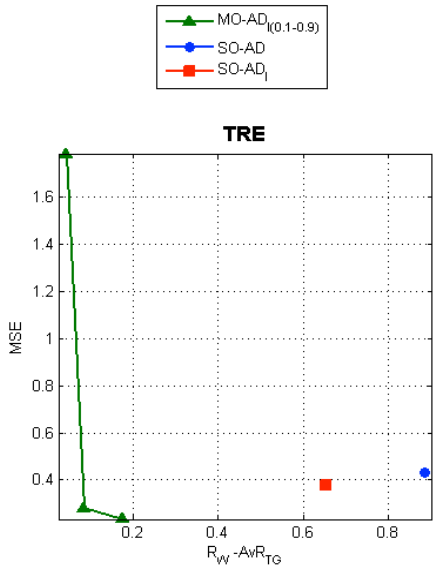

Fig. 3. Average pareto fronts obtained for TRE data sets in the accuracy-interpretability plane using WM rule bases.

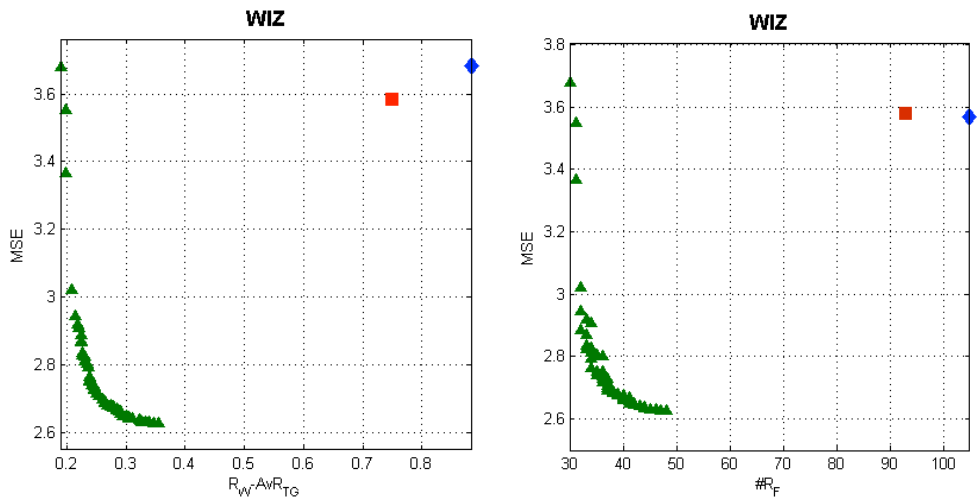

Fig. 4. Example Pareto fronts obtained in WIZ problem.

4.3. *Second part of the experimental study*

In this subsection we analyse the practical behaviour of the second or adaptive thresholds model proposed.

Table 8 summarizes the FRBSs considered. We compare the adaptive interpretability improvement mechanism approach against two of the previously employed fixed threshold approaches, T_L=0.1 and T_H=0.9, and T_L=0.3 and T_H=0.7. Regarding the MOEA set-up, the initial population was obtained initializing all the individuals randomly (between 0 to 1 in C_D part and within the corresponding variation intervals defined in C_T part) except those whose genes all have value '1' in the C_D part in order to begin the evolutionary process with all the rules activated and without weights, and the C_T part with β_T=1, which is T_L=0 and T_H=1.

The rest of the set-up of the MOEA used is the same employed in section 4.1 for the first part of the experimental study.

Table 8. FRBSs considered for comparison

FRBSs	Description
	Multi-Objective FRBSs
MO-AD$_I$	Adaptive Defuzzification with ***fixed*** mechanism to improve interpretability
MO-AD$_{I(A)}$	Adaptive Defuzzification with ***adaptable*** mechanism to improve interpretability

Results and analysis

Tables 9 and 10 show the results for the three representative points: the most accurate, the median, and the most interpretable. Viewing them, we can see that the most accurate solution of the adaptive mechanism MO-AD$_{I(A)}$ is very close to the most accurate fixed thresholds approach MO-AD$_{I(0.1\text{-}0.9)}$ and usually, the most interpretable (using the index proposed) obtained by MO-AD$_{I(A)}$ is normally more interpretable than the most interpretable of MO-AD$_{I(0.3\text{-}0.7)}$, but sometimes the most interpretable solution obtained may be unusable because its accuracy is very bad.

However, we actually consider that the aforementioned three representative points do not show the usefulness of this adaptive approach. This time, we needed to use other tools to study the behaviour of the new adaptive interpretability improvement mechanism for adaptive defuzzification systems in order to compare it with the non adaptive approach.

Figure 5 shows an example of the Pareto fronts obtained by the MO-AD$_{I(A)}$, MO-AD$_{I(0.1\text{-}0.9)}$ and MO-AD$_{I(0.3\text{-}0.7)}$. It reveals the interest of the solutions obtained by the MO-AD$_{I(A)}$ because the curve is below the two other in most of them, i.e. it has more interpretable solutions taking the same level of accuracy.

For this reason, we used the Two Set Coverage[57] (CS) ratio previously cited. Their results are shown in Table 11. The best results obtained are highlighted in bold. It can be observed that MO-AD$_{I(A)}$ is better than both, MO-AD$_{I(0.1\text{-}0.9)}$ and MO-AD$_{I(0.3\text{-}0.7)}$ for almost all problems.

Figure 6 shows an example of the thresholds found by the adaptive width mechanism. Probably the most interesting zones of solutions are those with the lower interpretability index (R_W_AvR_{TG} around 0.04) which keep the error at the lower values (close to 0) and thresholds are between T_L=0.25 and T_H=0.75, and T_L=0.3 and T_H=0.7.

Therefore, we can conclude the following:

- The proposal based on the use of adaptive mechanism to improve the interpretability of FRBSs with adaptive defuzzification, MO-AD$_{I(A)}$, allow us to obtain, in a single execution of the algorithm, a set of solutions ranging from the most interpretable to the most accurate without the need to set up thresholds to obtain a more interpretable or more accurate set of solutions.
- The set of solutions obtained with the MO-AD$_{I(A)}$ approach improves on those obtained with the fixed thresholds mechanism approaches, thus constituting a preferable design option.

5. Conclusions

Classically, linguistic fuzzy modelling has focused on the improvement of system accuracy. Recently, there has been a growing interest in interpretability, initially in not significantly affecting the interpretability while improving the accuracy, and now improving the accuracy together with the interpretability.

This evolution has also required an understanding of the different factors and slopes involving the interpretability, which is a subjective concept and thus difficult to evaluate. At present, some works[30,31-33] propose different taxonomies and key considerations to better achieve and deal with interpretability.

Table 9. Results obtained by MO-AD$_I$ for different configurations of thresholds using the WM rule base

		MAX $_{INT}$						MEDIAN ($_{INT\ /\ ACC}$)						MAX $_{ACC}$					
Dataset	MO-AD$_I$ T_L - T_H	MSE$_{tra}$	MSE$_{tst}$	#R$_F$	R$_W$_AvR$_{TG}$	(#R$_W$	AvR$_{TG}$)	MSE$_{tra}$	MSE$_{tst}$	#R$_F$	R$_W$_AvR$_{TG}$	(#R$_W$	AvR$_{TG}$)	MSE$_{tra}$	MSE$_{tst}$	#R$_F$	R$_W$_AvR$_{TG}$	(#R$_W$	AvR$_{TG}$)
PLA	0.1 - 0.9	6.947800	6.940028	6.1	0.128	(0.00	1.02)	2.073691	2.091053	10.6	0.385	(2.23	2.48)	1.699226	1.759935	14.0	0.771	(10.80	3.25)
	0.3 - 0.7	7.042636	7.042633	6.0	0.125	(0.00	1.00)	2.543587	2.545364	10.4	0.309	(0.60	2.31)	1.830941	1.900387	12.4	0.551	(6.13	2.75)
	Adaptive	3.516398	3.587155	5.00	0.132	(0.00	1.05)	2.157240	2.168258	9.67	0.344	(1.70	2.30)	1.690599	1.745825	14.07	0.783	(11.13	3.26)
QUA	0.1 - 0.9	0.023570	0.024276	22.7	0.179	(4.93	2.12)	0.020914	0.022153	27.2	0.287	(10.60	3.01)	0.020675	0.021944	32.6	0.448	(22.67	3.79)
	0.3 - 0.7	0.026361	0.027337	17.7	0.114	(0.13	1.80)	0.021700	0.022975	24.7	0.193	(3.13	2.62)	0.020897	0.022139	31.4	0.380	(17.37	3.49)
	Adaptive	0.071487	0.071611	6.10	0.018	(0.00	0.28)	0.036666	0.037420	9.97	0.063	(0.23	0.97)	0.020688	0.022138	32.97	0.450	(23.70	3.66)
ELE	0.1 - 0.9	57867	60608	35.8	0.167	(7.37	3.10)	35428	39094	38.6	0.285	(16.30	4.48)	32518	35633	45.2	0.469	(33.27	5.96)
	0.3 - 0.7	143152	150408	20.0	0.063	(0.03	1.77)	64459	70994	27.6	0.086	(1.17	2.16)	33433	35677	45.4	0.421	(25.97	6.19)
	Adaptive	4515380	4510411	13.03	0.014	(0.00	0.39)	308668	326805	17.60	0.049	(0.00	1.39)	32809	35539	45.00	0.482	(32.57	6.49)
AU6	0.1 - 0.9	3.487613	6.568578	84.6	0.214	(6.60	6.07)	2.746415	6.482094	73.4	0.293	(28.57	5.57)	2.568845	6.457802	73.6	0.387	(48.77	5.77)
	0.3 - 0.7	5.551319	9.068401	40.9	0.086	(0.27	2.78)	3.326684	7.252765	52.2	0.132	(3.43	3.84)	2.687347	6.430410	72.5	0.325	(35.53	5.61)
	Adaptive	4.825753	8.955610	47.60	0.109	(1.70	3.33)	3.024116	7.687152	56.40	0.189	(13.70	4.26)	2.609892	6.387879	78.97	0.441	(58.43	6.20)
AU8	0.1 - 0.9	11.281517	13.152617	32.1	0.132	(3.00	3.93)	9.102294	11.28545	33.9	0.201	(10.80	4.45)	8.897037	10.759520	36.2	0.303	(23.00	4.99)
	0.3 - 0.7	21.088986	23.062541	14.8	0.054	(0.00	1.93)	10.672853	12.84438	21.4	0.090	(1.13	2.91)	9.127924	11.186454	35.9	0.255	(17.23	4.73)
	Adaptive	19.514876	20.801083	12.77	0.045	(0.30	1.52)	9.739728	11.83919	22.57	0.120	(5.23	2.97)	8.887143	10.698300	37.17	0.325	(25.63	5.13)
ANA	0.1 - 0.9	0.647718	0.646577	63.7	0.190	(1.77	1.43)	0.107990	0.109300	67.1	0.267	(3.67	1.94)	0.007488	0.009588	67.5	0.659	(59.50	1.98)
	0.3 - 0.7	0.955815	0.960777	45.6	0.137	(0.03	1.09)	0.438822	0.440663	50.6	0.195	(0.70	1.52)	0.007551	0.009691	64.8	0.531	(41.37	1.96)
	Adaptive	0.360127	0.360551	38.87	0.054	(0.00	0.43)	0.104234	0.110433	49.33	0.160	(2.07	1.17)	0.007347	0.009452	66.80	0.656	(59.13	1.98)
ABA	0.1 - 0.9	7.613683	7.712487	15.2	0.083	(2.00	2.58)	4.836747	4.858605	19.8	0.142	(6.23	3.66)	4.753375	4.789237	26.5	0.225	(13.47	4.79)
	0.3 - 0.7	14.830294	14.893969	10.3	0.044	(0.00	1.66)	6.432483	6.432756	13.4	0.066	(0.37	2.42)	4.821530	4.845469	23.1	0.184	(9.37	4.39)
	Adaptive	11.779818	11.777811	5.30	0.027	(0.03	1.01)	5.504223	5.571520	9.70	0.066	(1.20	2.19)	4.761915	4.779487	25.87	0.232	(13.97	4.91)
CON	0.1 - 0.9	70.753300	76.129910	83.8	0.131	(6.30	10.97)	55.1029	61.1530	71.5	0.169	(20.23	9.65)	50.3219	57.5966	59.9	0.219	(37.07	8.39)
	0.3 - 0.7	128.97494	134.53928	25.9	0.030	(0.03	3.06)	73.2893	83.2969	33.2	0.049	(1.60	4.37)	52.9665	59.9898	60.0	0.198	(31.23	8.43)
	Adaptive	121.13485	127.93996	28.87	0.037	(0.53	3.53)	62.424558	69.86624	38.17	0.071	(4.97	5.38)	50.580760	57.292015	64.60	0.254	(44.00	9.34)
STP	0.1 - 0.9	3.708981	3.794866	25.7	0.061	(3.20	5.04)	2.129880	2.227016	29.8	0.092	(7.83	6.25)	2.052056	2.154450	36.7	0.139	(15.43	7.94)
	0.3 - 0.7	8.334997	8.401997	10.5	0.024	(0.07	2.44)	3.113456	3.240279	15.9	0.039	(1.13	3.58)	2.191832	2.272137	33.8	0.106	(9.67	6.91)
	Adaptive	15.327830	15.680952	11.33	0.019	(0.00	1.96)	2.568037	2.731294	19.27	0.048	(2.27	4.00)	2.080961	2.147385	36.77	0.135	(14.63	7.88)
WAN	0.1 - 0.9	6.463753	7.172746	38.8	0.115	(12.30	13.24)	6.033333	6.780082	43.1	0.146	(18.50	15.22)	5.983361	6.674879	48.6	0.190	(27.90	17.52)
	0.3 - 0.7	16.437768	17.227865	19.2	0.029	(0.03	5.03)	7.376043	8.159211	27.6	0.052	(0.67	8.66)	6.130478	6.859088	46.0	0.140	(14.50	16.28)
	Adaptive	13.890007	13.886866	26.77	0.052	(0.67	8.81)	6.702816	7.416840	34.43	0.085	(5.10	11.93)	6.033621	6.791114	46.00	0.165	(21.27	16.96)
WIZ	0.1 - 0.9	3.846971	4.359892	30.5	0.147	(7.90	10.00)	2.718277	3.149809	36.1	0.211	(14.73	12.86)	2.629827	3.068601	43.2	0.297	(27.57	15.13)
	0.3 - 0.7	24.760387	25.488658	14.8	0.034	(0.03	3.11)	5.940635	6.631707	19.3	0.062	(0.73	5.35)	2.734843	3.302554	45.1	0.265	(20.30	15.39)
	Adaptive	19.506770	20.197547	17.27	0.056	(0.23	5.01)	3.220616	3.690454	25.80	0.115	(3.57	8.98)	2.643355	3.061785	44.60	0.311	(29.27	15.63)
MOR	0.1 - 0.9	1.590566	1.564395	9.2	0.042	(0.77	2.97)	0.201851	0.219410	12.9	0.079	(2.83	4.83)	0.153782	0.165901	16.5	0.144	(9.57	6.50)
	0.3 - 0.7	3.180415	3.323322	5.6	0.021	(0.00	1.64)	0.551852	0.574422	8.5	0.039	(0.43	2.86)	0.163368	0.175955	18.2	0.137	(7.90	6.86)
	Adaptive	2.080311	2.089618	5.23	0.020	(0.03	1.60)	0.339201	0.355028	8.27	0.041	(0.53	3.01)	0.165634	0.186429	17.07	0.148	(9.60	6.80)
TRE	0.1 - 0.9	1.720608	1.786207	10.3	0.045	(0.67	2.96)	0.248659	0.280729	14.2	0.086	(3.10	4.75)	0.202813	0.232601	20.3	0.177	(12.23	7.00)
	0.3 - 0.7	4.611414	4.668308	5.5	0.025	(0.00	1.80)	1.005509	1.023034	7.9	0.039	(0.23	2.72)	0.212469	0.238197	18.9	0.144	(7.93	6.68)
	Adaptive	3.887324	3.855478	5.20	0.019	(0.00	1.41)	0.796229	0.820500	7.90	0.040	(0.33	2.74)	0.223408	0.229770	22.17	0.199	(13.30	8.09)

Table 10. Results obtained by MO-AD$_I$ for different configurations of thresholds using the FS_MOGUL rule base

		MAX $_{INT}$					MEDIAN ($_{INT / ACC}$)					MAX $_{ACC}$				
Dataset	MO-AD$_I$ / T_L - T_H	MSE$_{tra}$	MSE$_{tst}$	#R$_F$	R$_W$_AvR$_{TG}$	(#R$_W$ AvR$_{TG}$)	MSE$_{tra}$	MSE$_{tst}$	#R$_F$	R$_W$_AvR$_{TG}$	(#R$_W$ AvR$_{TG}$)	MSE$_{tra}$	MSE$_{tst}$	#R$_F$	R$_W$_AvR$_{TG}$	(#R$_W$ AvR$_{TG}$)
PLA	0.1 - 0.9	1.574572	1.623150	21.9	0.130	(4.80 4.68)	1.171741	1.207446	29.3	0.216	(11.57 6.61)	1.163733	1.201293	40.5	0.379	(28.03 9.16)
	0.3 - 0.7	5.290166	5.346323	7.0	0.029	(0.00 1.38)	1.754391	1.810790	13.2	0.063	(0.10 2.97)	1.166274	1.204826	33.5	0.251	(14.87 7.24)
	Adaptive	3.231820	3.335011	12.67	0.059	(0.50 2.65)	1.281801	1.301098	21.33	0.129	(4.40 4.73)	1.163860	1.200922	41.47	0.387	(28.50 9.41)
QUA	0.1 - 0.9	0.021684	0.022334	199.7	0.292	(13.63 26.19)	0.017291	0.018149	177.8	0.385	(71.50 22.83)	0.017111	0.018167	170.8	0.553	(141.50 24.26)
	0.3 - 0.7	0.017608	0.018610	86.4	0.129	(13.13 10.08)	0.017135	0.018257	98.0	0.174	(22.40 12.52)	0.017088	0.018183	117.2	0.268	(48.00 16.29)
	Adaptive	0.017483	0.018617	99.30	0.160	(16.63 12.46)	0.017136	0.018311	105.77	0.205	(28.47 14.31)	0.017086	0.018236	122.67	0.311	(62.93 17.35)
ELE	0.1 - 0.9	64241	67141	56.0	0.129	(4.23 4.10)	42156	45526	49.5	0.228	(17.10 5.09)	39495	42072	52.0	0.352	(33.80 6.28)
	0.3 - 0.7	108723	113721	23.2	0.048	(0.07 1.85)	61099	65626	29.1	0.078	(2.07 2.56)	40766	43063	48.5	0.258	(22.10 5.18)
	Adaptive	4026527	4027142	29.33	0.028	(0.40 0.99)	157856	155209	34.07	0.076	(1.97 2.51)	39974	42599	58.60	0.434	(43.93 7.25)
AU6	0.1 - 0.9	3.821884	6.764193	143.4	0.197	(4.50 10.80)	3.296278	6.516126	129.9	0.266	(32.60 9.97)	2.941543	6.263927	111.0	0.389	(79.53 9.18)
	0.3 - 0.7	4.202431	7.836509	58.7	0.083	(3.43 4.31)	3.124690	7.149861	69.9	0.126	(10.77 5.52)	2.976153	6.502033	88.3	0.226	(34.93 7.27)
	Adaptive	4.124317	8.088891	66.67	0.104	(6.47 4.94)	3.086823	7.151888	77.37	0.176	(24.03 6.14)	2.949025	6.371336	109.33	0.379	(77.10 8.94)
AU8	0.1 - 0.9	10.610777	11.814385	24.6	0.120	(3.97 3.30)	7.526326	9.129575	27.7	0.185	(11.33 3.95)	7.386325	9.000788	33.3	0.283	(22.67 4.81)
	0.3 - 0.7	19.910480	20.495670	14.7	0.056	(0.00 2.00)	9.872984	11.83105	19.3	0.081	(0.37 2.76)	7.505406	9.082736	32.1	0.238	(17.17 4.52)
	Adaptive	18.357268	19.391021	10.00	0.036	(0.27 1.20)	8.196575	9.935652	19.33	0.106	(4.20 2.76)	7.398018	8.961176	34.93	0.305	(24.73 5.10)
ANA	0.1 - 0.9	0.044437	0.047909	174.4	0.188	(12.07 4.67)	0.013463	0.015529	154.4	0.250	(44.00 4.24)	0.007158	0.009335	133.0	0.358	(91.53 4.12)
	0.3 - 0.7	0.126768	0.126480	85.7	0.094	(8.33 2.16)	0.015763	0.018568	92.5	0.125	(10.17 2.93)	0.007180	0.009461	108.3	0.216	(34.20 3.92)
	Adaptive	0.143600	0.143536	115.67	0.078	(3.40 2.05)	0.033059	0.036042	122.23	0.119	(11.07 2.72)	0.007096	0.009207	125.23	0.321	(77.90 3.97)
ABA	0.1 - 0.9	10.034018	9.980401	15.6	0.098	(1.10 2.33)	6.252071	6.261997	20.2	0.180	(5.57 3.34)	6.198179	6.219666	24.9	0.311	(15.07 4.32)
	0.3 - 0.7	13.471095	13.490648	9.1	0.058	(0.00 1.56)	6.933849	6.956984	13.8	0.099	(0.70 2.45)	6.204145	6.222632	22.9	0.220	(7.90 3.77)
	Adaptive	11.046818	11.051219	5.73	0.036	(0.07 0.94)	6.615973	6.628668	10.77	0.088	(1.67 1.92)	6.197562	6.219388	22.13	0.276	(13.50 3.80)
CON	0.1 - 0.9	67.672232	71.789422	86.8	0.143	(5.13 12.02)	59.010937	65.03955	66.0	0.201	(25.23 9.74)	56.380503	62.489223	62.6	0.257	(39.37 9.66)
	0.3 - 0.7	116.57328	123.10188	27.7	0.036	(0.30 3.45)	66.961798	73.82797	36.8	0.066	(2.67 5.44)	57.450287	63.305839	58.3	0.189	(24.53 8.79)
	Adaptive	102.84723	110.31467	26.50	0.038	(0.47 3.56)	61.63860	67.58694	37.37	0.078	(4.90 5.72)	56.389478	62.40864	58.00	0.227	(33.40 9.06)
STP	0.1 - 0.9	5.405339	5.474713	13.3	0.096	(1.33 3.21)	4.259753	4.362481	18.6	0.167	(4.63 4.54)	4.159594	4.255350	25.0	0.287	(11.73 6.19)
	0.3 - 0.7	8.748136	8.816542	7.9	0.058	(0.00 2.25)	4.455629	4.560821	14.7	0.125	(2.67 3.75)	4.194782	4.309624	24.9	0.252	(8.50 6.22)
	Adaptive	12.726416	12.816972	5.17	0.028	(0.00 1.07)	4.374793	4.494771	13.10	0.110	(2.67 3.18)	4.165515	4.247484	24.87	0.284	(11.53 6.17)
WAN	0.1 - 0.9	26.310868	26.725976	9.6	0.088	(0.53 2.86)	14.522565	14.77936	13.3	0.193	(4.40 4.46)	13.774738	14.009191	16.1	0.361	(12.27 6.19)
	0.3 - 0.7	32.988571	34.115293	7.9	0.065	(0.00 2.26)	15.540758	16.02159	11.5	0.122	(0.93 3.81)	13.948231	14.207306	15.2	0.302	(9.47 5.61)
	Adaptive	63.395682	62.167748	5.17	0.036	(0.03 1.20)	14.83103	15.04641	9.77	0.141	(2.57 3.46)	13.770163	13.98595	15.83	0.363	(12.60 6.25)
WIZ	0.1 - 0.9	12.545764	13.543774	15.7	0.107	(2.57 4.18)	3.881428	4.467590	19.7	0.193	(7.50 6.11)	3.413844	3.856717	25.0	0.351	(19.80 8.12)
	0.3 - 0.7	34.337238	36.299840	10.5	0.054	(0.00 2.74)	6.706427	7.391861	14.5	0.091	(0.87 4.18)	3.547122	4.014734	24.5	0.291	(14.73 7.61)
	Adaptive	68.862557	68.324376	7.23	0.029	(0.03 1.50)	11.29783	11.72132	11.77	0.093	(1.80 3.86)	3.419498	3.854768	24.50	0.358	(20.27 8.28)
MOR	0.1 - 0.9	3.056375	3.052106	6.0	0.060	(0.03 1.89)	0.423308	0.439410	8.6	0.134	(2.57 2.93)	0.267971	0.275199	12.8	0.305	(9.03 5.06)
	0.3 - 0.7	3.689980	3.747015	5.8	0.050	(0.00 1.58)	0.548342	0.553402	7.8	0.109	(1.50 2.69)	0.283597	0.290467	13.0	0.293	(8.13 5.15)
	Adaptive	2.081246	2.046311	5.07	0.044	(0.00 1.38)	0.364971	0.368820	7.30	0.137	(3.07 2.77)	0.293933	0.306642	12.93	0.321	(9.23 5.47)
TRE	0.1 - 0.9	4.887197	5.066371	5.8	0.054	(0.17 1.80)	0.806103	0.792245	8.7	0.104	(0.87 3.17)	0.448043	0.436540	10.4	0.229	(7.20 4.17)
	0.3 - 0.7	4.886871	4.991007	5.7	0.046	(0.00 1.62)	0.991394	0.975439	8.0	0.084	(0.23 2.81)	0.455389	0.437021	10.3	0.211	(6.17 4.10)
	Adaptive	3.762724	3.686490	5.00	0.043	(0.00 1.49)	0.906458	0.912706	6.73	0.080	(0.17 2.68)	0.449993	0.436562	10.90	0.246	(7.93 4.38)

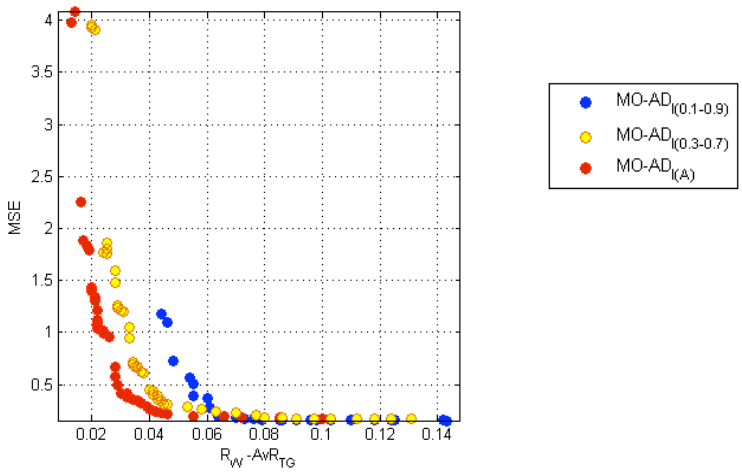

Fig. 5. Example Pareto fronts and Thresholds obtained in MOR problem

Table 11 CS ratios obtained in the accuracy-interpretability plane

Datasets	*WM*				*FS_MOGUL*			
	Adaptive vs. 0.1_0.9	*0.1_0.9 vs. Adaptive*	*Adaptive vs. 0.3_0.7*	*0.3_0.7 vs. Adaptive*	*Adaptive vs. 0.1_0.9*	*0.1_0.9 vs. Adaptive*	*Adaptive vs. 0.3_0.7*	*0.3_0.7 vs. Adaptive*
PLA	**0.733**	0.205	**0.788**	0.119	**0.825**	0.373	0.363	**0.708**
QUA	0.278	**0.671**	0.416	**0.491**	**0.815**	0.200	0.381	**0.875**
ELE	0.263	**0.554**	0.243	**0.641**	**0.563**	0.321	0.144	**0.815**
AU6	**0.675**	0.445	**0.811**	0.211	**0.605**	0.502	0.299	**0.815**
AU8	**0.683**	0.363	**0.709**	0.447	**0.806**	0.337	**0.803**	0.399
ANA	**0.830**	0.051	**0.966**	0.001	**0.904**	0.008	**0.613**	0.227
ABA	**0.726**	0.157	**0.906**	0.079	**0.822**	0.140	**0.922**	0.112
CON	**0.643**	0.182	**0.554**	0.424	**0.912**	0.063	**0.678**	0.397
STP	**0.470**	0.329	**0.595**	0.420	**0.675**	0.307	**0.845**	0.180
WAN	**0.654**	0.267	**0.531**	0.478	**0.910**	0.149	**0.937**	0.099
WIZ	**0.613**	0.297	**0.591**	0.253	**0.807**	0.198	**0.886**	0.105
MOR	**0.627**	0.187	**0.794**	0.164	**0.792**	0.154	**0.860**	0.125
TRE	**0.439**	0.292	**0.677**	0.297	**0.772**	0.105	**0.802**	0.127

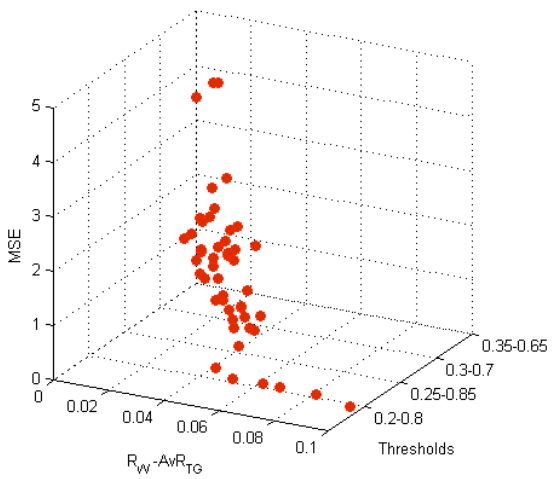

Fig. 6. Example of thresholds found by the adaptive width mechanism in MOR problem.

The most spread adaptive defuzzification, based on the use of rule weights, improves significantly the accuracy of the FRBSs, but reduces the system interpretability noticeably. In this work, and making use of an important instrument nowadays in the development of accurate and interpretable FRBSs such as multi-objective evolutionary algorithms, we propose a way to improve the interpretability of this kind of adaptive defuzzification systems also improving their accuracy. This is carried out by introducing a mechanism to reduce the number of rules, the number of rules with weights and proposing an interpretability index that also involves the measure of rules triggered

jointly, to reach different slopes of the interpretability together. The methodology proposed can be self-adaptive, and produces a set of solutions with different balance, from the most interpretable to the most accurate, also improving the accuracy of this kind of classical adaptive defuzzification accuracy oriented methods, as shown by developing an experimental study with thirteen data sets, two different rule bases, statistical tests and a measure to compare multi-objective Pareto fronts.

It is important to note that adaptive defuzzification, and thus the proposal developed in this paper, can be combined with other methodologies to design or improve the accuracy or interpretability of fuzzy rule based systems. It may be viewed as an additional element to use within this context.

We consider that together with interpretability comprehension and advances, methods to improve interpretability could be enhanced in the near future, based on the fine balanced combination of measures and slopes of interpretability.

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Appendix A.

Statistical analysis of the comparison between the different thresholds

Tables A.1 to A.6 show the rankings of the different methods considered in this study for WM and FS_MOGUL rule bases, respectively. This study also took into account the $\#R_F$ measure in order to confirm the relation between $R_W\_AvR_{TG}$ index and $\#R_F$ measure. The Friedman test tells us that there are significant differences among the results observed in all data-sets when the p-fried <0.05. Indeed, for MAX ACC, MEDIAN INT/ACC and the MAX INT there are significant differences between the uses of different thresholds. However, in all points the best ranking is obtained by $MO\text{-}AD_{I(0.1\text{-}0.9)}$ for the accuracy and the $MO\text{-}AD_{I(0.3\text{-}0.7)}$for the index of $R_W\_AvR_{TG}$ and $\#R_F$ measure.

In almost all cases (excepting MAX ACC for $\#R_F$) we now can apply Finner post-hoc procedure to compare the best ranking method in each case with the remaining methods. Tables A.7 to A.12 present these results. In these tables, the algorithms are ordered with respect to the p-value obtained with respect to control algorithm. Finner's test rejects the hypothesis of equality when the p-Finner is < 0.05. Indeed, the Finner's test rejects the hypothesis of equality for MSE_{TST} for MAX ACC, MAX INT and MEDIAN INT/ACC point when the control algorithm is $MO\text{-}AD_{I(0.1\text{-}0.9)}$. It also rejects the hypothesis with $MO\text{-}AD_{I(0.3\text{-}0.7)}$ methods in $R_W\_AvR_{TG}$ for all representative points.

On the other hand, it also rejects the hypothesis with $MO\text{-}AD_{I(0.3\text{-}0.7)}$ methods in $\#R_F$ in MAX INT and MEDIAN INT/ACC point but not in MAX ACC. Nevertheless, in all cases the best algorithm (control algorithm) is $MO\text{-}AD_{I(0.3\text{-}0.7)}$..

Table A.1 Rankings obtained through Friedman's test for different values of thresholds on MSE_{TST}, $R_W\_AvR_{TG}$ and $\#R_F$ measures using WM rule bases for MAX ACC point.

Algorithm	Ranking on MSE_{TST} (p-fried: 0,000196)	Ranking $R_W\_AvR_{TG}$ (p-friedn:0,000006)	Ranking on R_F (p-fried: 0,125315)
$MO\text{-}AD_{I(0.1\text{-}0.9)}$	1.1538	3	2.4615
$MO\text{-}AD_{I(0.2\text{-}0.8)}$	2.0769	1.9231	1.7692
$MO\text{-}AD_{I(0.3\text{-}0.7)}$	2.7692	1.0769	1.7692

Table A.2 Rankings obtained through Friedman's test for different values of thresholds on MSE_{TST}, $R_W\_AvR_{TG}$ and $\#R_F$ measures using FS_MOGUL rule bases for MAX ACC point.

Algorithm	Ranking on MSE_{TST} (p-fried: 0,0000091)	Ranking $R_W\_AvR_{TG}$ (p-fried: 0,000002)	Ranking on R_F (p-fried: 0,001366)
$MO\text{-}AD_{I(0.1\text{-}0.9)}$	1.1538	3	2.8077
$MO\text{-}AD_{I(0.2\text{-}0.8)}$	2	2	1.7308
$MO\text{-}AD_{I(0.3\text{-}0.7)}$	2.8462	1	1.4615

Table A.3 Rankings obtained through Friedman's test for different values of thresholds on MSE_{TST}, $R_W\_AvR_{TG}$ and $\#R_F$ measures using WM rule bases for MAX INT point.

Algorithm	Ranking on MSE_{TST} (p-fried: 0,000004)	Ranking $R_W\_AvR_{TG}$ (p-fried: 0,000004)	Ranking on R_F (p-fried: 0,000004)
$MO\text{-}AD_{I(0.1\text{-}0.9)}$	1	3	3
$MO\text{-}AD_{I(0.2\text{-}0.8)}$	2.0385	1.9615	1.9615
$MO\text{-}AD_{I(0.3\text{-}0.7)}$	2.9615	1.0385	1.0385

Table A.4 Rankings obtained through Friedman's test for different values of thresholds on MSE_{TST}, $R_W\_AvR_{TG}$ and $\#R_F$ measures using FS_MOGUL rule bases for MAX INT point.

Algorithm	Ranking on MSE_{TST} (p-fried: 0,000912)	Ranking $R_W\_AvR_{TG}$ (p-fried: 0,000002)	Ranking on R_F (p-fried: 0,000012)
$MO\text{-}AD_{I(0.1\text{-}0.9)}$	1.3077	3	3
$MO\text{-}AD_{I(0.2\text{-}0.8)}$	1.9231	2	1.8462
$MO\text{-}AD_{I(0.3\text{-}0.7)}$	2.7692	1	1.1538

Table A.5 Rankings obtained through Friedman's test for different values of thresholds on MSE_{TST}, $R_W\_AvR_{TG}$ and $\#R_F$ measures using WM rule bases for MEDIAN INT/ACC point.

Algorithm	Ranking on MSE_{TST} (p-fried: 0,000002)	Ranking $R_W\_AvR_{TG}$ (p-fried: 0,000002)	Ranking on R_F (p-fried: 0,000002)
$MO\text{-}AD_{I(0.1\text{-}0.9)}$	1	3	3
$MO\text{-}AD_{I(0.2\text{-}0.8)}$	2	2	2
$MO\text{-}AD_{I(0.3\text{-}0.7)}$	3	1	1

Table A.6 Rankings obtained through Friedman's test for different values of thresholds on MSE_{TST}, $R_W\_AvR_{TG}$ and $\#R_F$ measures using FS_MOGUL rule bases MEDIAN INT/ACC point.

Algorithm	Ranking on MSE_{TST} (p-fried: 0,000006)	Ranking $R_W\_AvR_{TG}$ (p-fried: 0,000002)	Ranking on R_F (p-fried: 0,000002)
$MO\text{-}AD_{I(0.1\text{-}0.9)}$	1.0769	3	3
$MO\text{-}AD_{I(0.2\text{-}0.8)}$	1.9231	2	2
$MO\text{-}AD_{I(0.3\text{-}0.7)}$	3	1	1

Table A.7 Finner Table with $\alpha = 0.05$ for the methods on MSE_{TST}, $R_W\_AvR_{TG}$ and $\#R_F$ on MAX ACC point using WM rule bases.

MSE_{TST}				$R_W\_AvR_{TG}$				$\#R_F$			
i	Algorithm	p-finner	Hypot	i	Algorithm	p-finner	Hypot	i	Algorithm	p-finner	Hypot
1	$MO\text{-}AD_{I(0.3\text{-}0.7)}$	0.000076	Rejec.	1	$MO\text{-}AD_{I(0.1\text{-}0.9)}$	0.000002	Rejec.	1	$MO\text{-}AD_{I(0.1\text{-}0.9)}$	0.149097	Accep.
2	$MO\text{-}AD_{I(0.2\text{-}0.8)}$	0.018603	Rejec.	2	$MO\text{-}AD_{I(0.2\text{-}0.8)}$	0.030984	Rejec.	2	$MO\text{-}AD_{I(0.2\text{-}0.8)}$	1	Accep.

Table A.8 Finner Table with $\alpha = 0.05$ for the methods on MSE_{TST}, $R_W\_AvR_{TG}$ and $\#R_F$ measures on MAX ACC point using FS_MOGUL rule bases

MSE_{TST}				$R_W\_AvR_{TG}$				$\#R_F$			
i	Algorithm	p-finner	Hypot	i	Algorithm	p-finner	Hypot	i	Algorithm	p-finner	Hypot
1	$MO\text{-}AD_{I(0.3\text{-}0.7)}$	0.000032	Rejec.	1	$MO\text{-}AD_{I(0.1\text{-}0.9)}$	0.000001	Rejec.	1	$MO\text{-}AD_{I(0.1\text{-}0.9)}$	0.001179	Rejec.
2	$MO\text{-}AD_{I(0.2\text{-}0.8)}$	0.030984	Rejec.	2	$MO\text{-}AD_{I(0.2\text{-}0.8)}$	0.010187	Rejec.	2	$MO\text{-}AD_{I(0.2\text{-}0.8)}$	0.492457	Accep.

Table A.9 Finner Table with $\alpha = 0.05$ for the methods on MSE_{TST}, $R_W\_AvR_{TG}$ and $\#R_F$ measures on MAX INT point using WM rule bases

MSE_{TST}				$R_W\_AvR_{TG}$				$\#R_F$			
i	Algorithm	p-finner	Hypot	i	Algorithm	p-finner	Hypot	i	Algorithm	p-finner	Hypot
1	$MO\text{-}AD_{I(0.3\text{-}0.7)}$	0.000001	Rejec.	1	$MO\text{-}AD_{I(0.1\text{-}0.9)}$	0.000001	Rejec.	1	$MO\text{-}AD_{I(0.1\text{-}0.9)}$	0.000001	Rejec.
2	$MO\text{-}AD_{I(0.2\text{-}0.8)}$	0.008107	Rejec.	2	$MO\text{-}AD_{I(0.2\text{-}0.8)}$	0.018603	Rejec.	2	$MO\text{-}AD_{I(0.2\text{-}0.8)}$	0.018603	Rejec.

Table A.10 Finner Table with α = 0.05 for the methods on MSE_{TST}, $R_W\_AvR_{TG}$ and $\#R_F$ measures on MAX INT point using FS_MOGUL rule bases

MSE_{TST}				RW_AvR_{TG}				$\#R_F$			
i	Algorithm	p-finner	Hypot	i	Algorithm	p-finner	Hypot	i	Algorithm	p-finner	Hypot
1	$MO\text{-}AD_{I(0.3\text{-}0.7)}$	0.000389	Rejec.	1	$MO\text{-}AD_{I(0.1\text{-}0.9)}$	0.000001	Rejec.	1	$MO\text{-}AD_{I(0.1\text{-}0.9)}$	0.000005	Rejec.
2	$MO\text{-}AD_{I(0.2\text{-}0.8)}$	0.116664	Accep	2	$MO\text{-}AD_{I(0.2\text{-}0.8)}$	0.010187	Rejec.	2	$MO\text{-}AD_{I(0.2\text{-}0.8)}$	0.047556	Rejec.

Table A.11 Finner Table with α = 0.05 for the methods on MSE_{TST}, $R_W\_AvR_{TG}$ and $\#R_F$ measures on MEDIAN INT/ACC point using WM rule bases

MSE_{TST}				RW_AvR_{TG}				$\#R_F$			
i	Algorithm	p-finner	Hypot	i	Algorithm	p-finner	Hypot	i	Algorithm	p-finner	Hypot
1	$MO\text{-}AD_{I(0.3\text{-}0.7)}$	0.000001	Rejec.	1	$MO\text{-}AD_{I(0.1\text{-}0.9)}$	0.000001	Rejec.	1	$MO\text{-}AD_{I(0.1\text{-}0.9)}$	0.000001	Rejec.
2	$MO\text{-}AD_{I(0.2\text{-}0.8)}$	0.010187	Rejec.	2	$MO\text{-}AD_{I(0.2\text{-}0.8)}$	0.010187	Rejec.	2	$MO\text{-}AD_{I(0.2\text{-}0.8)}$	0.010787	Rejec.

Table A.12 Finner Table with α = 0.05 for the methods on MSE_{TST}, $R_W\_AvR_{TG}$ and $\#R_F$ measures on MEDIAN INT/ACC point using FS_MOGUL rule bases

MSE_{TST}				RW_AvR_{TG}				$\#R_F$			
i	Algorithm	p-finner	Hypot	i	Algorithm	p-finner	Hypot	i	Algorithm	p-finner	Hypot
1	$MO\text{-}AD_{I(0.3\text{-}0.7)}$	0.000002	Rejec.	1	$MO\text{-}AD_{I(0.1\text{-}0.9)}$	0.000001	Rejec.	1	$MO\text{-}AD_{I(0.1\text{-}0.9)}$	0.000001	Rejec.
2	$MO\text{-}AD_{I(0.2\text{-}0.8)}$	0.030984	Rejec.	2	$MO\text{-}AD_{I(0.2\text{-}0.8)}$	0.010187	Rejec.	2	$MO\text{-}AD_{I(0.2\text{-}0.8)}$	0.010787	Rejec.

Appendix B.

Statistical analysis of the comparison between single-objective and multi-objective approaches.

Tables B.1 and B.2 show the rankings (through Friedman's test) for both WM and FS_MOGUL rule bases, respectively. The p-value computed using the Friedman test implies that there are statistical differences among the results on MSE_{TST}, $R_W\_AvR_{TG}$, and $\#R_F$ respectively. In all cases $MO\text{-}AD_{I(0.1\text{-}0.9)}$ is the best in the ranking. In all cases, Finner test (Tables B.3, B.4 and B.5) rejects the null hypothesis with all single-objective methods.

Table B.1 Rankings obtained through Friedman's test for different values of thresholds on MSE_{TST}, $R_W\_AvR_{TG}$ and $\#R_F$ measures using WM rule bases

Algorithm	Ranking on MSE_{TST} (p-fried: 0,024914)	Ranking on $R_W\_MR_{TG}$ (p-fried: 0,000002)	Ranking on R_F (p-fried: 0,000002)
$MO\text{-}AD_{I(0.1\text{-}0.9)}$	1.3846	1	1
$SO\text{-}AD_{I(0.1\text{-}0.9)}$	2.3077	2	2
SO-AD	2.3077	3	3

Table B.2 Rankings obtained through Friedman's test for different values of thresholds on MSE_{TST}, $R_W\_AvR_{TG}$ and $\#R_F$ measures using FS_MOGUL rule bases

Algorithm	Ranking on MSE_{TST} (p-fried: 0,000072)	Ranking on $R_W\_MR_{TG}$ (p-fried: 0,000002)	Ranking on R_F (p-fried: 0,000002)
$MO\text{-}AD_{I(0.1\text{-}0.9)}$	1.0769	1	1
$SO\text{-}AD_{I(0.1\text{-}0.9)}$	2.1538	2	2
SO-AD	2.7692	3	3

Table B.3 Finner Table with $\alpha = 0.05$ for the methods on MSE_{TST} , $R_W\_AvR_{TG}$ and $\#R_F$ measures on MAX ACC point using WM rule bases

MSE_{TST}				$R_W\_MR_{TG}$				$\#R_F$			
i	Algorithm	p-finner	Hypot	i	Algorithm	p-finner	Hypot	i	Algorithm	p-finner	Hypot
1	SO-AD	0.03686	Rejec.	1	SO-AD	0.000001	Rejec.	1	SO-AD	0.000001	Rejec.
2	$SO\text{-}AD_{I(0.1\text{-}0.9)}$	0.03686	Rejec.	2	$SO\text{-}AD_{I(0.1\text{-}0.9)}$	0.010787	Rejec.	2	$SO\text{-}AD_{I(0.1\text{-}0.9)}$	0.010787	Rejec.

Table B.4 Finner Table with $\alpha = 0.05$ for the methods on MSE_{TST} , $R_W\_AvR_{TG}$ and $\#R_F$ measures on MAX ACC point using FS_MOGUL rule bases

MSE_{TST}				$R_W\_MR_{TG}$				$\#R_F$			
i	Algorithm	p-finner	Hypot	i	Algorithm	p-finner	Hypot	i	Algorithm	p-finner	Hypot
1	SO-AD	0.000032	Rejec.	1	SO-AD	0.000001	Rejec.	1	SO-AD	0.000001	Rejec.
2	$SO\text{-}AD_{I(0.1\text{-}0.9)}$	0.00604	Rejec.	2	$SO\text{-}AD_{I(0.1\text{-}0.9)}$	0.010787	Rejec.	2	$SO\text{-}AD_{I(0.1\text{-}0.9)}$	0.010787	Rejec.

Table B.5 Finner Table with $\alpha = 0.05$ for the methods on MSE_{TST} , $R_W\_AvR_{TG}$ and $\#R_F$ measures on MAX ACC point using FS_MOGUL rule bases

MSE_{TST}				$R_W\_MR_{TG}$				$\#R_F$			
i	Algorithm	p-finner	Hypot	i	Algorithm	p-finner	Hypot	i	Algorithm	p-finner	Hypot
1	SO-AD	0.000032	Rejec.	1	SO-AD	0.000001	Rejec.	1	SO-AD	0.000001	Rejec.
2	$SO\text{-}AD_{I(0.1\text{-}0.9)}$	0.00604	Rejec.	2	$SO\text{-}AD_{I(0.1\text{-}0.9)}$	0.010787	Rejec.	2	$SO\text{-}AD_{I(0.1\text{-}0.9)}$	0.010787	Rejec.