A NEW RELIABILITY INDEX BASED ON FUZZY PROCESS CAPABILITY INDEX FOR TRAVEL TIME IN MULTIMODAL NETWORKS

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Abstract

The variability in travel times is a very important feature in determining the arrival of the passenger at the destination within a given travel time threshold and can be noted as one of the most important characteristic for travel time reliability that is an increasing concern of travelers, because it allows travelers to make better use of their own time. In this paper reliability in multimodal travel by consideration of focuses on the passenger travel times is investigated. For this aim, the variations passengers encounter in a multimodal network with respect to the capabilities of different services is analyzed. In order to identify the sources of variations and improve the service in an integrated manner a methodology based on process capability indices is presented. The fuzzy set theory is also integrated with process capability indices to increase the sensitiveness and flexibility of route and station reliability and to ease definition of specification limits. The proposed methodology is applied in Istanbul to investigate the reliability of travel times of multimodal travels of passengers using a specific subway line.

Keywords: Reliability, Transportation systems, Fuzzy Set theory, process capability index.

1. Introduction

The travel of passengers have commonly been examined in terms of average travel time or/and the duration of the travel time delays. A passenger may experience different travel times when undertaking a given journey on different occasions[1]. This variability in travel times is a very important feature in determining the arrival of the passenger at the destination within a given travel time threshold[2]. Travel time reliability is an increasing concern of travelers, because it allows travelers to make better use of their own time. It is well realized that delays and unreliability on the network have direct costs to people and businesses, increasing business costs and affecting productivity and innovation[4.] Thereby, promoting reliable multimodal transport systems is also emphasized in developing policies of a range of countries[3].

Travel time variations which are the reasons of delays and unreliability are due to different sources of network uncertainty. These sources can be broadly divided into two categories, i.e., exogenous and endogenous sources[5]. Exogenous sources are referred to the capacity variations, which are mainly resulted from signalization, accidents, breakdowns and so on. Such capacity variations usually lead to the non-recurrent congestion[6]. Endogenous sources are regarded as the travel demand fluctuations. Travel demand between a specified origin–destination (OD) pair varies between times of day, days of the week, and seasons of the year. Such demand

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variations always cause recurrent congestion[2,7]. Given the definitions, exogenous and endogenous sources of variations and their implications may vary in different travel modes. In terms of average delays, recurrent congestion seems to be more important than non-recurrent congestion in private transport. In public transport the balance between recurrent and non-recurrent (non-predictable) delays tends to differ when compared to road transport. The unpredictable component tends to be larger[8]. The analysis of delays in multimodal travels is more complex because both recurrent and non-recurrent congestions need to be considered. Most of the transit travels comprise feeder services which are commonly operated on the road (eg. bus, park&ride) to access high-speed transit services (eg. rail, metro) having their own right of way. Therefore, the total travel time for the transit travelers is affected not only by the transit performance, but also by the service characteristics of the feeder transit system[9]. From a multimodal travel perspective, the variation resulted more or less from exogenous and endogenous uncertainties exist in the multimodal transportation network and have important implications on the actual travel time.

Consideration of reliability in multimodal travel focuses on the passenger travel times rather than the vehicle arrival times. The difference between two is small in the case of uni-modal travels but is substantial in the case of multimodal travel due to the problem of missed connections[8]. For multimodal travels, the analysis of reliability from the perspective of the traveler is important to understand the passenger door-to-door travel experience.

The concepts embodied in the notion of reliability are various. Examining thoroughly, the statements of reliability and punctuality could be distinguished, defining the former as the rate of cancellations and the latter as the rate services exceed a given lateness standard[10]. Punctuality is examined explicitly under the notions of lateness and delay, such that lateness refers to the difference between the actual and publicly timetabled arrivals at destinations and delay is used to refer to the difference between the actual and working times to pass over short route sections[1]. Our analysis is mainly related to the notions of lateness and delays in the multimodal networks.

In this paper, we aim to analyze the variations passengers encounter in a multimodal network with respect to the capabilities of different services and present a methodology in order to identify the sources of variations and improve the service in an integrated manner. We offer to use process capability analysis (PCA) which provides a methodology to examine the variations in the travel time by using the parameters of travel time distribution. The fuzzy set theory is also combined with process capability indices (PCIs) to obtain sensitiveness and more flexibility in measuring the reliability of travel systems. We empirically investigate the process capability indices of multimodal travels, using archived data from electronic payment systems.

The rest of this paper is organized as follows: The methods used to assess reliability of transportation systems from literature are reviewed in Section 2. Section 3 includes a brief summary about PCA and the most popular PCIs. Then the fuzzy set theory is introduced and fuzzy process capability analysis is explained in Section 4. The travel time of the multimodal travels of passenger in Istanbul is investigated and an analysis of the reliability based on fuzzy PCIs for the selected routes in Istanbul are presented in Section 5. The obtained results and future research directions are discussed in Section 6.

2. Literature Review

In transportation literature, the notion of reliability is defined and employed in a wide range of studies. These studies extend from passenger satisfaction surveys to network assignment and traffic equilibrium models. This section presents examples of these various types of studies and provides a brief summary of the definition, assessment and use of the concepts related to reliability in these studies.

The notion of reliability is often studied in passenger surveys and satisfaction measurement. Reliability has been broadly considered as a service dimension and defined as the consistency of the service performance[11]. In transport research, surveys have shown that reliability is strongly related to passenger satisfaction and perceptions of service quality,[12] while stated preference experiments have found that passengers implicitly value reliability[13] and consider it in their mode choice decisions[14]. Unreliable service results in additional waiting time for passengers,[15,16,17] the unit cost of which has been estimated to exceed the cost of in-vehicle travel time by a factor of three[18]. Valuation of travel components which are related to reliability such as wait time, late time, delay time and interchange time has been investigated well within the transport research literature. Many empirical studies have been conducted in national, corridor or route level to estimate the value of the travel component times such as walk time, wait time in terms of in-vehicle time[19, 20].

The importance of reliability and on-time service has been analyzed by various methods. A common method is to use the stated passenger satisfaction valuations and weighting the importance of service dimensions with respect to satisfaction or an overall quality index[21, 22]. Such studies reveal the importance and the actual achieved performance of service dimensions such as reliability. These results help the service providers to identify the service dimension to be improved in order to obtain a higher impact on the overall quality.

Another stream of research in the literature is investigating the reliability of transport systems and the implications of the unreliable services by simulation studies[8,9]. In these studies, the data related to the lateness and delays are used which are collected through field studies. Recently, electronically archived data from automatic vehicle location, automatic passenger counter systems or payment collection systems have been available and provides a systematic and inexpensive source of data[23]. Simulation studies are used to uncover the impact of service reliability levels on waiting times, delays in the networks[9]. Turning around, the effects of daily variability in OD demand on travel time reliability have also been investigated by simulation studies[2, 24]. Simulation studies also help service providers to analyze the implications of policies on reliability levels[8].

Analytical methods for estimating the network service levels and network loadings are also a significant transport research area. In this stream of research, the probability distribution of the network travel times in light of the variations in travel demand is estimated and the traffic assignment problems are studied using equilibrium models. In various studies, the assignment problem is solved with stochastic travel times where reliability of travel times and demand are related to each other bidirectional[7,25]. The interested readers are recommended to refer Sheffi [26].

The variation of the travel time in the network is an important characteristic of transportation systems and needs to be handled in the transport problems. The probability distributions are commonly employed in order to analyze the variations. Even though probability distributions represent the nature of the process well and provide comprehensive information, a numeric measure to compare the service reliability of different modes, routes or corridors would provide a practical tool. In this paper, we present a methodology to develop such a measure by fuzzy process capability analysis in order to analyze and compare the services with respect to spatial and temporal variations in a practical manner.

3. Process Capability Analysis

The process capability index (PCI) is an approach for establishing the relationship between the actual process performance and the manufacturing specifications[27]. The process capability compares the output of a process to the specification limits by using capability indices. This comparison is made by forming the ratio of the width between the process specification limits to the width of the natural tolerance limits which is measured by 6 process standard deviation units. This method leads to make a statement about how well the process meets specifications[28]. A process is said to be capable if with high the real valued quality characteristic of the produced items lays between a lower and upper specification limits[29]. A process capability index (PCI) is a number which summarizes the behavior of a product or process characteristic relative to specifications. These indices help us to decide how well the process meets the specification limits[28]. Several PCI such as C_p, C_{pk}, C_{pm} and C_{pmk} are used to estimate the capability of process[29].

The index C_p is the first process capability index to appear in the literature. It can be defined as Eq. (1).

$$C_p = \frac{\text{Allowable Process Spread}}{\text{Actual Process Spread}} = \frac{\text{USL} - \text{LSL}}{6\sigma} \qquad (1)$$

where, σ is the standard deviation of the process, USL and LSL represent the upper and lower specification limits, respectively.

The proper use of PCIs is based on several assumptions. One of the most essential is that the process monitored is supposed to be stable and the output is approximately normally distributed. When the distribution of process characteristic is non-normal, the index C_p can be calculated as follows[30]:

$$C_p = \frac{\text{USL-LSL}}{(\text{upper } 0.135\% \text{ point})\text{-}(\text{lower } 0.135\% \text{ point})} = \frac{\text{USL-LSL}}{U_p - L_p} \quad (2)$$

where, $U_p(L_p)$ is the $99.865(0.135)$ percentile of observations.

The value of index C_p gives us an opinion about process' performance. For example if it is greater than $1.3\overline{3}$ which corresponds to 63 nonconforming parts per million (ppm) for a centered process, we conclude that process performance is satisfactory. The six quality conditions and the corresponding C_p values are summarized in Table 1[27].

The index C_p indicates how well the process fits between upper and lower specification limits. It never considers any process shift and simply measures the spread of the specifications relative to the six-sigma spread in the process as shown in Figure 1 and thus gives no indication of the actual process performance. Therefore the index C_{pk} defines in Eq. (2) is used.

Table 1 Quality Conditions and C_p Values

Quality Condition	C_p **value**
Super excellent	$2.00 \le C_P$
Excellent	$1.67 \le C_P < 2.00$
Satisfactory	$1.33 \le C_P < 1.67$
Capable	$1.00 \le C_P < 1.33$
Inadequate	$0.67 \le C_P < 1.00$
Poor	$C_P \prec 0.67$

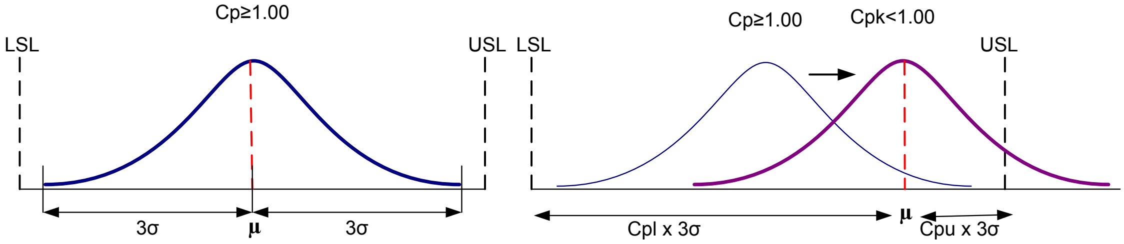

Figure 1 Process location and the indices C_p and C_{pk}[42]

$$C_{pk} = \min\{C_{pu}, C_{pl}\} = \frac{\min\{USL-\mu, \mu-LSL\}}{3\sigma} \quad (3)$$

The index C_p for non-normal process characteristic can be calculated as follows[30]:

$$C_{pu} = \frac{\text{USL} - \text{median}}{(\text{upper } 0.135\% \text{ point})\text{-median}} = \frac{\text{USL} - M}{U_p - M} \quad (4)$$

$$C_{pl} = \frac{\text{median-LSL}}{\text{median-}(\text{lower } 0.135\% \text{ point})\text{-}} = \frac{M \text{ - } LSL}{M \text{ - } L_p} \quad (5)$$

$$C_{pk} = \min\{C_{pu}, C_{pl}\} = \min\left\{\frac{\text{USL} - M}{U_p - M}, \frac{M \text{ - } LSL}{M \text{ - } L_p}\right\} \quad (6)$$

4. The Fuzzy Set Theory and Fuzzy Process Capability Analysis

The fuzzy set theory was specifically designed to mathematically represent uncertainty and vagueness and provide formalized tools for dealing with the imprecision intrinsic to many problems. The roots of fuzzy set theory go back to 1965 when Zadeh initiated the concept of Fuzzy Logic[31]. It uses approximate information and uncertainty to generate decisions. This is why it looks somewhat similar to human reasoning. Since knowledge can be expressed in a more natural way by using fuzzy sets, many engineering and decision problems can be greatly simplified. Fuzzy set theory implements groupings of data with loosely defined boundaries. Keeping this in mind, any methodology or theory implementing ‘‘crisp’’ definitions may be ‘‘fuzzified’’, if needed, by generalizing the concept of a crisp set to a fuzzy set with blurred boundaries. The main benefit of extending crisp analysis methods to fuzzy techniques is the strength in solving real-world problems, which has imprecision in the variables and parameters measured and processed for the application. To achieve this benefit, linguistic variables are used as a critical aspect of some fuzzy logic applications. After the inception of the notion of fuzzy sets, many authors have applied this approach to different areas such as statistics, quality control, and optimization techniques. These studies affected process capability analyses and process capability indices are analyzed under fuzziness such as C_p,[32,33,34,27,35,36,37,38,39,40,41,42,43,44] process accuracy index named as C_a,[45,46,41] C_{pk},[47,35,36,37,38,39,40,41,42,43,44] C_{pm},[48,35,36,37,49,50,41,51,44] C_{pmk}[35,52,41,44]. In this paper, unlike the previous studies, fuzzy process capability indices are produced to calculate transportation systems' reliability by using fuzzy set theory to express specification limits in linguistic terms. In the next sub-sections, PCIs are analyzed when the specification limits (SLs) are defined as triangular (TFN) or trapezoidal (TrFN) fuzzy numbers.

4.1. *Triangular Fuzzy Numbers (TFNs)*

Any $a \in$ F($\Re$) is called a fuzzy quantity on $\Re$ and any $TFN_{a_1,a_2,a_3} \in F(\Re)$ is called a triangular fuzzy number (TFN). The membership function of TFN is shown in Figure 2 and can be calculated as follows[53]:

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x \le a_1, \\ \dfrac{x-a_1}{a_2-a_1}, & a_1 < x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & a_1 < x \le a_2 \\ 0, & x > a_3 \end{cases} \tag{7}$$

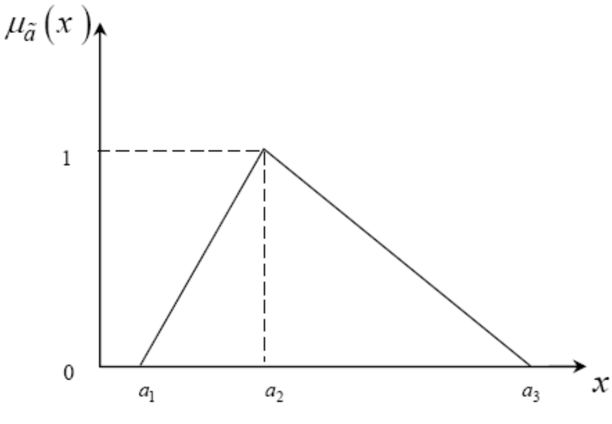

Figure 2. A triangular fuzzy number $\tilde{a}$

Let $\tilde{A} = TFN(a_1,a_2,a_3)$, $\tilde{B} = TFN(b_1,b_2,b_3)$ be positive fuzzy numbers. The main operators for TFN can be defined as follows[53]:

Addition

$$\tilde{A} \oplus \tilde{B} = TFN(a_1,a_2,a_3) \oplus TFN(b_1,b_2,b_3) = TFN(a_1+b_1, a_2+b_2, a_3+b_3) \tag{8}$$

Subtraction

$$\tilde{A} \ominus \tilde{B} = TFN(a_1-b_3, a_2-b_2, a_3-b_1) \tag{9}$$

Division

$$\tilde{A} \oslash \tilde{B} = TFN(a_1,a_2,a_3) \oslash TFN(b_1,b_2,b_3) = TFN\left(\frac{a_1}{b_3},\frac{a_2}{b_2},\frac{a_3}{b_1}\right) \tag{10}$$

$$\tilde{A} \oslash \mathbf{m} = TFN(a_1,a_2,a_3) \oslash \mathrm{m} = TFN\left(\frac{a_1}{m},\frac{a_2}{m},\frac{a_3}{m}\right) \tag{11}$$

and it is called "division" of $TFN(a_1,a_2,a_3)$ by *m*.

Suppose we have a fuzzy process with fixed σ, for which the upper and lower specification limits are the fuzzy sets $U\tilde{S}L = \mathrm{TFN}(a_1,a_2,a_3)$, $L\tilde{S}L = \mathrm{TFN}(b_1,b_2,b_3)$. The width between fuzzy process specification limits is a triangular fuzzy number $\tilde{w}_{SL}$, defined by $\tilde{w}_{SL} = U\tilde{S}L(a_1,a_2,a_3) \ominus L\tilde{S}L(b_1,b_2,b_3)$. The fuzzy process capability index is a TFN, $\tilde{C}_p = \tilde{w}_{SL} \oslash 6\sigma$. Therefore,

$$\tilde{C}_p = TFN\left(\frac{a_1-b_3}{6\sigma},\frac{a_2-b_2}{6\sigma},\frac{a_3-b_1}{6\sigma}\right) \tag{12}$$

$$\tilde{C}_{pu} = TFN\left(\frac{a_1-\mu}{3\sigma},\frac{a_2-\mu}{3\sigma},\frac{a_3-\mu}{3\sigma}\right) \tag{13}$$

$$\tilde{C}_{pl} = TFN\left(\frac{\mu-b_3}{3\sigma},\frac{\mu-b_2}{3\sigma},\frac{\mu-b_1}{3\sigma}\right) \tag{14}$$

$$\tilde{C}_{pk} = \min\left(\tilde{C}_{pu},\tilde{C}_{pl}\right) \tag{15}$$

If the distribution of process characteristic is non-normal, the fuzzy PCIs for TFN can be calculated as follows:

$$\tilde{C}_p^{nn} = TFN\left(\frac{a_1-b_3}{d},\frac{a_2-b_2}{d},\frac{a_3-b_1}{d}\right) \tag{16}$$

where d is the distance between upper 0.135% point and lower 0.135% point of process distribution $\left(d=(\text{upper 0.135\% point})\text{-}(\text{lower 0.135\% point})\right)$.

$$\tilde{C}_{pu}^{nn}=TFN\left(\frac{a_1-M}{d_u},\frac{a_2-M}{d_u},\frac{a_3-M}{d_u}\right) \tag{17}$$

where d_u is the distance between upper 0.135% point and median of process distribution $\left(d_u=(\text{upper 0.135\% point})\text{-}M\right)$.

$$\tilde{C}_{pl}^{nn}=TFN\left(\frac{M-b_3}{d_l},\frac{M-b_2}{d_l},\frac{M-b_1}{d_l}\right) \tag{18}$$

where d_l is the distance between median and lower 0.135% point of process distribution $\left(d_l=M\text{-}(\text{lower 0.135\% point})\right)$.

$$\tilde{C}_{pk}=\min\left(\tilde{C}_{pu}^{nn},\tilde{C}_{pl}^{nn}\right) \tag{19}$$

4.2. *Trapezoidal Fuzzy Numbers (TrFNs)*

Another shape of fuzzy numbers is trapezoidal fuzzy numbers (TrFN). This shape is originated from the fact that there are several points whose membership degree is maximum ($\alpha=1.00$).We can define TrFN as $\tilde{a}=(a_1,a_2,a_3,a_4)$ whose membership function is shown in Figure 3. In this paper, PCIs have been also obtained by using TrFNs. The membership function of this fuzzy number is as follows[53]:

$$\mu_{\tilde{a}}(x)=\begin{cases}0, & x<a_1\\ \dfrac{x-a_1}{a_2-a_1}, & a_1\le x\le a_2\\ 1, & a_2\le x\le a_3\\ \dfrac{a_4-x}{a_4-a_3}, & a_3\le x\le a_4\\ 0, & x>a_4\end{cases} \tag{20}$$

The graphical representation for the membership function of TrFN is shown in Fig. 3.

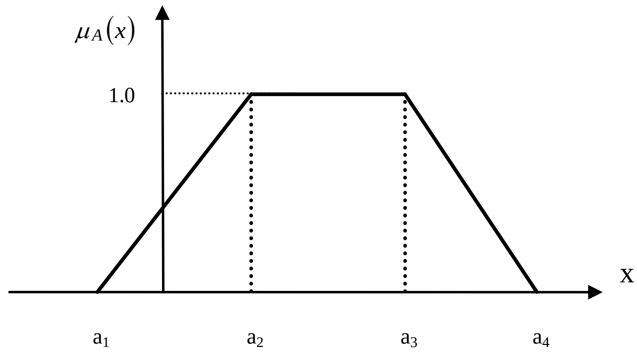

Figure 3 A Trapezoidal Fuzzy Number $\tilde{a}$

The main operators for TrFN can be defined as follows[53]:

Addition

$$\tilde{A}\oplus\tilde{B}=TrFN(a_1,a_2,a_3,a_4)\oplus TrFN(b_1,b_2,b_3,b_4)$$
$$=TrFN(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4) \tag{21}$$

Subtraction

$$\tilde{A}\ominus\tilde{B}=TrFN(a_1,a_2,a_3,a_4)\ominus TrFN(b_1,b_2,b_3,b_4)$$
$$=TrFN(a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1) \tag{22}$$

Division

$$\tilde{A}\oslash\tilde{B}=TrFN(a_1,a_2,a_3,a_4)\oslash TrFN(b_1,b_2,b_3,b_4)$$
$$=TrFN\left(\frac{a_1}{b_4},\frac{a_2}{b_3},\frac{a_3}{b_2},\frac{a_4}{b_1}\right) \tag{23}$$

$$\tilde{A}\oslash \mathbf{m}=TrFN(a_1,a_2,a_3,a_4)\oslash m=$$
$$TrFN\left(\frac{a_1}{m},\frac{a_2}{m},\frac{a_3}{m},\frac{a_4}{m}\right) \tag{24}$$

and it is called "division" of $TrFN(a_1,a_2,a_3,a_4)$ by m.

Sometimes specification limits can be represented by TrFNs. Suppose that $U\tilde{S}L$ and $L\tilde{S}L$ are defined as follows:

$U\tilde{S}L=TrFN(a_1,a_2,a_3,a_4)$,
$L\tilde{S}L=TrFN(b_1,b_2,b_3,b_4)$.

The width between fuzzy process specification limits is a trapezoidal fuzzy number $\tilde{w}_{SL}\in F_T(\Re)$, defined by $\tilde{w}_{SL}=TrFN(a_1,a_2,a_3,a_4)\ \ominus\ TrFN(b_1,b_2,b_3,b_4)$. The fuzzy process capability index is a trapezoidal fuzzy number, $\tilde{C}_p = \tilde{w}_{SL}$ Ø 6σ. Therefore,

$$\tilde{C}_p = TrFN\left(\frac{a_1-b_4}{6\sigma},\frac{a_2-b_3}{6\sigma},\frac{a_3-b_2}{6\sigma},\frac{a_4-b_1}{6\sigma}\right) \quad (25)$$

$$\tilde{C}_{pu} = TrFN\left(\frac{a_1-\mu}{3\sigma},\frac{a_2-\mu}{3\sigma},\frac{a_3-\mu}{3\sigma},\frac{a_4-\mu}{3\sigma}\right) \quad (26)$$

$$\tilde{C}_{pl} = TrFN\left(\frac{\mu-b_4}{3\sigma},\frac{\mu-b_3}{3\sigma},\frac{\mu-b_2}{3\sigma},\frac{\mu-b_1}{3\sigma}\right) \quad (27)$$

$$\tilde{C}_{pk} = \min\left(\tilde{C}_{pu},\tilde{C}_{pl}\right) \quad (28)$$

If the distribution of process characteristic is non-normal, the fuzzy PCIs for TrFN can be calculated as follows:

$$\tilde{C}_p^{nn} = TrFN\left(\frac{a_1-b_4}{d},\frac{a_2-b_3}{d},\frac{a_3-b_2}{d},\frac{a_4-b_1}{d}\right) \quad (29)$$

$$\tilde{C}_{pu}^{nn} = TrFN\left(\frac{a_1-M}{d_u},\frac{a_2-M}{d_u},\frac{a_3-M}{d_u},\frac{a_4-M}{d_u}\right) \quad (30)$$

$$\tilde{C}_{pl}^{nn} = TrFN\left(\frac{M-b_4}{d_l},\frac{M-b_3}{d_l},\frac{M-b_2}{d_l},\frac{M-b_1}{d_l}\right) \quad (31)$$

$$\tilde{C}_{pk} = \min\left(\tilde{C}_{pu}^{nn},\tilde{C}_{pl}^{nn}\right) \quad (32)$$

4.3. *Comparison of fuzzy numbers*

If two or more competitive fuzzy processes exist, a criterion is needed to compare these processes. For this aim, the literature has different approaches. In this paper, the total integral value method with an index of optimism has been applied to compare fuzzy values to determine the index $\tilde{C}_{pk}$. Also this method is used for defuzzification of fuzzy PCIs. Liou and Wang[54] proposed the total integral value method with an index of optimism $\omega \in [0,1]$. Let $\tilde{A}$ be a fuzzy number with left membership function $f_{\tilde{A}}^L$ and right membership function $f_{\tilde{A}}^R$. Then the total integral value is defined as[55]:

$$E_\omega\left(\tilde{A}\right) = \omega E_R\left(\tilde{A}\right) + (1-\omega)E_L\left(\tilde{A}\right) \quad (33)$$

where

$$E_R\left(\tilde{A}\right) = \int_\alpha^\beta x f_{\tilde{A}}^R(x)\,dx \quad (34)$$

and

$$E_L\left(\tilde{A}\right) = \int_\gamma^\delta x f_{\tilde{A}}^L(x)\,dx \quad (35)$$

where, $-\infty < \alpha \le \beta \le \gamma \le \delta < \infty$. For a triangular fuzzy number, $\tilde{A} = (a,b,c)$, the total integral value is obtained by

$$E_\omega\left(\tilde{A}\right) = \frac{1}{2}\left[\omega(a+b) + (1-\omega)(b+c)\right] \quad (36)$$

5. An Application

In this study, the travel time of the multimodal travels of passenger in Istanbul is investigated. The present number of vehicles in Istanbul is 2.5 million that is 20% of Turkey. The total number of private cars is 1.8 million that is 29 % of Turkey and the daily number of vehicles in traffic is 1.7 million that 1.6 million of them are private cars and daily number of vehicles joining Istanbul traffic is 400. The total length of highway is nearly 12,000 kilometers. Approximately 15 million journeys are made every day in the metropolitan area of which 71% is made by public transportation and 29% by private cars[56]. There are two bridges in Istanbul connecting the two continents, Asia and Europe. The former bridge which is called Boğaziçi was built about 30 years ago. Due to heavy traffic between the two continents, the latter bridge, Fatih Sultan Mehmet, was built on 1988. The daily number of travels between Asia and Europe is 1.3 million which are made by private cars (34%), public transportation vehicles (49%), and waterborne (17%)[57].

The average time of travels in the metropolitan area has increased 20% in the last ten years and reached 48 minutes[56]. 90% of transportation in Istanbul is made by land-based transportation which includes private cars, public buses, minibuses, metro buses, taxis, and vehicles for company services; 3.5% is made by waterborne which includes ships, fast ferries and private motorboats and 6.5% is made by rail transportation which includes trains, subways, tramcars, IETT nostalgic tramcars and tunnel, light metro.

Istanbul’s traffic is managed by 3 local organizations administrated by Istanbul Metropolitan Municipality

(IMM) and 2 central governmental organizations. The largest and first local organization which controls highway and railway public transportation mostly is Istanbul Establishment of Electricity, Trams and Tunnel (IETT). The other 2 local organizations are Istanbul Fast Ferries Co. (IDO) which is in charge of sea transportation since 1988 and Istanbul Ulasim AS which is responsible for the metro lines. The governmental organizations are Turkish Republic Railways Organization (TCDD) and Turkish Sea Transportation Organization (TDI). Besides, transportation is also met by the private sector in Istanbul. Taxis, minibuses, private motorboats, and public buses carry a high percentage of passengers. Tables 2-4 show distributions of highway transportation, sea transportation and railway transportation, respectively[57].

Table 2. The distribution of highway transportation.

Type of Vehicle	Number of Vehicles	Daily Passenger
Private cars	1,600,000	2,100,000
IETT Buses	2,500	1,700,000
Public Buses	1,300	1,100,000
Minibuses	7,000	2,000,000
Dolmuş	590	80,000
Taxis	18,000	1,100,000
Company Services	35,500	2,150,000
TOTAL	1,664,890	10,230,000

According to Table 2, the most important part of the vehicles in traffic belongs to company services, private cars, minibuses and IETT Buses while the TDI ships are the most important vehicles among sea based transportation as can be seen from Table 3.

Table 3. The distribution of sea transportation.

Type of Vehicle	Number of Vehicles	Daily Passenger
TDI Ships	74	250,000
IDO Ferries	26	30,000
Motorboats	236	160,000
TOTAL	336	440,000

According to Table 4, all vehicle types except tunnel and nostalgic tramcar are almost equally important among railway transportation vehicles.

Table 4. The distribution of railway transportation

Type of Vehicle	Number of Vehicles	Daily Passenger
TCDD Trains	72	125,000
Light Metro	19	190,000
Tramcars	11	160,000
Subway	9	130,000
IETT Tunnel & Nostalgic Tramcar	5	11,000
TOTAL	116	616,000

Our approach is to investigate the reliability of travel times of multimodal travels of passengers using a specific subway line in this study. The multi-modal travels are identified as the routes which different passengers follow integrated with the specified subway line. These routes include various modes such as trams, funicular, buses, sea ferries that integrate at specific stations of the chosen subway line. The selected subway line is named as M2 operating with 11 stations in a highly populated business district. The map of Istanbul is given in Figure 4, showing the zones which are formed such that zone populations are almost similar to each other. Thus, the zones with smaller areas have a denser population. The subway line M2, which is presently the only subway line in Istanbul, goes through one of the business districts in the city center. Line M2 is interconnected with many bus and minibus lines in all 11 stations. However the majority of the interchanges are made at 3 main stations which are connected to high capacity modes which include two bus rapid transportation (BRT) line named as 34 and 34A and one funicular line F1. Interchanges between the BRT lines (34 and 34A) and the subway line (M2) are made at two stations: Sisli and Gayrettepe. The connection between the funicular line (F1) and subway line (M2), offering interchanges, is available at one station: Taksim.

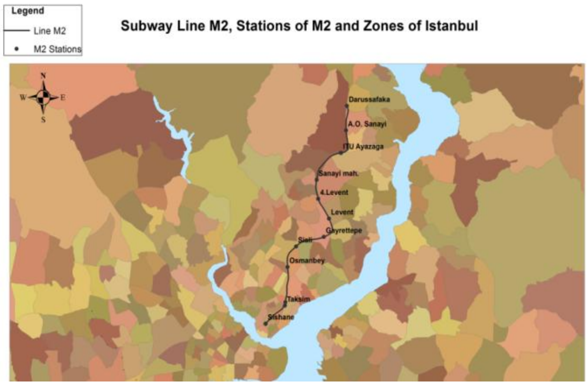

Fig. 4. Subway line M2, stations of line M2 and the zones of Istanbul.

We analyze the passenger travel time reliability of multimodal trips in two steps. In the first step, we investigate the travel times of lines which are prior trip legs of the subway line M2 trip. We limited our analysis with the BRT and funicular lines due to data availability. In the second step, we analyze the travel times of trips that contain two interchanges. These travel chains are identified with 3 trip legs which are subway line M2, BRT (34or 34A) or funicular line (F1) plus the trip leg prior to the BRT or funicular lines. The initial trip leg of the travels with two interchanges includes tram lines T1, T2 or T4; light rail line M2 or ferry lines FBSN and FUSA. The multimodal network constituted with the integrated lines is shown in Figure 5.

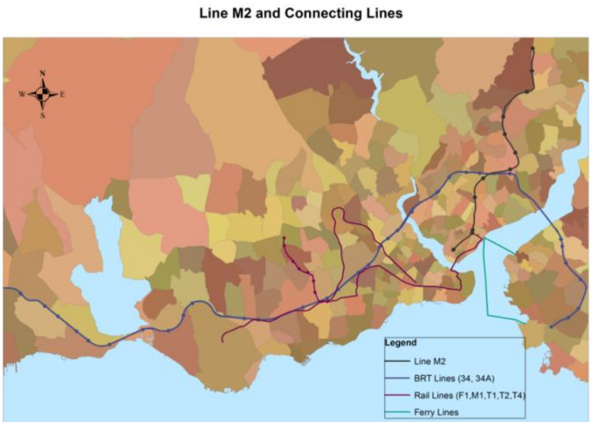

Fig. 5. Multimodal network of line M2 and the specified connecting lines.

The hourly number of passengers travelling through the routes integrated with subway line M2 at the morning peak period is listed at Tables 5 and 6.

Table 5. Hourly number of passengers interchanging to subway line M2 on the listed routes.

Route	**Number of Passengers**
34-M2	5265
34A-M2	2548
F1-M2	1694
Bus lines-M2	602

Table 6. Hourly number of passengers interchanging to subway line M2 with minimum two interchanges on the listed routes.

Travel chain	**1. Trip Leg**	**2.Trip Leg**	**Number of Passengers**
T4-34-M2	T4	34	1487
M1-34-M2	M1	34	697
T2-34-M2	T2	34	456
T1-34-M2	T1	34	252
Bus Line-34-M2	Bus Line	34	294
Bus Line-34A-M2	Bus Line	34A	846
Train-34A-M2	Train	34A	92
T1-F1-M2	T1	F1	751
FBSN-F1-M2	FBSN	F1	549
FUSA -F1-M2	FUSA	F1	215

The travel time data have been obtained through the integrated electronic payment system used in the public transportation modes in Istanbul. Each passenger has a payment chip with an identification number which is permanently used by the same person. As the person uses this chip for the payment, the system records the data related to the payment to a database. The recorded data contains the identification number of the payment chip, the station/vehicle it is used, the day and time of use, the mode, line and the operator related to the payment station. Using this database, the travel chains of passengers and the travel times of a chip in different days can be inferred.

We use the travel time data of the weekdays between the dates of November 22nd, 2010 and February 8th, 2011. The travels made during the morning peak hours, 7 and 8 am, have been investigated in order to include only daily variations in the dataset instead of hourly variations. We adopted this approach due to the fact that a person, who uses the same route regularly, would probably travel at the same time of day. For example, a person travelling between home and work places would regularly use the same route almost at the same time of day. So, we chose to analyze the daily variability of travel time and eliminated the hourly variability by using the data of the same time period of the day.

We have drawn a representative sample of persons, regularly using the same travel chain almost every day. Then, we computed the total travel times of passengers by calculating the time between the payments of each passenger. The time span between two payments related to the legs of a travel chain consists of the access time from the payment toll to the platform, the waiting time for the vehicle, the in-vehicle time and the access time from alighting the vehicle of the prior leg to the payment toll at the station of the latter leg.

The data is checked for the two assumptions of process capability to be satisfied in order to produce reliable results. The first condition is that the process is under control statistically. This has been verified by applying control charts to the travel time datasets. The control charts are depicted for the travel time dataset of each route. In Figure 6, an example of the control charts is given for the route starting from the station named as EKP of line 34, BRT to the station named as Sisli of the subway line M2.

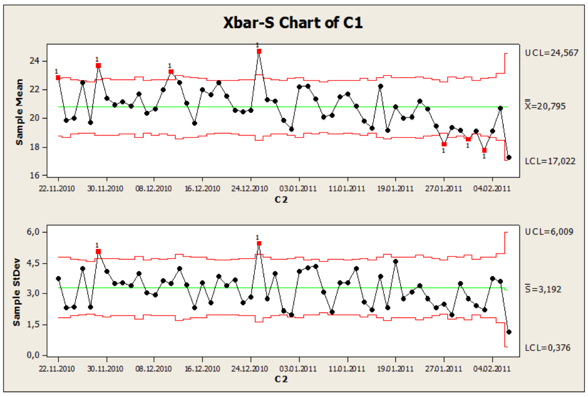

Fig. 6. Control chart for the travels times of the route starting from station EKP of the BRT line 34 to the interchange station of line M2.

As seen in the Figure 6, process is shown to be out of control in some days. In order to obtain reliable parameters from the travel time distribution of the dataset, the observations causing the process to be out of control are eliminated before applying the process capability methodology. However, the eliminated observations are investigated specially in order to explain why the process is out-of control on these days. Taking a close look at the out-of control observations, the common characteristic of them is that these observations are collected on Monday, the first day of the week. So, service providers should realize the extraordinary situation of Mondays and need to take extra measures for this day.

The second assumption of process capability analysis is to ensure that data is distributed normally. The basis for this condition is to obtain reliable parameters of the distribution that are the average and the standard deviation for calculating the process capability index. Unless the distribution of the dataset is not normal, the average and standard deviation values will not represent the dataset's central tendency and the percentile to be accepted. The histogram of the same data set used for drawing control chart has been plotted in Figure 7 to visualize the distribution of the data.

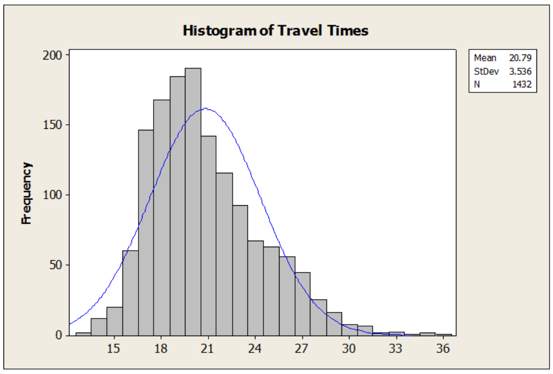

Fig. 7. Histogram for the travels times dataset of the route starting from station EKP of the BRT line 34 to the interchange station of line M2.

The travel time datasets of different routes have been found to be similar to the distribution in Figure 7 such that the data of travel times has typically a positive skewness. Thus, the average and the standard deviation parameters used in the formulation of process capability index are not representative. We tried to apply data transformation methods Box-Cox and Johnson to the datasets. However, the transformed data has not fit normal distribution well enough. Finally, we adopted the probability plotting approach offered to be used for the process capability analysis of non-normal data. Figure 8 shows the value corresponding to the 99,865% percentile of the distribution of the dataset.

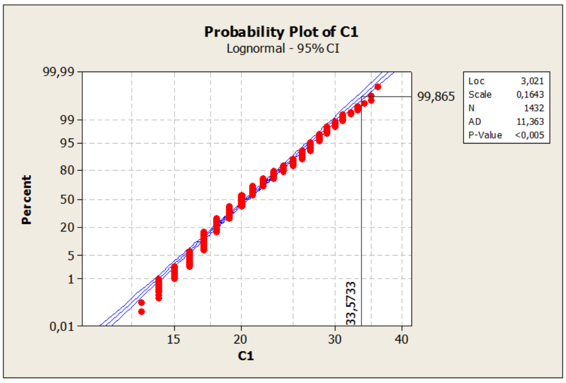

Fig. 8. Value corresponding to the 99,865% percentile of the distribution.

In some applications of process capability, the process tolerances need to be investigated from only one side, upper limits or lower limits. Then, one sided process capability ratios are analyzed. In our application case, we also made a one-sided process capability analysis with the upper specification limits. The investigated quality characteristic is travel time. Thus, it has been assumed that passengers favor lower travel times but become annoyed when they encounter higher travel times. Travelers sometimes partly dislike lower travel times because it causes longer waiting times during interchanges when the headways between the two successive vehicles are long. However, our application case is taken from the city center of the Istanbul metropolitan area so the headways of the investigated lines are rather short. Thereby, we assumed that passengers are not disturbed with lower travel times and examined the process capability for the upper tolerance limits of the travel time.

The passenger travel times are mainly related to the station from which she starts her journey and the route she follows. So, the process capability analysis of travel times is made for each major route and its starting station. The upper specification limits are set for each of the route-station analysis unit. The specification limits are used as fuzzy numbers in order to handle the natural variability of multimodal passenger travel times. The travel times used in our example includes the access time, in-vehicle time and waiting time of the analysis route. When working with access times, the common practice is to specify the access time based on the access distance and an average walk speed. And the waiting time is usually accepted as the half of the headway that is the expectation of the passenger waiting time given the time between two successive vehicles. We have chosen to use fuzzy process capability analysis to handle the vagueness and ambiguity caused by different walking speeds within the observation set and the different passenger arrival times with respect to the vehicle arrival time. We set triangular fuzzy numbers as upper specification limits to obtain more sensitive and informative results. For example the specification limit for the route 34-M2 starting from station EKP is determined as $\widetilde{USL}$ = Approximately 36 = (30; 36; 45).

Using TFNs for specification limits, the process capability analysis is extended to the fuzzy case. The FPCIs for passenger travel times for arriving the line M2 using a specific route are calculated. One-sided process capability analysis has been made with a probability plotting approach in order to handle the non-normal data. The FPCI for the travel time of route 34-M2 starting from station EKP is derivate as $\tilde{C}_{pu}^{nn} = (0.73,\ 1.17,\ 1.84)$ in Figure 9, the membership function of the given $\tilde{C}_{pu}^{nn}$ is plotted.

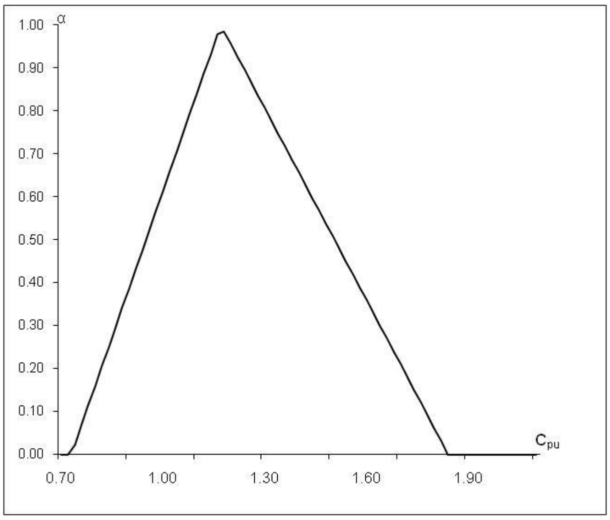

Fig. 9. The membership function of the index $\tilde{C}_{pu}^{nn}$ for 34-M2.

As seen in Figure 9 the fuzzy value for process capability contain more information than a point or interval estimate. This gives an advantage to investigate reliability of travel times more deeply. According to Fig. 9, the defuzzified value of C_{pu} is 1.23. We can conclude that the travelers starting their journey from station EKP on this route are served a capable process taken the travel time reliability into consideration.

In order to develop an overall capability ratio for a route in terms of the capabilities of the process on every station, we calculated an average capability ratio weighted with the passenger numbers served at each station of the route. The fuzzy and defuzzified process capability indices of each station and the weighted average capability index of route 34-M2 have been listed in Table 7.

Table 7. Fuzzy process capability indices for route 34-M2.

Station	Nr. of Pass-enger	Fuzzy PCIs	Defuzzi-fied PCIs	Capability /Quality Condition
Z2C-Z2C	10	(0.92 ; 1.47 ; 2.31)	1.54	Satisfactory
PRP-PRP	3	(0.92 ; 1.47 ; 2.31)	1.54	Satisfactory
HLO-HLO	3	(0.92 ; 1.47 ; 2.31)	1.54	Satisfactory
SSK-SSK	4	(0.73 ; 1.17 ; 1.84)	1.23	Capable
EKP-EKP	49	(0.73 ; 1.17 ; 1.84)	1.23	Capable
AYV-AYV	25	(0.63 ; 1.01 ; 1.58)	1.06	Capable
CVG-CVG	37	(0.63 ; 1.01 ; 1.58)	1.06	Capable
MTP-MTP	8	(0.61 ; 0.98 ; 1.54)	1.03	Capable
ZBM-ZBM	43	(0.61 ; 0.98 ; 1.54)	1.03	Capable
MTR-MTR	10	(0.59 ; 0.95 ; 1.48)	0.99	Capable
SG2-SG2	12	(0.59 ; 0.95 ; 1.48)	0.99	Inadequate
INC-INC	70	(0.48 ; 0.77 ; 1.21)	0.81	Inadequate
BHE-BHE	18	(0.48 ; 0.77 ; 1.21)	0.81	Inadequate
SRE-SRE	76	(0.48 ; 0.77 ; 1.21)	0.81	Inadequate
YSN-YSN	31	(0.49 ; 0.78 ; 1.23)	0.82	Inadequate
SFA-SFA	33	(0.49 ; 0.78 ; 1.23)	0.82	Inadequate
KCM-KCM	14	(0.49 ; 0.78 ; 1.23)	0.82	Inadequate
CNN-CNN	11	(0.49 ; 0.78 ; 1.23)	0.82	Inadequate
FLR-FLR	14	(0.47 ; 0.76 ; 1.18)	0.79	Inadequate
SKR-SKR	25	(0.47 ; 0.76 ; 1.18)	0.79	Inadequate
IKP-IKP	14	(0.47 ; 0.76 ; 1.18)	0.79	Inadequate
UNV-UNV	124	(0.47 ; 0.76 ; 1.18)	0.79	Inadequate
Average PCI		(0.54 ; 0.87; 1.36)	0.91	Inadequate

According to Table 7, the capability level of three stations of route 34-M2 can be classified as "Satisfactory" and seven stations' capability can be classified as "Capable". These stations should be more chosen for a reliable travel. The membership functions of stations, "SSK-SSK", "AYV-AYV" and "MTR-MTR" are plotted in Figure 10. As it is seen in Figure 10, the station "SSK-SSK" is more suitable for a reliable travel.

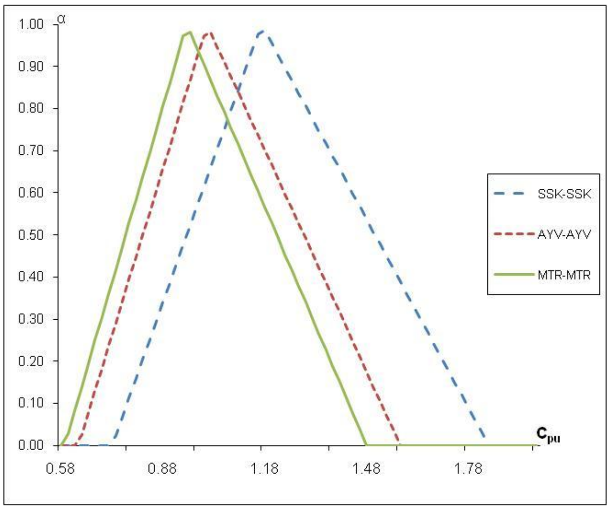

Fig. 10. The membership functions of the index $\tilde{C}_{pu}^{nn}$ for three stations of route 34-M2.

The funicular line F1 which is a one-station line has a higher capability index in terms of travel reliability than BRT line M2. The fuzzy capability ratios are found as (0.97, 1.65, 1.95) and the defuzzified capability ratio as 1.56.

The examples of 34-M2 and F1-M2 routes are routes with 2 legs. In the second step, we analyzed the travel time reliability for the passengers that have a travel chain with 3 legs having 2 interchanges between these legs. Such travels have been identified with the boarding station of the first leg of the travel, the interchange stations and the lines they are following. At Table 8, the fuzzy and defuzzified capability indices calculated for the route starting with tram line T1, then following to funicular line F1 from station KBT and finally connecting to the subway line M2 at the interchange station TAK.

Table 8. Fuzzy process capability indices for route T1-F1-M2.

Station of T1	Station of F1	Fuzzy PCIs	Defuz-zified PCIs	Capability /Quality Condition
TPH-KBT	KBT	(0.91 ; 1.53 ; 2.39)	1.59	Satisfactory
KAR-KBT	KBT	(0.91 ; 1.53 ; 2.39)	1.59	Satisfactory
SRK-EMN	KBT	(0.87 ; 1.48 ; 2.30)	1.53	Satisfactory
GLH-EMN	KBT	(0.87 ; 1.48 ; 2.30)	1.53	Satisfactory
SAH-EMN	KBT	(0.89 ; 1.51 ; 2.35)	1.57	Satisfactory
CTS-EMN	KBT	(0.94 ; 1.59 ; 2.48)	1.65	Satisfactory
BYZ-EMN	KBT	(0.94 ; 1.59 ; 2.48)	1.65	Satisfactory
	Average PCI	**(0.90 ; 1.52 ;2.37)**	**1.58**	**Satisfactory**

According to Table 8, the capability level of all stations of route T1-F1-M2 can be classified as "Satisfactory". This route offers a reliable travel for the passengers who are able to access to the route in Istanbul. The membership functions of stations, "TPH-KBT", "GLH-EMN" and "CTS-EMN" are plotted in Figure 11. As it is seen from Figure 11, the station "CTS-EMN" is more suitable for a reliable travel.

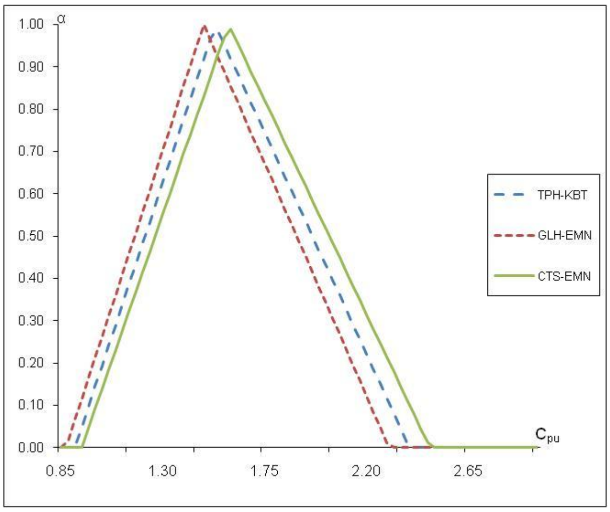

Fig. 10. The membership functions of the index $\tilde{C}_{pu}^{nn}$ for three stations of route T1-F1-M2.

The average PCI values for the routes listed at Table 9 are calculated in the similar way. The results show that waterborne modes are highly capable in terms of travel time reliability, as well as funicular line and the modes which have their own right of way and are integrated with the funicular line are capable. The process capability of bus rapid transport lines is concluded as capable only when the line length is not long. As the line length gets longer, the value of capability ratios decreases dramatically.

Table 9. PCI values for routes integrated with line M2

Route	Fuzzy PCIs	Defuzzified PCIs	Capability / Quality Condition
34-M2	(0.54 ; 0.87 ; 1.36)	0.91	Inadequate
34A-M2	(0.93 ; 1.50 ; 2.35)	1.57	Satisfactory
F1-M2	(0.97 ; 1.65 ; 1.95)	1.56	Satisfactory
T4-34-M2	(0.74 ; 1.20 ; 1.77)	1.23	Capable
M1-34-M2	(0.87 ; 1.40 ; 2.07)	1.44	Satisfactory
T2-34-M2	(0.79 ; 1.27 ; 1.87)	1.30	Capable
T1-F1-M2	(0.90 ; 1.52 ; 2.37)	1.58	Satisfactory
F.BSN-F1-M2	(0.97 ; 1.73 ; 2.39)	1.71	Excellent
F.USA -F1-M2	(0.88 ; 1.52 ; 2.26)	1.55	Satisfactory

The membership functions of three route alternatives named as "34-M2", "F1-M2" and "T2-34-M2" are plotted in Figure 12. As it clearly seen from Figure 12, the route "F1-M2" is more preferable.

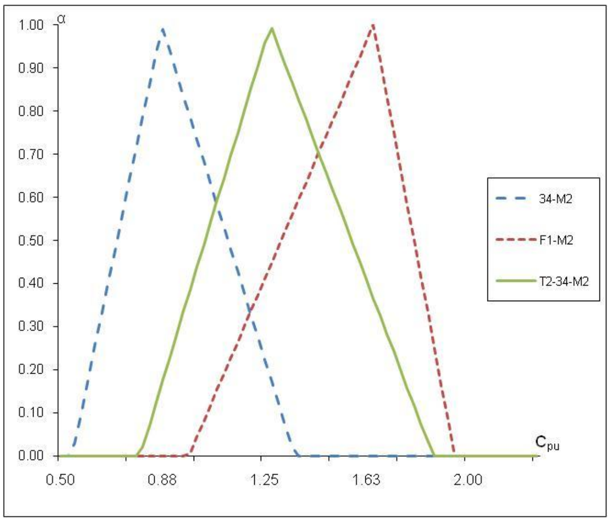

Fig. 11. The membership functions of the index $\tilde{C}_{pu}^{nn}$ for three route alternatives.

6. Conclusion

The variation of the travel time in the network is an important characteristic of transportation systems and needs to be handled in the transport problems. The probability distributions are commonly employed in order to analyze the variations. Even though probability distributions represent the nature of the process well and provide comprehensive information, a numeric measure to compare the service reliability of different modes, routes or corridors would provide a practical tool. In this paper, a methodology to develop such a measure by fuzzy process capability analysis in order to analyze and compare the services with respect to spatial and temporal variations is presented.

Process capability analysis produces statistics called PCIs to summarize process performance. The process can be classified as capable if the PCIs are greater than predetermined critical values. Otherwise they can be labeled as incapable. PCIs are rapidly becoming a standard tool for quality reporting. In this paper an empirical analysis of the reliability PCIs based on fuzzy set theory for the selected routes including various modes such as trams, funicular, buses, sea ferries that integrate at specific stations with the chosen subway line in Istanbul is presented.

Fuzzy sets bring an advantage to the flexible definition and evaluation of PCIs and this effect the reliability measurement of travel time directly. Fuzzy number allows to analyze the passenger travel times by which consumer-oriented performance of the transportation services was represented incorporating the vagueness among the passenger travels. Our approach in this study also presents the use of a valuable and economical source of data for obtaining passenger travel times from electronic payment systems.

The results of our analysis show that the causes of unreliability which is measured with process capability analysis are various. One of the main factors that determine the unreliability is the type of travel mode. Results show that some of the travel modes such as funicular and ferries have higher reliability compared to other modes. On the other side, different lines of the same type of travel mode can yield different reliability levels. For example, the capability of the BRT line 34's reliability is obtained as inadequate whereas line 34A's reliability level is satisfactory. These two lines are significantly different with respect to the route length. Thus, we can conclude that route characteristics play an important role on the reliability of the travels. The sources of unreliability on different routes can be determined effectively with the proposed analysis to improve the design, planning, operating conditions and the service levels of the transportation systems.

In future studies, the variability caused by operating conditions such as traffic volume, service frequency and vehicle operators can be investigated with different experimental designs. Moreover, the reliability levels of routes which are summarized with numerical indices by process capability analysis can be incorporated into the route choice and network assignment models used in transportation planning.

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