Study on Quantum Particle Swarm Optimization Algorithm to Win-win Situation for the Physical Distribution Routing Problem

Wenxue Ran^{1,2} Dongfeng Zhao² Zhilan Song¹ Yuan Zhou¹

¹Department of Business, Yunnan University of Finance and Economics, Kunming, Yunnan, P. R. China

²Department of communication and information, Yunnan University, Kunming, Yunnan, P. R. China

Abstract

This paper, based on the order with which the theory of random character combined, offers random distribution ordering assembly system model on the basis of multi-distribution route. In addition, this model's sorting capacity and its impact of time delay characteristic on sorting capacity of system are presented, with its strength and weakness is analyzed as well. For that reason, we come up with the scientific foundation for new thinking and in-depth research of planning and construction of distribution center.

Keywords: Win-win Situation, The Physical Distribution Routing Problem, Quantum Particle Swarm Optimization Algorithm

1. Introduction

The physical distribution routing optimization problem is NP (Non-deterministic Polynomial) problem and this is the typical vehicle routing optimization problem. VRP(Vehicle Routing Problem), by Dantzig and Ramser suggested first in 1959[1], is an important content in the study for the delivery line at the end of Modern logistics distribution center.

Through dozens of years of research, VRP has become a hot topic in Operational Research and Combinatorial Optimization filed. It is usually can be described as how to organize the appropriate route and the vehicles ordered move through a series of unloading point problem on distribution transport route, organizing the traffic lines, vehicles and orderly through these points. In certain constraints (such as goods delivery demand, sending volume, the delivery time and delivery vehicles carrying capacity, etc), it will reach certain goals, which usually are the shortest route, the least cost and time, at least vehicles. This article regards the optimization goal as "win-win" pursuit of the goal under the condition of market economy, based on both customer satisfaction maximization conditions and the benefit maximization of the distribution center.

Quantum Optimization Algorithm is an exploring arithmetic which use the unique nature of quantum computing theory, for example, quantum coherent, quantum stack, quantum entanglement, etc. Comparing other classic optimization algorithms, its essential difference is quantum parallelism.

This paper advocates applying quantum optimization algorithm and opti-

mized algorithm of artificial intelligence, such as neural network algorithm, evolutionary algorithm, the ant colony algorithm, particle swarm algorithm, to improve algorithm performance, broaden algorithm application, improve and perfect the algorithm system.

2. The system model for pursuing win-win distribution route

2.1. Customer satisfaction model

The traditional vehicle routing problem regards time window as transportation service time constraint. Combined with practical conditions, customers tend to get service in a certain period[2]. Let us suppose that the customer satisfaction is described as the trapezoid fuzzy time window in figure 1, $[\tau_{1i}, \tau_{2i}]$ shows the service time scope which customers can tolerate, $[a_i, b_i]$ shows the service time scope which customers expect. The satisfaction function of customer i described in figure 1 is as follows:

Fig. 1: trapezoid fuzzy number time window of customer satisfaction.

$$\varphi_i(t_i) = \begin{cases} 0; t_i < \tau_{1i} \\ \frac{t_i - \tau_{1i}}{a_i - \tau_{1i}}; \tau_{1i} < t_i < a_i \\ 1; a_i \leq t_i \leq b_i \\ \frac{\tau_{2i} - t_i}{\tau_{2i} - b_i}; b_i < t_i < \tau_{2i} \\ 0; t_i > \tau_{2i} \end{cases} \quad (1)$$

Obviously, in the above formula, for the checking of lead-time, let us suppose

$$k_{1i} = \frac{1}{a_i - \tau_{1i}}, k_{1i} \text{ is smaller and the}$$

scope in advance which customers can tolerate is longer. Conversely, for the checking of delay, suppose,

$$k_{2i} = -\frac{1}{\tau_{2i} - b_i}, k_{2i} \text{ is bigger and the}$$

scope in advance which customers can tolerate is shorter. Meanwhile, k_{1i} is smaller and the scope in advance which customers can tolerate is longer.

Suppose that the distribution center prepares $m(m=1,2,\dots,m)$ delivery trucks for $i(i=1,2,\dots,n)$ customers to distribute, $m=0, i=0$ means delivery goods are trapped in the warehouse of distribution center. Each of the distribution vehicles carrying $l_m(m=1,2,\dots,m)$. Each customer demand is $b_n(n=1,2,\dots,n)$. The shipping cost for customer i to customer j is c_{ij} . Each time the distribution vehicle has completed task distribution and come back distribution center service for customer i , the begin service time is t_i , Travel time for customer i to customer j is t_{ij} . $w_i(t_i)$ means the waiting time for customer i , s_i shows the service time for customer i . $\varphi_i(t_i)$ is satisfaction of customer i . We will define variables as follows:

$$y_{im} = \begin{cases} 1; \text{Vehicle } m \text{ distribute customer } i \\ 0; \text{Others} \end{cases} \quad (2)$$

$$x_{ijm} = \begin{cases} 1; \text{Vehicle } m \text{ from customers } i \text{ to } j \\ 0; \text{Others} \end{cases} \quad (3)$$

Objective function of distribution route based on customer satisfaction is as follows:

(1) Maximize customer satisfaction on average:

$$\max \frac{1}{n} \sum_{i=1}^n \varphi_i(t_i) \quad (4)$$

The objective function is equivalent to minimize customer unsatisfaction:

$$\min(1 - \frac{1}{n} \sum_{i=1}^n \varphi_i(t_i)) \quad (5)$$

(2) Minimize vehicle number:

$$\min \sum_{j=1}^n \sum_{k=1}^m x_{0jk} \quad (6)$$

(3) Minimize driving distance:

$$\min \sum_{k=1}^m \sum_{i=0}^n \sum_{j=0}^n c_{ij} x_{ijk} \quad (7)$$

(4) Minimize waiting time:

$$\min \sum_{i=1}^n w_i(t_i) \quad (8)$$

The constraint conditions are as follows:

(1) To ensure that every customer satisfaction is not below a certain value $\eta_i (0 < \eta_i < 1)$, This value is given according to experience of distribution center's business decision-makers, that is:

$$\varphi_i(t_i) > \eta_i \quad (9)$$

(2) Capacity constraints of each of the distribution vehicle:

$$\sum_{i=1}^n d_i y_{im} \leq l_m; \forall m \quad (10)$$

(3) To ensure that every customer can get services:

$$\sum_{j=1}^m y_{im} = 1; \forall i \quad (11)$$

(4) To ensure that every customer is serviced only by a distribution vehicle:

$$\sum_{i=1}^n x_{ijm} = y_{jm}; \forall j, m \quad (12)$$

$$\sum_{j=1}^m x_{ijm} = y_{im}; \forall i, m \quad (13)$$

(5) There are not circuits in distribution process:

$$\sum_{i,j \in S \times S} x_{ijk} \leq |S| - 1; S \in (1, 2, \dots, n), \forall m \quad (14)$$

(6) To ensure that every customer is serviced in fuzzy time window $[\tau_{1i}, \tau_{2i}]$:

$$t_j = \max\{\tau_{1j}, t_i + s_i + t_{ij}\} \quad (15)$$

$$t_j < \tau_{2j} \quad (16)$$

$$w_j(t_j) = \max\{0, t_j - \tau_{1j}\} \quad (17)$$

2.2. The economic benefit model of distribution center

Distribution transport task of distribution center is always around the economic benefits, even free service; also other business income of the distribution center must offset the distribution transport costs. Therefore, according to the calculation

model of Fisher market equilibrium price[3], suppose that the distribution center prepares $m(m=1,2,\dots,m)$ delivery trucks for $i(i=1,2,\dots,n)$ customers to distribute, every customer hold cash amount is $A=\{e_1, e_2, \dots, e_n\}$, $B=\{q_1, q_2, \dots, q_m\}$ means the service fee to the distribution vehicle. We consider the following logarithmic profit function:

$$u_i(x_{i,1}, x_{i,2}, \dots, x_{i,m}) = \sum_{j=1}^m \beta_{i,j} \ln\left(\frac{x_{i,j}}{\varepsilon_{i,j}} + 1\right) \quad (18)$$

For every customer i , $x_{i,j}$ is the receiving service quantity of distribution vehicle m . $\beta_{i,j} > 0$, $\varepsilon_{i,j} > 0$, they are experienced constant. Obviously, the service of every vehicle can be divided, $x_{i,1}$ is the real number that meets $x_{i,1} \in [0, q_j]$.

p_j shows the service price of distribution vehicle j , $P=(p_1, p_2, \dots, p_m) > 0$, each customer will maximize his returns:

$$\begin{aligned} & \max u_i(x_{i,1}, x_{i,2}, \dots, x_{i,m}) \\ & s.t. p_1 x_{i,1} + p_2 x_{i,2} + \dots + p_m x_{i,m} = e_i \\ & x_{i,j} \geq 0, j=1,2,\dots,m \end{aligned} \quad (19)$$

We apply Lagrange-multiplier method, suppose that:

$$F(x_{i,1}, x_{i,2}, \dots, x_{i,m}; \zeta) =$$

$$u_i(x_{i,1}, x_{i,2}, \dots, x_{i,m}) - \zeta(p_1 x_{i,1} + p_2 x_{i,2} + \dots + p_m x_{i,m} - e_i)$$

then:

$$\frac{\partial F}{\partial \zeta} = e_i - (p_1 x_{i,1} + p_2 x_{i,2} + \dots + p_m x_{i,m}) = 0$$

$$\frac{\partial F}{\partial x_{i,j}} = \frac{\beta_{i,j}}{\varepsilon_{i,j} + x_{i,j}} - \zeta p_j = 0, j=1,2,\dots,m$$

The solution to these equations is $(x_{i,1}^*(p), x_{i,2}^*(p), \dots, x_{i,m}^*(p), \zeta)$. After solving equations, we can get:

$$\zeta = \frac{\sum_{j=1}^m \beta_{i,j}}{e_i + \sum_{j=1}^m \varepsilon_{i,j} p_j} \quad (20)$$

$$x_{i,j}^* = \frac{\beta_{i,j}}{p_j} \cdot \frac{e_i + \sum_{j=1}^m \varepsilon_{i,j} p_j}{\sum_{j=1}^m \beta_{i,j}} - \varepsilon_{i,j},$$

and

$$x_{i,j}^* \geq 0, j=1,2,\dots,m \quad (21)$$

If $x_{i,j}^* < 0$, then $x_{i,j}^* = 0$.

And in distribution transport services, we shall assume these services have been accepted, and all of customers must pay the corresponding service fee, so there are some constraint conditions:

$$\begin{aligned} & \forall i \in A, p_1 x_{i,1}^*(P) + p_2 x_{i,2}^*(P) + \dots + p_m x_{i,m}^*(P) = e_i \\ & \forall j \in B, x_{i,1}^*(P) + x_{i,2}^*(P) + \dots + x_{i,m}^*(P) = q_j \end{aligned} \quad (22)$$

Then we can get the maximum income which are from distribution transport:

$$E_{\max} = \max u_i(x_{i,1}, x_{i,2}, \dots, x_{i,m}) = \max \sum_{j=1}^m \beta_{i,j} \ln\left(\frac{x_{i,j}^*}{\varepsilon_{i,j}} + 1\right) \quad (23)$$

The formula (23) can also be expressed as:

$$R_{\min} = (\max \sum_{j=1}^m \beta_{i,j} \ln(\frac{x_{i,j}^*}{\varepsilon_{i,j}} + 1))^{-1} \quad (24)$$

3. Quantum and Particle Swarm Optimization Algorithm

Quantum-inspired Evolution Algorithm, QEA, has been proved to be a better probability evolutionary algorithm in series optimization calculations. It is a result of combining the theory of quantum computing and evolutionary algorithms. In This paper we use the integration of quantum's powerful area search capability and PSO (Particle Swarm Optimization) tracking the extreme particle, proposed a Quantum Behaved Particle Swarm Optimization to solve the delivery route based on win-win approach.

(1) Initialize the population

Use the probability of quantum bits as a the code of particles(vehicles).The scenario of Initialization code is:

$$P_j = \begin{bmatrix} \cos(\theta_{j1}) & \cos(\theta_{j2}) & \dots & \cos(\theta_{jn}) \\ \sin(\theta_{j1}) & \sin(\theta_{j2}) & \dots & \sin(\theta_{jn}) \end{bmatrix} \quad (25)$$

$\theta_{ji} = 2\pi \times rnd$, rnd is random located between (0,1). $j = 1, 2, \dots, m$, $i = 1, 2, \dots, n$. m shows the population scale, n means the spatial dimension. Each particle in a population occupies and traverses two positions of the space. They correspond to the probability amplitude of quantum state $|0\rangle$ and $|1\rangle$.

$$P_{jc} = (\cos(\theta_{j1}), \cos(\theta_{j2}), \dots, \cos(\theta_{jn})) \quad (26)$$

$$P_{js} = (\sin(\theta_{j1}), \sin(\theta_{j2}), \dots, \sin(\theta_{jn})) \quad (27)$$

(2) The solution space transformation

Each of probability amplitude of quantum state has corresponding optimized variable in the solution space.

$[\alpha_j^i, \beta_j^i]^T$ is the i qubit on particle P_j ,

The corresponding solution space variables are:

$$P_{jlc} = (\cos(\theta_{j1}), \cos(\theta_{j2}), \dots, \cos(\theta_{jn})) \quad (28)$$

$$P_{jls} = (\sin(\theta_{j1}), \sin(\theta_{j2}), \dots, \sin(\theta_{jn})) \quad (29)$$

(3) Particle State Update

The regulations of particle state update:

$$\Delta\theta_{ji}(t+1) = \omega\Delta\theta_{ji}(t) + c_1r_1(\Delta\theta_l) + c_2r_2(\Delta\theta_g) \quad (30)$$

Meanwhile:

$$\Delta\theta_l = \begin{cases} 2\pi + \theta_{jli} - \theta_{ji}; & (\theta_{jli} - \theta_{ji} < -\pi) \\ \theta_{jli} - \theta_{ji}; & (-\pi \leq \theta_{jli} - \theta_{ji} \leq \pi) \\ \theta_{jli} - \theta_{ji}; & (\theta_{jli} - \theta_{ji} > \pi) \end{cases}$$

$$\Delta\theta_g = \begin{cases} 2\pi + \theta_{jgi} - \theta_{ji}; & (\theta_{jgi} - \theta_{ji} < -\pi) \\ \theta_{jgi} - \theta_{ji}; & (-\pi \leq \theta_{jgi} - \theta_{ji} \leq \pi) \\ \theta_{jgi} - \theta_{ji}; & (\theta_{jgi} - \theta_{ji} > \pi) \end{cases}$$

Then the update for probability amplitude of qubits based on the quantum revolving door is:

$$\begin{bmatrix} \cos(\theta_{ji}(t+1)) \\ \sin(\theta_{ji}(t+1)) \end{bmatrix} = \begin{bmatrix} \cos(\Delta\theta_{ji}(t+1)) & -\sin(\Delta\theta_{ji}(t+1)) \\ \sin(\Delta\theta_{ji}(t+1)) & \cos(\Delta\theta_{ji}(t+1)) \end{bmatrix} \begin{bmatrix} \cos(\theta_{ji}(t) + \Delta\theta_{ji}(t+1)) \\ \sin(\theta_{ji}(t) + \Delta\theta_{ji}(t+1)) \end{bmatrix} \quad (31)$$

The two new position of particle P_j :

$$P_{jc}^* = (\cos(\theta_{j1}(t) + \Delta\theta_{j1}(t+1)), \dots, \cos(\theta_{jn}(t) + \Delta\theta_{jn}(t+1))) \quad (32)$$

$$P_{js}^* = (\sin(\theta_{j1}(t) + \Delta\theta_{j1}(t+1)), \quad (33) \\ \dots, (\sin(\theta_{jn}(t) + \Delta\theta_{jn}(t+1)))$$

(4) Mutation processing

The process of mutation is realized by the quantum controlled NOT gate:

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \cos(\theta_{ji}) \\ \sin(\theta_{ji}) \end{bmatrix} = \begin{bmatrix} \sin(\theta_{ji}) \\ \cos(\theta_{ji}) \end{bmatrix} = \begin{bmatrix} \cos(\frac{\pi}{2} - \theta_{ji}) \\ \sin(\frac{\pi}{2} - \theta_{ji}) \end{bmatrix} \quad (34) \\ j \in \{1, 2, \dots, m\}, \quad i \in \{1, 2, \dots, n\}.$$

(5) The fitness of particle swarm

We define the fitness of quantum and particle swarm optimization based on the win-win distribution route as follows:

$$Z = \rho_1 \frac{VN - VN_{\min}}{VN_{\max} - VN_{\min}} + \rho_2 \frac{D - D_{\min}}{D_{\max} - D_{\min}} \quad (35) \\ + \rho_3 \frac{WT - WT_{\min}}{WT_{\max} - WT_{\min}} + \rho_4 \left(1 - \frac{1}{n} \sum_{i=1}^n \varphi_i(t_i)\right) \\ + \frac{\rho_5}{\sum_{j=1}^m \beta_{i,j} \ln\left(\frac{x_{i,j}^*}{\varepsilon_{i,j}} + 1\right)}$$

VN means the number of vehicles used, $VN_{\min}$ is the minimum number of vehicles used, $VN_{\max}$ is the maximum number of vehicles used, D shows the total driving distance of vehicles, $D_{\min}$ is the minimum driving distance of vehicles, $D_{\max}$ is the maximum driving distance of vehicles, WT means the total waiting time, $WT_{\min}$ is the minimum waiting time of former populations, $WT_{\max}$ is the maximum waiting time of former populations, and $\rho_1, \rho_2, \rho_3, \rho_4, \rho_5$ are weighting coefficients, $\rho_1 + \rho_2 + \rho_3 + \rho_4 + \rho_5 = 1$.

4. The description for solution of the process

The quantum and particle swarm optimization procedure is as follows:

Step1: According to the formula 25, initializing particle swarm, generating particle position and then composing the initial population ;

Step2: According to the formula 28 and 29, we can get the solution space transformation. The fitness of every particle can be calculated according to the formula 35. If the current position is superior to their memory, the optimal position will be replaced by current position, if current global optimal location is superior to the positions which can be searched, current global optimal location will replace;

Step3: We apply the formula 30 and 31 to realize particle state update;

Step4: The process of mutation is realized by formula 34;

Step5: Determining whether it is the optimal solution, if not, we will turn to Step2 and calculate circularly until we get the optimal solution.

5. Example and Conclusion

Hunan Baisha logistics distribution center is engaged in retail distribution of cigarette commodity. The working time of the year is 251 days/year (five days a week), 8 hours a day (one-shift workday system). Cigarettes are expected to 40 million big boxes in annual sales. There are 28000 retailers in jurisdiction areas. Customers provide detailed layout of retailers, location of distribution center, the distance and the traffic toward these retailers.

The optimization results: the number of distribution vehicles reduces from 58 to 28. The optimized route take the shape of "petals" around the distribution center, consistent with results of other optimiza-

tion algorithm. The operation cost for a carton of cigarette reduces to 0.26 Yuan, closing to actual effect.

Research shows that the quantum and particle swarm algorithm with advantages of high speed, good global performance and convergence speed. In this paper, the algorithm is a quantum optimization algorithm combined with the particle swarm optimization algorithm for solving the practical distribution routing optimization, furthermore, it will improve the superiority of two algorithms.

References

- [1] G. B. Dantzig, J. H. Ramser, The Truck Dispatching Problem. 1959.
- [2] Ning Chen, Xiaotie Deng, Xiaoming Sun, Andrew Chi-Chih Yao: Fisher Equilibrium Price with a Class of Concave Utility Functions, ESA 2004:169-179.
- [3] Finnila A B, Gomez M A, Sebenik C. Quantum annealing a new method for minimizing multidimensional functions[J]. Chemical Physics Letters, 1994, 219: 343-348.
- [4] Kadowaki T. Study of Optimization Problems by Quantum Annealing[D]. Japan: Department of Physics, Tokyo Institute of Technology, 2002.
- [5] Trugenberger C A. Quantum optimization for combinatorial searches[J]. New Journal of Physics, 2002, 4(26): 1-7.
- [6] Han K H, Kim J H. Quantum-inspired evolutionary algorithm with a new termination criterion, He-gate and two-phase scheme[J]. IEEE Trans on Evolutionary Computation, 2004, 8(2): 156-169.