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The Study on Multi-Attribute Decision-Making with Risk Based on Linguistic Variable*

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Abstract

An approach based on relative optimal membership degree is proposed to deal with multiple attribute decision-making (MADM) problems under risk with weight information unknown and attribute value as linguistic variable. Firstly, the operational laws of linguistic variable are introduced, and risk linguistic decision matrix is transformed into certain linguistic decision matrix by expectation value. Then, the ideal solution and negative ideal solution with linguistic variable are defined, and the attribute weight model is developed by relative optimal membership degree between alternatives and ideal solutions. In addition, the alternatives are ranked by relative optimal membership degree. Finally, illustrative example is provided to demonstrate the steps and effectiveness of the proposed approach.

Keywords: linguistic variable; risk decision; relative optimal membership degree; multiple attribute decision-making.

1. Introduction

Multiple attribute decision making (MADM) has been extensively applied to various areas such as society, economics, management, military and engineering technology. For example, Investment decision-making, project evaluation, economic evaluation, personnel evaluation, etc. The decision makers, evaluating some problems, often give the evaluation information in the form of linguistic term directly, such as good, medium good, poor, etc. For example, private morality, automobile performance etc. Therefore, the study on MADM problems with attribute value as linguistic variable has very important values on theoretical and practical application, and many achievements have been made[1-14]. In addition, in the real decision-making process, the decision makers often face some problems, such as the uncertain environment and random variable as the attribute value, which make the decision makers not to determine the further status, but they can give the possible status, and quantify this randomness by setting up the probability distribution. This called multiple

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attribute decision making with risk (MADMR)[15]. So the study on MADMR problems has the same important values on theoretical and practical applications. Since Zadeh (ref. 16) proposed the concept of a linguistic variable, the research on certain linguistic multiple attribute decision making problems has got rich achievements. Herrera et al. (refs. 1-3) presented the linguistic ordered weighted averaging (LOWA) operators to aggregate linguistic preference relations based on the ordered weighted averaging (OWA) operator defined by Yager (ref. 17) and the convex combination of linguistic labels defined by Delgado et al (ref. 18). Herrera and Herrera-Viedma (ref. 4) analyzed the problem of finding a solution set of alternatives from a collective linguistic preference relation, following two research directions: the choice functions and the mechanisms. Herrera et al. (ref. 5) introduced a framework to reach consensus in group decision making under linguistic assessments. Xu (ref. 19) proposed the linguistic order weighted geometric (LOWG) operators. Xu (ref. 20) studied the group decision making problems, in which all attribute weights, attribute values and the decision maker weights take the form of linguistic term, and the operational laws of the linguistic evaluation scales are defined and some new operators are developed, then a method based on the operators for multi-attribute group decision making under pure linguistic information is presented. Xu (ref. 21) presented an interactive procedure for linguistic multiple attribute decision making, in which the weight information is incomplete and the attribute values take the form of linguistic variable. Wei (ref. 22) studied the linguistic multiple attribute decision making problems, in which the attribute weights are completely unknown and the attribute values and the subjective preference values take the form of linguistic variable. The mathematical model is proposed to calculate the attribute weights, and the linguistic weighted arithmetic averaging (LWAA) operator is utilized to aggregate the linguistic decision-making information. Dong (ref. 23) studied the group decision-making problems with natural linguistic evaluation information, his method is that linguistic evaluation values are transformed into triangular fuzzy numbers, then the triangular fuzzy numbers are aggregated, finally, the best alternative is selected based on the aggregation results.

The research above is not studied the multiple attribute decision making problems with risk decision-making information, about the research on multiple attribute decision making problems with risk, some achievements have been gotten. At present, the main achievements are shown as follows: Yu (ref. 15) studies MADMR problems, in which the attribute weights are unknown and the attribute values are real numbers, and sets up the related mathematical model. Luo (ref. 24) studies MADMR problems in which the attribute weights are completely unknown and attribute values are interval numbers, and sets up two algorithms, grey fuzzy relationship method and two-basic-point method. Yao (ref. 25) and Konstantinos, et al (ref. 26) proposed the TOPSIS method for MADMR problems based on the continuous random variables. Rao and Xiao (ref. 27) proposed the method of dynamic hybrid multiple attribute decision making under risk, based on the unknown weight information and the attribute values which integrates with the precision number, interval number and linguistic fuzzy number. Jin, Zhang and Liu (ref. 27) proposed a rank approach based on projection model to deal with multiple attribute decision-making problems under risk and with attribute value as continuous random variable on bounded intervals. Firstly, risk decision matrix is normalized by density function, and weights of attributes are calculated based on exception value of random variable by using projection pursuit model and genetic algorithm. Then, through calculating weighted correlation coefficients between alternatives and ideal solutions, weighted grey correlation projection models on ideal solutions are developed by grey correlation projection method for every alternative, and alternatives are ranked by grey correlation projection value. Liu and Guan (ref. 28) proposed a grey correlation rank method to solve the problems of multiple attribute continuous decision-making under risk with weight unknown and attribute value as continuous random variable on bounded intervals. Firstly, risk decision matrix is normalized by density function, deviation between two random variables is defined by expectation value, and maximizing deviation rule is used to determine the weights of attributes. Then, ideal/negative ideal solutions are defined. Grey correlation degrees between alternatives and ideal/negative ideal solutions, and relative closeness coefficients are calculated. Furthermore, the alternatives are ranked by relative closeness coefficient of alternatives.

The research above is not studied the MADMR problems with weight information unknown and attribute value as linguistic variable. With respect to these decision making problems, firstly, we transformed the risk linguistic decision matrix into certain linguistic decision matrix by expectation value. Then the attribute weights are determined and the alternatives are ranked by relative optimal membership degree. In order to do so, the remainder of this paper is organized as follows. In the next section, the definition of the multiple attribute decision making problems with risk is briefly introduced. In section 3, linguistic variable set and its extension are introduced; the ideal solution and negative ideal solution with linguistic variable are defined. The attribute weight model is developed by relative optimal membership degree between alternatives and ideal solutions, and the alternatives are ranked by relative optimal membership degree. In section 4, illustrative example is provided to demonstrate the steps and effectiveness of the proposed approach. In Section 5, we concluded the paper and give some remarks.

2. Description of the Decision Making Problems

The MADMR problems are represented as follows:

Suppose that $A=(a_1,a_2,\cdots,a_m)$ is the set of alternatives, and $C=(c_1,c_2,\cdots,c_n)$ is the set of attributes. Let the vector of attributes $W=(w_1,w_2,\ldots,w_n)$ be unknown, and w_j represents the weight of attribute c_j , where $0\le w_j\le 1,\ \sum_{j=1}^{n} w_j=1$.Let $\Theta_j=(\theta_1,\theta_2,\ldots,\theta_{l_j})$ be the possible status which belongs to the attribute c_j , and p_j^t be the probability of the status θ_t occurred for the attribute c_j , where $0\le p_j^t\le 1,\ \sum_{t=1}^{l_j} p_j^t=1$. Let $x_{ij}^t\in S$ be the attribute value for the attribute c_j and status θ_t with respect to the alternative a_i . Let S be the linguistic assessment set, which is the ordered set with odd elements. For example, the linguistic assessment set S has seven elements, then S = (very poor, poor, medium poor, medium, medium good, good, very good). Then, we can evaluate the alternatives. (The MADMR data table is shown as Tab.1)

Table.1 risk multiple attribute decision-making table

	c_1				c_2				$\cdots$	c_n			
	θ_1	θ_2	$\cdots$	θ_{l_1}	θ_1	θ_2	$\cdots$	θ_{l_2}	$\cdots$	θ_1	θ_2	$\cdots$	θ_{l_n}
	p_1^1	p_1^2	$\cdots$	$p_1^{l_1}$	p_2^1	p_2^2	$\cdots$	$p_2^{l_2}$	$\cdots$	p_n^1	p_n^2	$\cdots$	$p_n^{l_n}$
a_1	x_{11}^1	x_{11}^2	$\cdots$	$x_{11}^{l_1}$	x_{12}^1	x_{12}^2	$\cdots$	$x_{12}^{l_2}$	$\cdots$	x_{1n}^1	x_{1n}^2	$\cdots$	$x_{1n}^{l_n}$
a_2	x_{21}^1	x_{21}^2	$\cdots$	$x_{21}^{l_1}$	x_{22}^1	x_{22}^2	$\cdots$	$x_{22}^{l_2}$	$\cdots$	x_{2n}^1	x_{2n}^2	$\cdots$	$x_{2n}^{l_n}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
a_m	x_{m1}^1	x_{m1}^2	$\cdots$	$x_{m1}^{l_1}$	x_{m2}^1	x_{m2}^2	$\cdots$	$x_{m2}^{l_2}$	$\cdots$	x_{mn}^1	x_{mn}^2	$\cdots$	$x_{mn}^{l_n}$

3. Evaluation Method

In this section, with respect to MADM problems under risk with weight information unknown and attribute value as linguistic variable, a decision making method is proposed and detailed steps of this method are given. Firstly, linguistic variable set and its extension are introduced; the ideal solution and negative ideal solution with linguistic variable are defined. Then, attribute weight model is developed by relative optimal membership degree between alternatives and ideal solutions, and the alternatives are ranked by relative optimal membership degree.

3.1. *Linguistic assessment set and extended linguistic assessment set*

Let $S=(s_0,s_1,\cdots,s_{l-1})$ be the linguistic assessment set with odd elements, where l is the odd number, generally, l is equal to 3, 5, 7 or 9. In this paper, let l be 7, then the assessment set is represented as $S=(s_0,s_1,s_2,s_3,s_4,s_5,s_6)$ = (very poor, poor, medium poor, medium, medium good, good, very good).

In these cases, it usually requires that s_i and s_j must satisfy the following additional characteristics:

(1) The set is ordered: $s_i \prec s_j$, if and only if $i<j$;

(2) There is the negation operator: $neg(s_i)=s_j$, and $j=l-i$;

(3) Maximum operator: $\max(s_i,s_j)=s_i$, if $s_i \ge s_j$;

(4) Minimum operator: $\min(s_i,s_j)=s_i$, if $s_i \le s_j$;

For each linguistic assessment set $S=(s_0,s_1,\cdots,s_{l-1})$, the relationship between the element s_i and its subscript i is strict monotonic increasing[30] .Therefore, we defined the function $f: s_i=f(i)$, obviously, the function $f(i)$ is the strict monotonic increasing function to the subscript i. In order to decrease the loss of the linguistic information, the discrete linguistic assessment set $S=(s_0,s_1,\cdots,s_{l-1})$ is extended to the continuous set $\bar{S}=\{s_\alpha \mid \alpha\in R\}$, where the continuous linguistic assessment set $\bar{S}$ still satisfies the strict monotonic-increasing relationship.

The operational laws are defined as follows[31]:

(1) $\beta s_i = s_{\beta\times i}$ (1)

(2) $s_i \oplus s_j = s_{i+j}$ (2)

(3) $s_i \oplus s_j = s_j \oplus s_i$ (3)

(4) $\lambda(s_i\oplus s_j)=\lambda s_i \oplus \lambda s_j$ (4)

(5) $(\lambda_1+\lambda_2)s_i=\lambda_1 s_i \oplus \lambda_2 s_i$ (5)

Definition 1[31]: Let s_α and s_β be two linguistic variables, and $s_\alpha, s_\beta \in \bar{S}$, then we defined the distance between s_α and s_β as:

$$d(s_\alpha,s_\beta)=|\alpha-\beta| \tag{6}$$

3.2. *Transform risk decision-making matrix into certain decision-making matrix*

Based on the formula (1) to (5), we calculate the expectation value of each status with respect to the alternative in the risk decision-making table (shown in table.1), and combine to a certain linguistic decision-making matrix Z.

$$Z=[z_{ij}]_{m\times n}=\begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_m \end{array}\begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1n} \\ z_{21} & z_{22} & \cdots & z_{2n} \\ \vdots & \cdots & \cdots & \vdots \\ z_{m1} & z_{m2} & \cdots & z_{mn} \end{bmatrix} \end{array}.$$

where $$z_{ij}=\sum_{t=1}^{l_j} p_j^t x_{ij}^t \tag{7}$$

3.3. *Positive Ideal Solution and Negative Ideal Solution*

Positive ideal solution (PIS) is the best alternative of all alternatives $a_i\ (i=1,2,\cdots,m)$, that is, each attribute value with respect to the PIS is the best value of this attribute with respect to all the alternatives in the decision matrix. While negative ideal solution (NIS) is the worst alternative, that is, each attribute value with respect to the NIS is the worst value of this attribute with respect to all the alternatives in the decision matrix.

$$\begin{aligned} V^+ &= (v_1^+, v_2^+, \cdots, v_n^+) \\ &= \left(\max_i(z_{i1}), \max_i(z_{i2}), \cdots, \max_i(z_{in})\right) \end{aligned} \tag{8}$$

$$\begin{aligned} V^- &= (v_1^-, v_2^-, \cdots, v_n^-) \\ &= \left(\min_i(z_{i1}), \min_i(z_{i2}), \cdots, \min_i(z_{in})\right) \end{aligned} \tag{9}$$

3.4. *Weight determined model*

The general weighted distance between each alternative a_i and the ideal solution is defined as:

$$D_i^+(a_i,V^+)=\sum_{j=1}^{n}[w_j d(v_j^+,z_{ij})]^2 \tag{10}$$

$$D_i^-(a_i,V^-)=\sum_{j=1}^{n}[w_j d(v_j^-,z_{ij})]^2 \tag{11}$$

where $d(v_j^+,z_{ij})$ is the distance between the linguistic variable v_j^+ and z_{ij}, and $d(v_j^-,z_{ij})$ is the distance between the linguistic variable v_j^- and z_{ij}.

The shorter the distance between each alternative a_i and the positive ideal solution V^+ is, the better the alternative is; the longer the distance between each alternative a_i and the negative ideal solution V^- is, the better the alternative is.

Therefore, for each alternative a_i, we construct the programming model as follows:

$$\begin{aligned} &min\ \ D_i^+(a_i,V^+)=\sum_{j=1}^{n}[w_j d(v_j^+,z_{ij})]^2 \\ &s.t\ \begin{cases}\sum_{j=1}^{n} w_j=1\\ w_j\ge 0\end{cases}\end{aligned} \tag{12}$$

$$\begin{aligned} &max\ \ D_i^-(a_i,V^-)=\sum_{j=1}^{n}[w_j d(v_j^-,z_{ij})]^2 \\ &s.t\ \begin{cases}\sum_{j=1}^{n} w_j=1\\ w_j\ge 0\end{cases}\end{aligned} \tag{13}$$

Due to each alternative is fair competition, and has none of the preference relationship, for formula (12), we can get the quadratic programming model as follows:

$$\begin{aligned} &min\ \ Z(w)=\sum_{i=1}^{m}D_i^+(a_i,V^+)=\sum_{i=1}^{m}\sum_{j=1}^{n}[w_j^+ d(v_j^+,z_{ij})]^2 \\ &s.t\begin{cases}\sum_{j=1}^{n} w_j^+=1\\ w_j^+\ge 0\end{cases}\end{aligned} \tag{14}$$

Construct the Lagrange function:

$$L=\sum_{i=1}^{m}\sum_{j=1}^{n}[w_j^+ d(v_j^+,z_{ij})]^2-2\lambda(\sum_{j=1}^{n}w_j^+-1)$$

Suppose that $\partial L/\partial w_j^+=0$, then:

$$2\sum_{i=1}^{m}d^2(v_j^+,z_{ij})w_j^+-2\lambda=0 \tag{15}$$

Namely, $$w_j^+=\frac{\lambda}{\sum_{i=1}^{m}d^2(v_j^+,z_{ij})} \tag{16}$$

Since $\sum_{j=1}^{n}w_j^+=1$, then $\sum_{j=1}^{n}\frac{\lambda}{\sum_{i=1}^{m}d^2(v_j^+,z_{ij})}=1$

So, $$\lambda=\left[\sum_{j=1}^{n}(\sum_{i=1}^{m}d^2(v_j^+,z_{ij}))^{-1}\right]^{-1} \tag{17}$$

Substitute the formula (17) into the formula (16), then:

$$w_j^+=\left[\sum_{j=1}^{n}(\sum_{i=1}^{m}d^2(v_j^+,z_{ij}))^{-1}\right]^{-1}\Big/\sum_{i=1}^{m}d^2(v_j^+,z_{ij}) \tag{18}$$

So we get: $W^+=\left(w_1^+,w_2^+,\cdots,w_n^+\right)$

Likewise, for formula (13), we can get the quadratic programming model as follows:

$$max\ Z(w)=\sum_{i=1}^{m}D_i^-(a_i,V^-)=\sum_{i=1}^{m}\sum_{j=1}^{n}[w_j^-d(v_j^-,z_{ij})]^2$$
$$s.t\begin{cases}\sum_{j=1}^{n}w_j^-=1\\ w_j^-\geq 0\end{cases} \tag{19}$$

Construct the Lagrange function:

$$L=\sum_{i=1}^{m}\sum_{j=1}^{n}[w_j^-d(v_j^-,z_{ij})]^2-2\lambda(\sum_{j=1}^{n}w_j^- -1)$$

Then

$$w_j^-=\frac{\left[\sum_{j=1}^{n}(\sum_{i=1}^{m}d^2(v_j^-,z_{ij}))^{-1}\right]^{-1}}{\sum_{i=1}^{m}d^2(v_j^-,z_{ij})} \tag{20}$$

So we get $W^-=\left(w_1^-,w_2^-,\cdots,w_n^-\right)$

Suppose that u_i is the relative optimal membership degree between the alternative a_i and the PIS, then $1-u_i$ is the relative optimal membership degree between the alternative a_i and NIS. The combined weighted distance between the alternative a_i and ideal solutions which contain the PIS and NIS, namely, the square sum of the distance between the alternative a_i and ideal solutions which contain the PIS and NIS, is defined as follows:

$$f_i(u_i)=u_i^2\times(\sum_{j=1}^{n}[w_jd(v_j^+,z_{ij})]^2)+(1-u_i)^2\times(\sum_{j=1}^{n}[w_jd(v_j^-,z_{ij})]^2) \tag{21}$$

Obviously, for $u=(u_1,u_2,\cdots,u_m)$, the smaller the value of the function $f_i(u_i)$ is, the better the alternative a_i is. So we can construct the multi-objective programming model:

$$min\{f(u)=(f_1(u_1),f_2(u_2),\cdots,f_m(u_m))\} \tag{22}$$

Satisfy the constraint conditions:

$$\begin{cases}\sum_{j=1}^{n}w_j=1,\ w_j\geq 0\ \ (j=1,2,\cdots,n)\\ 0\leq u_i\leq 1\ ,\ i=1,2,\cdots,m\end{cases} \tag{23}$$

Calculate the derivative of u_i for the formula (22):

$$\frac{\partial f_i(u_i)}{\partial u_i}=0 \tag{24}$$

Simplified the formulas, then we get:

$$u_i=\frac{D_i^-(a_i,V^-)}{D_i^+(a_i,V^+)+D_i^-(a_i,V^-)} \tag{25}$$

W^+ and W^-, determined by the formula (18)and (20) respectively, make that the alternative not only has the shortest distance from the PIS ,but also has the furthest distance from the NIS. Suppose that $W=\left(w_1,w_2,\cdots,w_n\right)$ be the weight vector. In order to get the biggest relative optimal membership degree, we construct the optimization model as follows:

$$max\quad u_i=\frac{D_i^-(a_i,V^-)}{D_i^+(a_i,V^+)+D_i^-(a_i,V^-)}=\frac{\sum_{j=1}^{n}[w_jd(v_j^+,z_{ij})]^2}{\sum_{j=1}^{n}[w_jd(v_j^+,z_{ij})]^2+\sum_{j=1}^{n}[w_jd(v_j^-,z_{ij})]^2} \tag{26}$$

$$s.t\begin{cases}\sum_{j=1}^{n}w_j=1\\ w_j\in[\min(w_j^-,w_j^+),\max(w_j^-,w_j^+)]\end{cases}$$

Since the formula (26) is a nonlinear programming model, and it is difficult to solve the model for using the traditional method, we can utilize the genetic algorithms to solve this problem. Then we can get the weight vector $W=\left(w_1,w_2,\cdots,w_n\right)$

3.5. *Rank the order of alternatives*

Substituting the weight vector $W=\left(w_1,w_2,\cdots,w_n\right)$ into the formula (25), we can get the relative optimal membership degree u_i between each alternative and the PIS. According to the relative optimal membership degree u_i, the ranking order of all the alternatives is determined. The more relative optimal membership degree u_i, the better the alternative is.

4. Illustrate Example

An enterprise will invest to construct a new factory. There are four alternatives. The four alternatives are assessed based on four attributes which are shown as follows: Direct efficiency (c_1), Indirect efficiency (c_2), Social efficiency (c_3), Pollution loss(c_4). Direct efficiency (c_1)and Indirect efficiency (c_2) are represented as four status, such as "very good(θ_1)", "good(θ_2)", "medium(θ_3)" and "poor(θ_4)", and Social efficiency (c_3),Pollution loss(c_4) are presented as three status, such as "very good(θ_1)", "good(θ_2)" and "medium(θ_3)". The attributes of each alternative take the form of linguistic variable from linguistic variable set $S=(s_0,s_1,s_2,s_3,s_4,s_5,s_6)$ = (very poor, poor, medium poor, medium, medium good, good, very good). Based on the risk decision-making table of each attribute (shown as Table.2), we can get the best alternative.

The decision-making steps are shown as follows:

(1) Transform the risk decision-making matrix into a certain decision-making matrix:

$$Z=\begin{bmatrix} s_{4.1} & s_{4.2} & s_{3.8} & s_{3.6} \\ s_{4.2} & s_{4} & s_{3.1} & s_{3.3} \\ s_{3.6} & s_{3.9} & s_{3.9} & s_{3.7} \end{bmatrix}$$

(2) Calculate the ideal solution of all alternatives:

$$V^{+}=(s_{4.2},s_{4.2},s_{3.9},s_{3.7})$$
$$V^{-}=(s_{3.6},s_{3.9},s_{3.1},s_{3.3})$$

(3) Calculate the attribute weights:

$$W^{+}=(0.1517,0.4318,0.0864,0.3302)$$
$$W^{-}=(0.0992,0.6052,0.0536,0.2421)$$
$$W=(0.1342,0.5011,0.1050,0.2597)$$

Table.2 risk decision-making table of each attribute

	c_1				c_2				c_3			c_4		
	θ_1	θ_2	θ_3	θ_4	θ_1	θ_2	θ_3	θ_4	θ_1	θ_2	θ_3	θ_1	θ_2	θ_3
	0.1	0.3	0.4	0.2	0.1	0.2	0.4	0.3	0.3	0.3	0.4	0.3	0.2	0.5
a_1	s_5	s_4	s_5	s_2	s_4	s_4	s_3	s_6	s_5	s_5	s_2	s_4	s_2	s_4
a_2	s_5	s_3	s_5	s_4	s_6	s_3	s_4	s_4	s_6	s_3	s_1	s_2	s_6	s_3
a_3	s_4	s_6	s_2	s_3	s_1	s_6	s_5	s_2	s_4	s_5	s_3	s_3	s_4	s_4

(4) Calculate the weighted distance between each alternative and the ideal solutions:

$$D^{+}=(0.0010,0.0279,0.0291)$$
$$D^{-}=(0.0386,0.0090,0.0178)$$

(5) Calculate the relative optimal membership degree:

$$u=(0.9756,0.2439,0.3802)$$

(6) Rank the order:

Based on the relative optimal membership degree, we can rank the order: $a_1 \succ a_3 \succ a_2$.

5. Conclusions

The MADMR problems are wildly used in various areas. This paper proposed a decision making method based on relative optimal membership degree for solving the MADMR problems in which the attribute weights are unknown and the attribute values are take the form of linguistic terms. Firstly, we transformed the risk linguistic decision matrix into certain linguistic decision matrix by expectation value. Then, the ideal solution and negative ideal solution with linguistic variable are defined, and the attribute weight model is developed by relative optimal membership degree between alternatives and ideal solutions. In addition, the alternatives are ranked by relative optimal membership degree. Then the attribute weights are determined and the alternatives are ranked by relative optimal membership degree. Finally, illustrative example is provided to demonstrate the steps and effectiveness of the proposed approach. This method is simple and easy to understand. This method constantly enriches and develops the theory and method of MADMR, and proposed a new idea for solving the MADMR problems. In the future, we shall continue working in the extension and application of the developed method to other domains.

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