

Study on circular loop source TEM with different solutions

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Abstract—The circular loop source is calculated by the method of inscribed polygon inner the circle or numerical calculation. The former uses decomposed polygon in each side with electric dipole, then stack all the electric dipole response; the latter adopts the method of digital filter to calculate the integral expression directly. There is some error between above methods and exact solutions in transient electromagnetic response values. This paper compares the two kinds of method. First, the equivalence of inscribed polygon is analyzed and the number of edges of the polygon, the proportion coefficient of transceiver distance and the length of the dipole are debated. Second, the contrast research in homogeneous half space with the two methods is carried on. Finally, layered medium is inverted by Artificial Bee Colony (ABC) algorithm and studying the fitting error. The results show that if inscribed polygon has few numbers of edges, the response error and fitting error calculated by the method of inscribed polygon inner the circle are higher than numerical calculation method in any conditions. But when the edge of inscribed polygon reach a certain number, the calculation accuracy is better than the numerical calculation method under a suitable parameter combination.

Keywords—circular loop source; inscribed polygon inner the circle; parameter combination; numerical calculation; inversion of layered medium

I INTRODUCTION

Transient electromagnetic method (TEM) is widely used in time domain electromagnetic detection problems; the shape of transmitter loop is mainly about rectangular loop^{[1]-[9]} and circular loop^{[10][11]}. For the circular loop, there are two computing methods. Based on the above work, further study is carried on inscribed polygon inner the circle, regard the proportion coefficient of transceiver distance and the length of the dipole as a function of the number of polygon edges and the concept of the smallest proportion coefficient with different number of edges is put forward. Under this premise, optimal parameter combination is selected to calculate electromagnetic response. Using the two methods respectively to calculate electromagnetic response in the homogeneous half space and using the two methods respectively to solve the inverse problem with Artificial Bee Colony algorithm in layered model. Comprehensive all the results, advantages and disadvantages of different approaches are acquired.

II THE FUNDAMENTAL THEORY

A. The Minimum Proportion Coefficient

The basic idea of inscribed polygon inner the circle as follows: according to cyclotomy in math, when the number of edges increases, the circumference of polygon is close to the perimeter of the circle. Therefore, inscribed polygon inner the circle can be used for a circular loop. Inside the loop, the electromagnetic field can be regarded as the combination of electromagnetic field from each side of the loop. When each side of the loop can be decomposed into multiple horizontal electric dipoles, the electromagnetic field at any point inside the loop can be regarded as superposition of many horizontal electric dipoles.

In homogeneous half space, the vertical magnetic field of polygon can be obtained by horizontal electric dipole:

$$h_{zm} = \sum_{i=1}^m \sum_{j=1}^n h_{zj} \quad (1)$$

$$= \sum_{i=1}^m \sum_{j=1}^n \frac{I ds y}{4\pi r^3} \times \left[\left(1 - \frac{3}{2\theta^2 r^2}\right) \text{erf}(\theta r) + \frac{3}{\theta r \sqrt{\pi}} \int_0^{\theta r} e^{-\theta^2 r^2} \right]$$

Assuming that R is the radius, m is the number of edge, L is the length of side and d is the distance between loop center and each edge. So, there are the follow relations:

$$L = 2R \sin(\pi / m) \quad (2)$$

$$d = R \cos(\pi / m) \quad (3)$$

Set each edge can be decomposed into n electric dipoles.

$$n = L / ds \geq 1 \quad (4)$$

Set

$$A_{\min} = r_{\min} / ds = d / ds \quad (5)$$

So

$$A_{\min} \geq \cot(\pi / m) / 2 \quad (6)$$

Select $m = 4, 8, 16, 32, 64, 128, 256, 512$, we can obtain the curve of $A_{\min} = \cot(\pi / m) / 2$ (Fig. 1) and different A_{\min} corresponding with m (0, integrate A_{\min}).

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are significant differences in results obtained by different

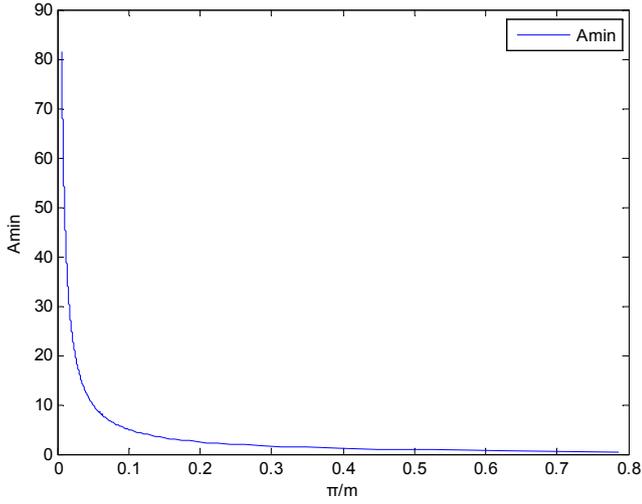


Fig. 1. The curve of $A_{\min} = \cot(\pi / m) / 2$

TABLE I. THE MINIMUM PROPORTION COEFFICIENT

m	4	8	16	32	64	128	256	512
A_{\min}	1	2	3	6	11	21	41	82

TABLE II. THE OPTIMAL COMBINATION BETWEEN m AND A

m	4	8	16	32	64	128	256	512
A	100	50	5	10	20	30	50	100

B. The Optimal Combination

The analytical expressions of magnetic field deduced the circular loop in homogenous half space can be written as^[7]:

$$h_z = \frac{I}{2R} \left[\frac{3}{\sqrt{\pi\theta R}} e^{-\theta^2 R^2} + \left(1 - \frac{3}{2\theta^2 R^2}\right) \text{erf}(\theta R) \right] \quad (1)$$

As a general rule, the bigger A is, the better the results is under the premise of 0. But compared with analytic solution, there is an optimal A corresponding to m (TABLE II.).

C. The Method of Numerical Calculation

The vertical magnetic field intensity in the center of receiver can be written as^[11]:

$$H_z = Ia \int_0^\infty \frac{\lambda^2}{\lambda + u_1} J_1(\lambda a) d\lambda \quad (2)$$

Uses of Fast Hankel transform^[13] to calculate the Bessel function in (8) and Gaver-Stehfest inverse Laplace transform^[14] to realize the Frequency-Time Transformation.

III COMPARISON IN HOMOGENOUS HALF SPACE

Set $\rho = 100\Omega \bullet m, r = 50m, I = 1A$, and the following results are calculated according to TABLE II. .

Fig. 2 shows in the observation period, the attenuation curves from different methods have the same trend. And there

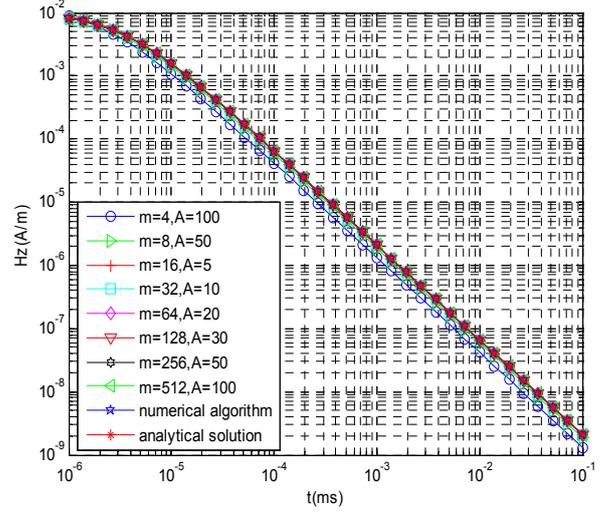


Fig. 2. The vertical magnetic field intensity

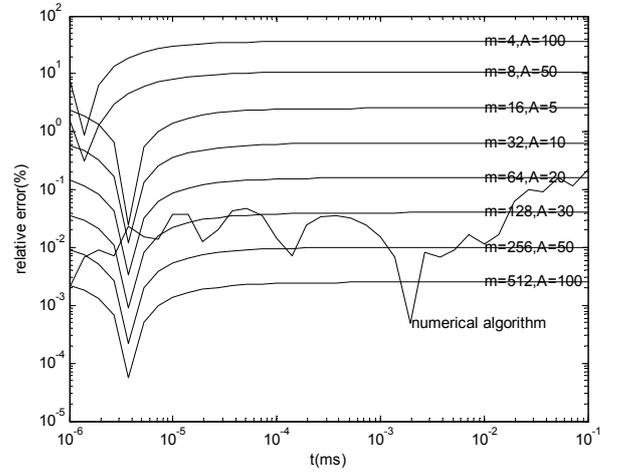


Fig. 3. The relative error of vertical magnetic field intensity

combination. Fig. 3 shows the vertical magnetic field intensity obtained by the method of numerical calculation maintains high accuracy in the observation time and the range of relative error is $3 \times 10^{-4}\% \sim 2 \times 10^{-1}\%$. For the method of inscribed polygon inner the circle, the precision enhances with the increase of the number of the polygon. Compared with the method of numerical calculation, if the number of edges of the polygon is less than a certain number (128), the calculated precision obtained was inferior to the numerical method. But if it reaches a certain number (256), the calculated precision obtained was superior to the numerical method as long as choose an appropriate combination of parameters.

IV COMPARISON IN LAYERED MODEL

Set the H type geoelectric model for example (0) and layered medium is inverted by Artificial Bee Colony algorithm (ABC). In order to make the results more representative, $m=128, A=30$ and $m=256, A=50$ are selected to calculate the results of the method of inscribed polygon inner the circle. The inversion results and curves of relative error as followed.

It shows that the relative error of each parameter with numerical algorithm is 0.135%, 1.2%, 0.103%, 0.54%, 1.26% and the method of inscribed polygon inner the circle is 0.242%, 8.71%, 0.258%, 1.88%, 9.43%, while $m=128, A=30$ and 0.014%, 0.99%, 0.597%, 0.064%, 1.48%, while $m=256, A=50$. Combined with the relative error of inversion result in Fig. 4, we can find that the numerical calculation method has a higher fitting precision before 10^{-2} ms than the method of inscribed polygon inner the circle when $m \leq 128$ and almost the same level in late time period, but the latter is lower in fitting precision in general within the calculate time. While $m \geq 256$, the latter has a higher fitting precision in general and this is same as the conclusion obtained from homogenous half space.

TABLE III. THE INVERSION RESULTS OF H TYPE GEOELECTRIC MODEL

Parameters	ρ_1	ρ_2	ρ_3	h_1	h_2
Theoretical value	100	10	300	150	50
Hunting zone	50-150	5-15	200-400	100-200	10-100
Inversion of numerical algorithm	100.135	10.120	299.691	149.185	50.629
Inversion of $m=128, A=30$	100.242	10.871	299.227	147.176	54.716
Inversion of $m=256, A=50$	99.986	10.099	301.791	149.903	50.739

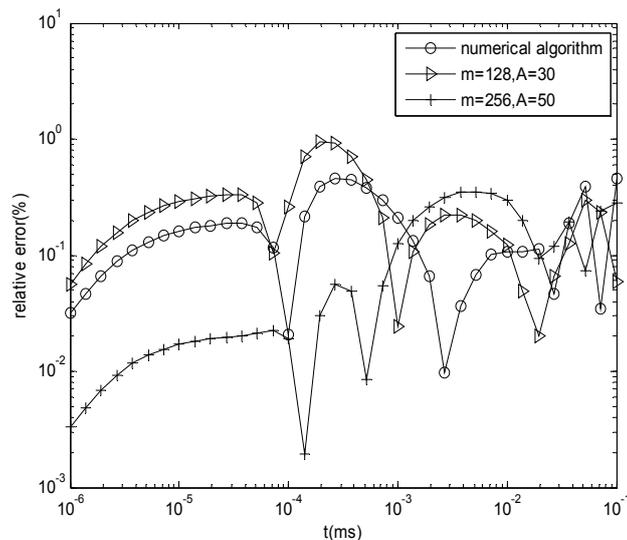


Fig. 4. The relative error of inversion result for magnetic field intensity

V CONCLUSIONS

For the method of inscribed polygon inner the circle, m and A must be combined. There is the smallest proportion coefficient A and the optimal combination between m and A .

For the method of inscribed polygon inner the circle, the more number of polygon edges, the better accuracy of the results.

Under the condition of $m \leq 128$, the fitting error calculated by the method of inscribed polygon inner the circle are higher than numerical calculation method; while $m \geq 256$, the former's calculation accuracy is better.

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