Dimensional Quality Oriented Reliability Modeling for Complex Manufacturing Processes

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Abstract

The stability of process is certainly one of the most important aspects to fulfill the task. Now, it is still a challenge to put forward a model to deal with the stability of manufacturing process mathematically. Aiming at solving the problem, this paper discusses the issues by the notion of manufacturing process reliability and its sensitivity. Based on state space model, the process performance function which is used to describe the relations between the key product characters (KPC) of the machined part and the key control characters (KCC) of multistage process in machining system has been built. Furthermore, the measure index to the process reliability of multistage machining system is proposed, which facilitates the forming of the ways to model the process reliability mathematically. To determine the weakest stage during machining process, the sensitivity of process reliability to KCC of the process has been put forward and the corresponding ways how to calculate it. Because of the multistage process is highly non-linear process, chaos optimization algorithm and mutative scale chaos optimization algorithm are used to calculate this kind of robust reliability index. Finally, A simple 2-D case study has been used to validate the proposed model. It shows that the process reliability can be calculated effectively.

Keywords: state space model, mutative chaos optimization algorithm, process reliability modeling, multistage machining process.

1. Introduction

Manufacturing process is the bridge to connect the design reliability and real product reliability. There are some quality instability phenomena due to the random variation in sources such as materials, machines, fixtures and machining methods in the process. Stability of any machining process is one of the most important key factors to measure process reliability. Zhang et al have showed that issues of quality and reliability count for 24 percent of the manufacturing processes in the industry [1]. Nowadays, the stability of machining process is not high which the requirement of certain industries such aviation industry due to the variation in the machining of products which involve complex machining process, special raw material and complex structure. Thus, this makes research on the machining process reliability one of the hottest research fields in modern machining research.


But those researchers are more concerned about the reliability of the machining system as the reliability of used machine and the fixtures, or the design of reliability [7-8]. Multiple stages machining process is one of
the important manufacturing processes in modern manufacturing field. Some researchers have done considerable amount of work on the product quality and machining system reliability [9-11] but the task reliability of the machining process, normally how to keep the quality of the machined products stable and how to establish the mathematical model to describe machining process reliability is not sufficient. So, this seems to be a prospective area to conduct research.

Now-a-days, there is a phenomenon that most machining processes are unstable processes and process reliability is not high. There have been many factors contributing to this situation, such as the lack of clarity in the relations between the Key process characters and the reliability of the machined parts. So, it is an important task to study the mechanism of the forming and representation of the machining process reliability.

This paper deals machining process reliability with the exact definition of process reliability, modeling of machining process reliability and sensitive analysis. The remainder of the paper is organized as follows; section 2 gives the linear state modeling of process reliability and the corresponding sensitivity determination. In section 3 a simple two-dimensional example is used to illustrate the implementation of the newly proposed method in section 2. Finally, the conclusion of the paper is drawn in section 4.

2. Modeling of Process Reliability

2.1 The Conception of Process Reliability

Process reliability has been studied against the process defects as well as product reliability against failure. The reliability of advanced manufacturing system can be dealt as basic reliability and task reliability. Here, the reliability of machining process is referred as the ability of process to meet the design requirement, namely the reliability of machining process - namely the ability of process to manufacture the quality bound product stability”. Process reliability consider the uncertainty of dimension, material and process parameters, and analyze how these random variables effect on the process reliability through numerical simulation and various resolution. The task of manufacturing system is to produce the quality bound part. So, quality parameter of produced part is one of the most important indexes to process reliability, which is used in this paper to formulate the model of machining process reliability.

2.2 Performance Function of Machining Process

The machined part of machining process is modeled as a vector by stacking up all part feature vectors, so we have

\[ X^a(k) = \left[ X^1_1(k) \ X^1_2(k) \ \ldots \ X^1_n(k) \right] \]

where \( X^1_i(k) \) denotes a part feature parameter. According to the coordinate system transformation theory, part feature representation can be transformed from coordinate system LCS to coordinate system GCS as follows:

\[ X^c(k) = T_{L-e}(k)X^a(k) + R^c_e(k) \]

where \( T_{L-e}(k) \) denotes the transformation matrix transforming vector representation in coordinate system LCS into the coordinate system GCS. \( R^c_e(k) \) denotes the representation of the original coordinate system LCS in coordinate system GCS. In the same way part feature representation can be transformed from coordinate system WCS to coordinate system LCS as follows:

\[ X^w(k) = T_{w-e}(k)X^c(k) + R^w_e(k) \]

where \( T_{w-e}(k) \) denotes the matrix that transforms vector representation in coordinate system WCS into the coordinate system LCS, while \( R^w_e(k) \) denotes the representation of the original coordinate system WCS in coordinate system LCS.

Combining Eq. (1) and (2) we have

\[ X^w(k) = T_{w-e}(k)T_{L-e}(k)X^a(k) - T_{w-e}(k)R^c_e(k) \]

where \( T_{w-e}(k) \), \( T_{L-e}(k) \) are inverse matrices of \( T_{w-L}(k) \), \( T_{L-e}(k) \) and denotes inverse transformation of vector representation from LCS to WCS and GCS to LCS respectively.

The above part feature positions are nominal value where no factual errors are considered. As many factors
will cause the deviation of workpiece features, so the dimensional errors emerge which causes failure of the machining process. The emergence and stacking up of the dimensional errors can be described by Stream of Variation (SOV) model.

The performance function of process is used to describe the relations of key control characters (KCC) of the process and key product characters (KPC) of the machined part mathematically. In fact machining process can be reckoned as discrete time-varying system, while the machining stage is used as time index and the dimensional errors as outputs. In this condition, state space model can be used to construct performance function of process.

\[
\Delta X[k] = A[k] \Delta X[k-1] + B[k] \epsilon[k] + W[k]
\]  
\[\text{(5)}\]

where \(\Delta X[k]\) are the dimensional deviation vector of the k stage, which is one of the important parts of KPC, while \(\Delta X[k-1]\) is that of the k-1 stage. \(A[k]\) is the dimensional deviation state matrix which reflects the influence of feature state vector at the k-1 stage on dimensional deviation state vector of the k stage; \(B[k]\) shows how errors \(\Delta X[k]\) depend on the newly introduced machining errors \(U[k]\), which is the one part of KCC. And vectors \(W[k]\) take into account the residuals after linearization and un-modeled effects. Fig.1 illustrates the multi-stage machining system. The following sections will show the derivation of the state space model for multi-stage machining system.

\[
\begin{align*}
&\Delta X_k[k] = A[k] \Delta X_{k-1}[k] + B[k] \epsilon[k] + W[k] \\
\text{(6)}
\end{align*}
\]

Fig. 1. Diagram of traditional multi station machining process

Actually the position and orientation of coordinate system LCS in coordinate system GCS is determined by fixture parameters. In that situation \(T_{L-G}[k]\) and \(R_{L-G}[k]\) which transfer the expression of position and orientation of coordinate system LCS into coordinate system GCS are determined by fixture parameters vectors \(L'_{k}\) and can be written as follows

\[
T_{L-G}[k] = f_s(L'_{k}), R_{L-G}[k] = f_s(L'_{k})
\]

\[\text{(6)}\]

It should also be noticed that previous machined features denoted as \(X^w[k-1]\), especially those features used as locating datum, may cause a deviation of workpiece and the attached coordinate system WCS in coordinate system LCS in the machining station where operation k is performed, thus influencing dimensional errors. So \(X^w[k-1]\) will determine the transformation matrix \(T_{L-G}[k]\) and the vector \(R_{L-G}[k]\) which describe the orientation and position of coordinate system WCS in coordinate system LCS. Therefore, there exist functions \(f_s(X^w[k-1])\) and \(f_s(X^w[k-1])\), such that

\[
T_{L-G}[k] = f_s(X^w[k-1]), R_{L-G}[k] = f_s(X^w[k-1])
\]

\[\text{(7)}\]

Substituting Eq. (6) and (7) into Eq. (4) gives

\[
X^w[k] = f_s(X^w[k-1])f_s(L_{k})X^w[k] - f_s(X^w[k-1])f_s(L_{k})f_s(L_{k})
\]

\[\text{(8)}\]

Differentiating Eq. (8), finally we have

\[
\Delta X^w[k] = A[k] \Delta X^w[k-1] + B[k] \epsilon[k] + \epsilon[k]
\]

\[\text{(9)}\]

where

\[
A[k] = \frac{\partial f_s}{\partial X_{k-1}}(f_s(L_{k})X^w[k] - f_s(L_{k})f_s(L_{k})) - f_s(X^w[k-1]) - \frac{\partial f_s}{\partial X_{k-1}}f_s(X^w[k-1])
\]

\[\text{(9)}\]

\[
B[k] = \frac{\partial f_s}{\partial L_{k}}(f_s(X^w[k-1])X^w[k] - f_s(X^w[k-1])f_s(L_{k})) - f_s(L_{k})f_s(L_{k}) - \frac{\partial f_s}{\partial L_{k}}f_s(X^w[k-1])f_s(L_{k})
\]

\[\text{(9)}\]

\[
U[k] = \Delta L_{k}^w
\]

\[\text{(9)}\]

and vectors \(\epsilon[k]\) takes into account the residuals after linearization and un-modeled effects. Eq. (9) is the needed linear state space model of dimensional variation in multi-station machining systems. \(A[k]\) is state transition matrix and transfer process deviations \(\Delta X^w[k-1]\) to state vector \(\Delta X^w[k]\). While \(B[k]\) is input matrix and combines the incoming part deviation into .

The key issue to establish the Eq. (9) is to find the four expressions:

\[
f_s(L_{k}), f_s(L_{k}), f_s(X^w[k-1]), f_s(X^w[k-1]),
\]

\[\text{(9)}\]

which were given in[12]. According to the procedures, one can derive the above SOV model for any machining system, provided that adequate CAD/CAPP data about the machining process are available. This makes the process plan evaluation possible prior to the real ma-
chining process conducted, so that supports the optimum of the process design.

It is noticed that the model only concerns about the influence of fixture parameters errors, previous machined features errors, no other process control characteristics is considered. One can employ the same procedure to establish the state space model concerning more other process control characteristics.

Given the tolerance of the feature machined in this stage is represented as \(T(k)\), and then the performance function of the process can be written as:

\[
g(\Delta L(k)) = \Delta X^W(k) - T(k)
\]  

(10)

2.3 The Model of Process Reliability

The calculation of process reliability is to determine the relations of the machined part quality statistical feature and the statistical feature of process character (KCC). The calculation can be fulfilled by the performance function of the process. From the equations (9) and (10), we can easily see that the mathematical function used to describe the variables \(\Delta X(k)\) and \(U(k)\) is nonlinear relation, namely the performance function of the machining process is nonlinear function under the condition that only locator errors are considered. From the theory of reliability, when performance function \(g(x)\) is not linear function, we can deal it by linearization. Since the process variables are independent, so standard deviation \(\sigma^2_g\) can be samplized as:

\[
\sigma^2_g = \sum_{i=1}^{n} a_i^2 \sigma^2_{x_i}
\]  

(14)


where \(u_x = u_{x_1}, u_{x_2}, \ldots, u_{x_n}\) is the average point of the performance function, and \(\left(\frac{\partial g}{\partial X_i}\right)\) is the value of the the differential coefficient function of the performance function at the average point of \(u_x\). It can be seen that the equation (11) is linear function.

Then mean value \(\mu_g\) and standard deviation \(\sigma^2_g\) can be written as:

\[
\mu_g = g(\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n})
\]  

(12)

\[
\sigma^2_g = \sum_{i=1}^{n} a_i^2 \sigma^2_{x_i} + \sum_{i=j=1}^{n} \sum_{i=j=1}^{n} a_i a_j Cov(X_i, X_j)
\]  

(13)

where, \(Cov(X_i, X_j)\) is covariance, and \(\rho_{X_i, X_j}\) is correlation coefficient of variables \(X_i\) and \(X_j\). When \(g(x)\) is not linear function, we can deal it by linearization.

According to that the combination of normal distribute variables is also obey the normal distribution, which mean value and standard deviation can be determined by basic variables, the performance function obeys the following normal distribution:

\[
g \sim N(\mu_g, \sigma^2_g)
\]  

(15)

The following index \(\beta\) can be designed to value the process reliability:

\[
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\[ \beta = \frac{\mu_g}{\sigma_g} = \frac{a_0 + \sum a_i \mu_i}{\sqrt{\sum_i a_i^2 \sigma_i^2 + \sum_{i,j} a_i a_j \text{Cov}(X_i, X_j)}} \]  \hspace{1cm} (16)

So process reliability \( R_g \) and failure probability \( F_g \) in first order and second moment (FOSM) can be written as follows:

\[ P_g = P[g \leq 0] = P \left( \frac{g - \mu_x}{\sigma_x} \leq \frac{\mu_x}{\sigma_x} \right) = \Phi(-\beta) \]  \hspace{1cm} (17)

\[ P_f = P[g > 0] = P \left( \frac{g - \mu_x}{\sigma_x} > \frac{\mu_x}{\sigma_x} \right) = \Phi(\beta) \]  \hspace{1cm} (18)

where, \( \Phi(\cdot) \) is normal standard variable cumulating distribution function.

### 2.4. Process Reliability Sensitive Analysis

Process reliability sensitivity is defined as the derivative of process failure probability \( u_x \) to basic variables (including mean value, standard and correlation coefficient). From the relations between index \( \beta \) and failure probability \( P_f \), and between index \( \beta \) and basic distribution variables, the process sensitivity can be formulated as follows by means composite deviation:

\[ \frac{\partial P_f}{\partial \mu_{x_i}} = \frac{\partial P_f}{\partial \beta} \frac{\partial \beta}{\partial \mu_{x_i}} \]  \hspace{1cm} (19)

\[ \frac{\partial P_f}{\partial \sigma_{x_i}} = \frac{\partial P_f}{\partial \beta} \frac{\partial \beta}{\partial \sigma_{x_i}} \]  \hspace{1cm} (20)

\[ \frac{\partial P_f}{\partial \rho_{x_i x_j}} = \frac{\partial P_f}{\partial \beta} \frac{\partial \beta}{\partial \rho_{x_i x_j}} \]  \hspace{1cm} (21)

Because:

\[ P_f = \Phi(-\beta) = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} \exp\left(-\frac{1}{2} t^2\right) dt \]  \hspace{1cm} (22)

So:

\[ \frac{\partial P_f}{\partial \beta} = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \beta^2\right) \]  \hspace{1cm} (23)

and represented as mean value and standard deviation:

\[ \frac{\partial P_f}{\partial \beta} = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \beta^2\right) \]  \hspace{1cm} (24)

From the above equations, the locator errors which contribute the product quality fluctuant largely can be diagnosed, and determine the sensitivity of the product quality to locator errors.

Because of the multistage process is highly nonlinear process. The above calculation procedure is very complex, it is very difficult to achieve the process reliability index \( \beta \) and following reliability sensitive analysis. Here chaos optimization algorithm and mutative scale chaos optimization algorithm are used to calculate the robust reliability index of non—probability model. By use of the idea of mutative scale, the feasible region is reduced, thereby the searching efficiency is improved and the iteration of optimization is reduced. The detail procedure of the use of chaos optimization algorithm can be seen [13].

### 2.5. Summary of the modeling Procedures

The modeling procedures described in this section can be summarized as follows:

(i) Formula the performance function of the station \( k \), which is the machining feature errors and the caused factors, fixture errors is used in this paper.

(ii) Obtain the distribution feature value \( \mu_{x_i}, \sigma_{x_i} \) of the produced KPC at the stage \( k \) by means of the distribution feature value \( \mu_{x_i}, \sigma_{x_i} \) of the KCC at the stage \( K \).

(iii) Construct an index to formulate the process reliability \( R_g \) and failure probability \( P_f \).

(iv) Perform the process reliability sensitive analysis by means of chaos optimization algorithm and mutative scale chaos optimization algorithm.

Figure 2 illustrates the modeling the process reliability for the multiple machined process.
Note that the above ways to formulate the process reliability model is called the first order and second moment (FOSM) method which will give the approximate value of the process reliability because of the used linearization method. If the more accuracy value is wanted advanced FOSM is desired.

Fig. 2. Flow-chart of the modeling of multistage machining process.

3. A Simple Two Dimensional Examples

A simple 2D example is employed to illustrate the procedure outlined in section 2. A rectangular part shown in Fig.3 (a) is machined at the fixture scheme shown in Fig.3 (b). It is assumed that features 1 and 2 have been machined in previous stations and are used as the locating datum at this station. Feature 3 is machined in the station whose locating scheme is shown in Fig.2 (b). Fig.2(c) demonstrates the setup of the workpiece and fixture system. The coordinate systems WCS, LCS and GCS are also shown in Fig.3, respectively.

![Diagram](attachment:image.png)

(a) 2D part  (b) Setup scheme  (c) Part in fixture

Fig 3. Simple two-dimensional example

The components of the deviation vector of fixture parameters errors are

\[ \Delta \psi^G(k) = [\Delta \nu_1^G(k) \Delta \nu_2^G(k) \Delta \nu_3^G(k) \Delta \nu_4^G(k) \Delta \nu_5^G(k) \Delta \nu_6^G(k)] \]

Assuming

\[ \Delta \nu_1^G(k) \Delta \nu_2^G(k) \Delta \nu_3^G(k) \Delta \nu_4^G(k) \Delta \nu_5^G(k) \Delta \nu_6^G(k) \]

obey \( \mathcal{N}(0.02,0.15)N(0,1.2)N(0,1)N(0.05,1)N(0.03,2) \) respectively. By using the equations presented in section 2, the deviation \( \Delta \psi^W(k) \) at the stage is in line with the normal distribution with mean value \( \mu_{\psi(k)} = 0.05 \), and stand deviation \( \sigma_{\psi(k)} = 0.021 \). So the process reliability sensitivity for the six KPC are calculated as Table 1:

<table>
<thead>
<tr>
<th>KPC</th>
<th>Sensitivity Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1/\psi_1]</td>
<td>1.579x10^-3</td>
</tr>
<tr>
<td>[1/\psi_2]</td>
<td>0.05x10^-3</td>
</tr>
<tr>
<td>[1/\psi_3]</td>
<td>0.063x10^-3</td>
</tr>
<tr>
<td>[1/\psi_4]</td>
<td>1.893x10^-3</td>
</tr>
<tr>
<td>[1/\psi_5]</td>
<td>0.035x10^-3</td>
</tr>
<tr>
<td>[1/\psi_6]</td>
<td>2.33x10^-3</td>
</tr>
</tbody>
</table>

From the table, we can see that the contribution of the different process parameters (KPC) to the station part feature quality (KCC) differs. The deviation \( \Delta \psi^W(k) \) of the feature is more sensitive to \( \Delta \nu_1^G(k) \) \( \Delta \nu_2^G(k) \) \( \Delta \nu_3^G(k) \) any \( \Delta \nu_4^G(k) \) than other process parameters, which reveals that we should pay more attention to \( \Delta \psi^G(k) \) \( \Delta \psi^W(k) \) and \( \Delta \psi^W(k) \) when the \( \Delta \psi^W(k) \) controlled.

4. Conclusion

Process reliability is one of the important factors which influence the stability of product quality during machining process. How to describe the forming of the process reliability and to calculate it mathematically is an important research Scenario. By means of the state space model of machining process and the performance function, this paper puts forward the method to calculate process reliability. Furthermore, the sensitivity of process reliability is given to facility the diagnosis of the weak points during the multistage machining process. This paper hopes to improve the stability of the machining process. The procedures are essential in the derivation of process reliability and keep the stability of the process.
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