Identification of Time Series Transfer Function Parmeter

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Abstract—Transfer functions are commonly used in the analysis of systems such as single-input single-output filters, typically within the fields of signal processing, communication theory, and control theory. Transfer function models is of considerable interest in economics, engineering, biology, and many other fields. Models of this kind can describe not only the behavior of industrial processes but also that of economic and business systems. Transfer function model building is important because it is only when the dynamic characteristics of a system are understood that intelligent direction, manipulation, and control of the system is possible. Engineering methods for estimating transfer functions are usually based on the choice of special inputs to the system such as step and sine wave inputs and "pulse" inputs. These methods have been useful when the system is affected by small amounts of noise but are less satisfactory otherwise. In this paper we show procedure and methods for estimating the transfer function parameters.

Keywords- transfer function models; frequency response function; identification procedure; least squares estimation , autocovariance function

I. INTRODUCTION

In engineering, a transfer function (also known as the system function or network function and, when plotted as a graph, transfer curve) is a mathematical representation for fit or to describe inputs and outputs of black box models.

Transfer functions are commonly used in the analysis of systems such as single-input single-output filters, typically within the fields of signal processing, communication theory, and control theory. The term is often used exclusively to refer to linear, time-invariant systems (LTI), as covered in this article. Most real systems have non-linear input/output characteristics, but many systems, when operated within nominal parameters (not "over-driven") have behavior that is close enough to linear that LTI system theory is an acceptable representation of the input/output behavior.

A topic of considerable industrial interest is the study of process dynamics. Such a study is made (1) to achieve better control of existing plants and (2) to improve the design of new plants. In particular, several methods have been proposed for estimating the transfer function of plant units from process records consisting of an input time series X_t and an output time series Y_t.

As shown in Fig .1, where the input X_t and the output Y_t. A hypothetical impulse response function v_j, j = 0, 1, 2, . . . , which determines the transfer function for the system through a dynamic linear relationship between input X_t and output Y_t of the form

$$Y_t = \sum_{j=0}^{\infty} v_j X_{t-j} \tag{1}$$

is also shown in the figure as a bar chart. Transfer function models that relate an input process X_t to an output process Y_t are introduced in this paper and many of their properties are examined.

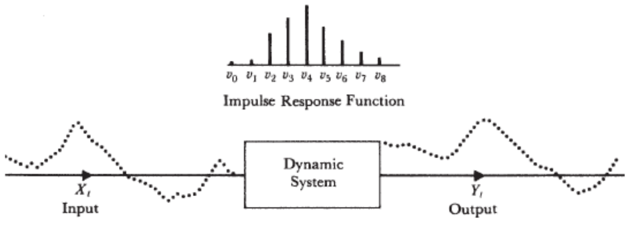

Figure 1. Input and output time series in relation to a dynamic system

With suitable inputs and outputs, the dynamic system of Fig .1 might represent an industrial process, the economy of a country, or the behavior of a particular corporation or government department.

Technically it is a representation in terms of spatial or temporal frequency, of the relation between the input and output of a linear time-invariant system with zero initial conditions and zero-point equilibrium. With optical imaging devices, for example, it is the Fourier transform of the point spread function (hence a function of spatial frequency) i.e. the intensity distribution caused by a point object in the field of view.

II. Outline of the Identification Procedure

Suppose that the transfer function model

$$Y_t = v(B)X_t + N_t \tag{2}$$

Can be parsimoniously parameterized in the form

$$Y_t = \delta^{-1}(B)\omega(B)X_{t-b} + N_t \tag{3}$$

Where

$$\delta(B) = 1 - \delta_1 B - \delta_2 B^2 - \cdots - \delta_\tau B^\tau$$

$$\omega(B) = \omega_0 - \omega_1 B - \omega_2 B^2 - \cdots - \omega_s B^s$$

The identification procedure is as follows:

1. Derive rough estimates $\hat{v}_j$ of the impulse response weights v_j in (2).

2. Use the estimates $\hat{v}_j$ so obtained to make guesses of the orders r and s of the denominator and numerator operators in (3) and of the delay parameter b.

3. Substitute the estimates $\hat{v}_j$ in the equations with values of r, s, and b obtained from initial estimates of the parameters δ and ω in (3).

Knowing the $\hat{v}_j$, values of b, r, and s may be determined by using the following facts established in the model of the form of (3) the impulse response weights vj consist of:

1. b zero values $v_0, v_1, \ldots, v_{b-1}$.

2. A further $s - r + 1$ values $v_b, v_{b+1}, \ldots, v_{b+s-r}$ following no fixed pattern (no such values occur if $s < r$).

3. Values v_j with $j \geqslant b + s - r + 1$ that follow the pattern dictated by an rth-order difference equation that has r starting values $v_{b+s}, \ldots, v_{b+s-r+1}$. starting values v_j for $j < b$ will, of course, be zero.

The basic tool that is employed here in the identification procedure is the cross-correlation function between input and output.

When the processes are nonstationary, it is assumed that stationarity can be induced by suitable differencing. Nonstationary behavior is suspected if the estimated autoand cross-correlation functions of the (X_t, Y_t) series fail to damp out quickly. It assume that a degree of differencing d necessary to induce stationarity has been achieved when the estimated auto- and cross correlations $r_{xx}(k)$, $r_{yy}(k)$, and $r_{xy}(k)$ of $x_t = \nabla^d X_t$ and $y_t = \nabla^d Y_t$ damp out quickly. In practice, d is usually 0, 1, or 2.

Some general remarks can be made concerning the procedure for identifying transfer function and noise models that just described

1. For many practical situations, when the effect of noise is appreciable, a delayed first- or second-order system or some simplification of it, would often provide as elaborate a model as could be justified for the data. In practice, the output Y could not be expected to follow exactly the pattern determined by the transfer function model, even if that model were entirely adequate. Disturbances of various kinds other than X normally corrupt the system.

2. To start off the recursion we need to know certain initial values. This need is not, of course, a shortcoming of the method of calculation but comes about because with a transfer function model, the initial values of Y will depend on values of X that occurred before observation was begun. In practice, when the necessary initial values are not known, we can substitute mean values for unknown Y's and X's (zeros if these quantities are considered as deviations from their means). The early calculated values will then depend upon this choice of the starting values. However, for a stable system, the effect of this choice will be negligible after a period sufficient for the impulse response to become negligible.Efficient estimation is only possible assuming the model form to be known. The estimates given are in general necessarily inefficient therefore. They are employed at the identification stage because they are easily computed and can indicate a form of model worthy to be fitted by more elaborate means.

3. Even if these were efficient estimates, the number required to trace out the impulse response function fully would typically be considerably larger than the number of parameters in a transfer function model. In cases where the δ and ω in an adequate transfer function model could be estimated accurately, nevertheless, the estimates of the corresponding v's could have large variances and be highly correlated .

A nonlinear least squares algorithm, analogous to that given for fitting the stochastic model can be used to obtain the least squares estimates and their approximate standard errors. The algorithm will behave well when the sum-of-squares function is very roughly quadratic. However, the procedure can sometimes run into trouble, in particular if the parameter estimates are very highly correlated (if, e.g., the model approaches singularity due to near-common factors in the factorizations of the operators), or in some cases, if estimates are near a boundary of the permissible parameter space. In difficult cases the estimation situation may be clarified by plotting sums-of-squares contours for selected two-dimensional sections of the parameter space.

As with stochastic models, the derivatives may be computed recursively. However, it seems simplest to work with a standard nonlinear least squares computer program in which derivatives are determined numerically and an option is available of "constrained iteration" to prevent instability

4.Usually, only very rough estimates are possible with the available data. However, some kind of rudimentary modeling may be possible by postulating a plausible but simple transfer function/noise model, fitting directly by the least squares procedures , and applying diagnostic checks leading to elaboration of the model when this proves necessary.

5. Since rather simple transfer function models of first or second order, with or without delay, are often adequate, iterative model building should begin with a fairly simple model, looking for further simplification if this is possible,and reverting to more complicated models only as the need is demonstrated.

6. When simplification by factorization is possible, but is overlooked, the least squares estimation procedure may become extremely unstable since the minimum will tend to lie on a line or surface in the parameter space rather than at a point. Conversely, instability in the solution can point to the possibility of simplification of the model. One reason for carrying out the identification procedure before fitting the model is to avoid redundancy or, conversely, to achieve parsimony in parameterization.

III. Identification of the Impulse Response Function without Prewhitening

After differencing d times, the model (3) can be written in the form:

$$y_t = v_0 x_t + v_1 x_{t-1} + v_2 x_{t-2} + \cdots + n_t \tag{4}$$

Where

$$y_t = \nabla^d Y_t$$

$$x_t = \nabla^d X_t$$

$$n_t = \nabla^d N_t$$

are stationary processes with zero means. Then, on multiplying throughout in (4) by x_{t-k} for k $\geqslant$0, we obtain

$$x_{t-k} y_t = v_0 x_{t-k} x_t + v_1 x_{t-k} x_{t-1} + \cdots + x_{t-k} n_t \tag{5}$$

assumption that x_{t-k} is uncorrelated with n_t for all k, taking expectations in (5) yields the set of equations

$$\gamma_{xy}(\mathrm{k}) = v_0 \gamma_{xx}(\mathrm{k}) + v_1 \gamma_{xx}(\mathrm{k}-1) + \cdots \quad \mathrm{k} = 0,1,2 \cdots \tag{6}$$

Suppose that the weights v_j are effectively zero beyond k = K. Then the first $K + 1$ of the equations (6) can be written

$$\gamma_{xy} = \Gamma_{xx} v \tag{7}$$

Where

$$\gamma_{xy} = \begin{bmatrix} \gamma_{xy}(0) \\ \gamma_{xy}(1) \\ \vdots \\ \gamma_{xy}(K) \end{bmatrix} \qquad v = \begin{bmatrix} v_0 \\ v_1 \\ \vdots \\ v_k \end{bmatrix}$$

$$\Gamma_{xx} = \begin{bmatrix} \gamma_{xx}(0) & \gamma_{xx}(1) & \cdots & \gamma_{xx}(K) \\ \gamma_{xx}(1) & \gamma_{xx}(0) & \cdots & \gamma_{xx}(k-1) \\ \vdots & \vdots & \vdots & \vdots \\ \gamma_{xx}(K) & \gamma_{xx}(k-1) & \cdots & \gamma_{xx}(0) \end{bmatrix}$$

Substituting estimates $c_{xx}(k)$ of the autocovariance function of the input x_t and estimates $c_{xy}(k)$ of the cross-covariance function between the input x_t and output y_t , (7) provides $K + 1$ linear equations for the first $K + 1$ weights. However, these equations, which do not in general provide efficient estimates, are cumbersome to solve for large K and in any case require knowledge of the point K beyond which the v_j are effectively zero. The sample version of equations (7) represents essentially, apart from “end effects,” the least squares normal equations from linear regression of y_t on x_t, x_{t-1}, . . . , x_{t-K}, in which it is assumed, implicitly, that the noise nt in (4) is not autocorrelated. This is one source of the inefficiency in this identification method, which may be called the regression method. This method of identification of impulse response functions to the case with multiple input processes can be generalization .

IV. Identification of Transfer Function Models by Prewhitening the Input

Considerable simplification in the identification process would occur if the input to the system were white noise. When the choice of the input is at our disposal, there is much to recommend such an input. When the original input follows some other stochastic process, simplification is possible by prewhitening.

Suppose that the suitably differenced input process xt is stationary and is capable of representation by some member of the general linear class of autoregressive–moving average models. Then, given a set of data, we can carry out our usual identification and estimation methods to obtain a model for the x_t process.

$$a_t = \theta_x^{-1}(B) \varphi_x(B) x_t \tag{8}$$

uncorrelated white noise series a_t . At the same time, we can obtain an estimate s_a^2 from the sum of squares of the$\hat{a}_t$. If we now apply this same transformation to y_t to obtain

$$\beta_t = \theta_x^{-1}(B) \varphi_x(B) y_t$$

then the model (4) may be written

$$\beta_t = v(B) a_t + \varepsilon_t \tag{9}$$

where ε_t is the transformed noise series defined by

$$\varepsilon_t = \theta_x^{-1}(B) \varphi_x(B) \eta_t \tag{10}$$

On multiplying (9) on both sides by a_{t-k} and taking expectations, we obtain

$$\gamma_{\alpha\beta}(k) = v_k \sigma_a^2 \tag{11}$$

where

$$\gamma_{\alpha\beta}(k) = E[a_{t-k} \beta_t]$$

is the cross covariance at lag +k between the series α_t and β_t. Thus

$$v_k = \frac{\gamma_{\alpha\beta}(k)}{\sigma_a^2}$$

in terms of the cross correlations

$$v_k = \frac{\rho_{\alpha\beta}(k)\sigma_\beta}{\sigma_a} \qquad k = 0,1,2,\cdots \qquad (12)$$

Hence, after prewhitening the input, the cross-correlation function between the prewhitened input and correspondingly transformed output is directly proportional to the impulse response function. We note that the effect of prewhitening is to convert the nonorthogonal set of equations (7) into the orthogonal set (11).

In practice, we must substitute the theoretical cross-correlation function $\gamma_{\alpha\beta}(k)$ in (12) to estimates give

$$\hat{v}_k = \frac{\gamma_{\alpha\beta}(k)s_\beta}{s_a} \qquad k = 0,1,2,\cdots \qquad (13)$$

The preliminary estimates $\hat{v}_k$ so obtained are again, in general, statistically inefficient but can provide a rough basis for selecting suitable operators $\delta_{(B)}$ and $\omega_{(B)}$ in the transfer function model. An additional feature of the prewhitening method is that because the prewhitened input series α_t is white noise, so that $\rho_{\alpha\beta}(k) = 0$ for all k _=0, there are considerable simplifications for var$[\gamma_{\alpha\beta}(k)]$. In particular, on the assumption that the series α_t and β_t are not cross correlated, the result applies to give simply

$$\gamma_{\alpha\beta}(k) \approx (n-k)^{-1}$$

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