Return Distribution under Behavioral Biases: A Numerical Simulation Study

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Abstract

Investors' overconfidence and regret aversion lead to behavioral biases, such as over-reaction、under-reaction and disposition effect. By constructing a numerical simulation model, this paper shows that, return distributions under the behavioral biases have higher peaks and fatter tails, and they are skew to left with the left tails thicker than the right ones, compared with normal distribution under Effective Market Hypothesis. Performances of return distributions under different degrees of the behavioral biases are also investigated.

Keywords: Behavioral Finance, Over-reaction, Under-reaction, Disposition Effect, Numerical Simulation.

1. Introduction

Behavioral finance believes that a common phenomenon in the security market[1] is that, an investor will unconsciously be influenced by various psychological factors in his cognitive and affective processes. He will exhibit behavioral biases in his investment judgment and decision, no matter whether he is an inexperienced investor, or an experienced fund manager, or a senior financial analyst. Moreover, due to limited arbitrage, those behavioral biases will be reflected in the return distribution[2]. Overconfidence and regret aversion are two typical behavioral biases; the former will cause an investor to over-react or under-react to information[3] [4], and the latter will lead an investor to a disposition effect [5]. This paper tries to explore the properties of the return distribution brought about by investors' overconfidence and regret aversion.

It is not easy to directly detect influence of the investors' behaviors on market return distributions. A market return distribution is not only a comprehensive reflection of investors' various behaviors, but also includes influence of other factors, such as trading system, information channel, and infrastructure of security markets. It is really a difficult job to quantify the influence of a certain kind of behavior on market return distribution. Thus, previous studies of behavioral finance generally tried to build the relationship between behaviors and anomalies in the security market, by qualitative deduction of human behavior's mechanisms and empirical test on anomalies in the security market. This paper attempts to explore the influence of investors' behaviors on the market return distribution by means of numerical simulation. According to EMH, return distributions are normal distributions if there are no behavioral biases. Starting for a normal distribution, we change individual return rate by simulating the investment strategies under the behavioral biases, hence we obtain the new distributions with behavioral biases. In this way, we try to investigate the characteristics of return distribution under the behavioral biases, and the qualitative changes of return distribution under different degrees of each behavioral bias. The motivation of this research is from physical experiments, in which the ideal environments are built to separate objective entities from interfering factors. We hope such a research will give a better understanding of the relationship between the investors' behaviors and the representation of the security market.

2. Simulation Procedure with the Behavioral Biases

Let us first construct a simple model as a framework to simulate investors' investment strategies under overconfidence and regret aversion.

We use $\hat{r}$ to represent the return distribution of an efficient security market. According to the EMH, the distribution of $\hat{r}$ is normal[6]. Now we simulate the changes of return rates when investors are under overconfidence and regret aversion psychology, and denote the new return rate by r.

Let τ_0 represent the current time period, at which the security prices are stable. In accordance with the behavioral biases caused by overconfidence and regret aversion, we further assume that:

(1) When there are n consecutive good news after τ_0, investors under-react to the first *n-1* ones; while they over-react to the last one. That is, $\hat{r}_{\tau_i} > 0$ (due to good news), $i = 1, 2, \cdots, n$; $r_{\tau_i} < \hat{r}_{\tau_i}$ (due to under-reaction) , $i = 1, 2, \cdots, n-1$; but $r_T > \hat{r}_T$ (due to over-reaction), where $T = \tau_1 + \tau_2 + \cdots \tau_n$.

(2) When there are N consecutive bad news after τ_0, investors under-react to the first *N-1* ones; while they over-react to the last one. That is, $\hat{r}_{\tau_i} < 0$ (due to bad news), $i = 1, 2, \cdots, N$; $r_{\tau_i} > \hat{r}_{\tau_i}$ (due to under-reaction), $i = 1, 2, 3 \cdots N-1$; but $r_T < \hat{r}_T$ (due to over-reaction), where $T = \tau_1 + \tau_2 + \cdots \tau_N$.

Based on the above framework, we further define some indices to reflect the degree of the behavioral biases as follows

1) Overconfidence and regret aversion will cause under-reaction of investors, which can be reflected by 3 indices: reaction delay time period n and N, and the degree of under-reaction $|r_{\tau_i}| < |\hat{r}_{\tau_i}|$. We use *n, N* and k to measure the degree of under-reaction, where $r_{\tau_i} = k * \hat{r}_{\tau_i}$, $0 \le k \le 1$.

2) Overconfidence and regret aversion will cause over-reaction of investors, which can be reflected by $|r_T| > |\hat{r}_T|$. We use j to measure the degree of over-reaction, where $r_T = \hat{r}_T + j * \hat{r}_T$.

3) The disposition effect caused by regret aversion is reflected in $N > n$, while h is used to measure the degree of the disposition effect, where $N = n + h$, $h = 1, 2, 3, \cdots$.

Thus, the influence of overconfidence and regret aversion on the return distribution is measured by the 4 variables: *n*, *k*, *j*, and *h*. In the sequel, we use numerical simulation to analyze the degree of the influence of overconfidence and regret aversion on the return distributions.

3. Numerical Simulation Results

According to the EMH, investors are rational and the return distribution is normal. Therefore, when simulating the influence of overconfidence and regret aversion on the return distribution, we investigate the difference between the distributions under overconfidence and regret aversion and a normal distribution. All the simulated return distributions are standardized to be compared with a normal distribution.

The simulation is conducted in two aspects: 1) to the influence of over-reaction, under-reaction and disposition effect on the return distributions with different time frequencies, i.e. daily return distribution, weekly return distribution, and monthly return distribution, to see what characteristics of the return distributions different from normal distribution; 2) to investigate the degrees of the influence of each behavioral bias in different levels on the return distributions.

3.1 Characteristics of Return Dis -tribution under the Behavioral Biases

A million return rates are generated from a normal distribution as daily market return rates of rational investors. And then we let $n = 3, h = 2$, $k = 0.5, j = 0.2$, that is, the return under under-reaction is half of the normal return; the investors will not realize that they under-react to the good news in the market until the third day; they will not adjust their under-reaction to the bad news in the market until the fifth day; and the stock prices will be 20% higher or lower than the normal prices, due to the investors' over-reaction.

As to different time horizons for return distributions, this paper chooses the daily, weekly and monthly distributions as their representatives. Five consecutive daily return rates make the weekly return rate; and 24 consecutive daily return rates make the monthly return rate. For comparison with the normal distribution, the daily, weekly and monthly return distributions are all standardized.

Comparing the curves obtained from the simulated return rates (figures in appendix), we find: 1) after standardization, the return distributions, daily weekly and monthly, are typically sharp-peaked with heavy tails; 2) the kurtosis of the daily return distribution is larger than that of the weekly one; the kurtosis of the weekly return distribution is larger than that of the monthly one, and the kurtosis of the monthly return distribution is larger than that of the standard normal distribution; 3) the both tails of the daily return distribution are thicker than these of the weekly; and the both tails of the weekly are thicker than these of the monthly; 4) the left tails of the daily, weekly and the monthly return distributions are all thicker than their right counterparts.

Table 1 Kurtosis and Skewness of Daily, Weekly, and Monthly Distributions at $n = 3, h = 2$, $k = 0.5, j = 0.2$.

	Daily	Weekly	Monthly
S	-0.4085*	-0.3515*	-0.2398*
K	10.6717*	5.6946*	4.3235*

" * " indicates, S, not being 0, and K significantly exceeding 3 at significant level of 1%. The same meaning holds in table2, table3, table4 and table5.

3.2 Influence of Biased Levels on Return Distributions

3.2.1 Influence of Under-reaction Level on Return Distribution

We use k to measure the degree of the investors' under-reaction. The smaller k is, the higher the degree of the investors' under-reaction is; while the bigger k is, the less the degree of investors' under-reaction is. Let us see the differences between the daily return distributions with $k = 50\%$, $k = 60\%$ and $k = 70\%$, while holding other things constant.

Table 2 Influence of Change in k on Daily Return Distributions

	k=50%	*k=60%*	*k=70%*
S	-0.4085*	-0.2463*	-0.1491*
K	10.6717*	6.4393*	4.5689*

The distribution curves from the simulation show that with the increasing degree of the investors' under-reaction (*k* decreasing from 70% to 50%), the kurtosis of the daily return distributions increases, and the tails of the distributions become heavier. Meanwhile, the skewness of the daily distributions increases with the increase of the degree of under-reaction.

3.2.2 Influence of Different Reaction Delay Time on Return Distribution

We use *n* to measure the delay time of reaction, and thus it is an aspect of the investors' under-reaction. According to the previous analysis, we know that the degree of the investors' under-reaction increases with the increasing value of *n*. As the value of *n* increases, the kurtosis of the return distribution will decrease, and the skewness will also decrease. With the increase of the value of *n*, the change in the adjustment will increase, but the frequency of the adjustment will decrease faster, and the tail will therefore tend to decrease, with the tendency especially obvious in the left tail. Now, let's see the differences between the daily return distributions with $n = 3$, $n = 4$ and $n = 5$, while holding other things constant.

The simulation reveals that the decrease in the kurtosis is very obvious and the skewness also decreases (Table 3). Comparing the curves from simulation, we find that *n* has a greater influence on the left tail than on the right one. With the increase of *n*, the left tail has an obvious tendency to decrease, but the influence on the right tail is not so remarkable.

Table 3 Influence of Change of n on Daily Return Distribution

	n=3	*n=4*	*n=5*
S	-0.4085*	-0.0830*	-0.0553*
K	10.6717*	19.6827*	26.9781*

3.2.3 Influence of Over-reaction Degree on Return Distribution

j measures the degree of over-reaction. Although over-reaction may occur in several continuous time periods, we assume it to occur in one time period to simplify the problem. The higher the degree of investors' over-reaction, the heavier the tails of the return distributions are. Let us see the differences between the daily return distributions with $j = 0.2$, $j = 0.3$ and $j = 0.4$, while holding other things constant.

Table 4 Influence of Change of j on Daily Return Distribution.

	j=0.2	*J=0.3*	*j=0.4*
S	-0.4085*	-1.5037*	-10.6404*
K	10.6717*	41.0476*	1.2543e+003*

The simulation reveals that with *j* increasing to the right end, the tail of the return distribution gets thicker and thicker. Compared with the left tail, the change in *j* has much more influence on the right tail than the right one. The kurtosis of the return distribution is also very sensitive to *j* (Table 4).

3.2.4 Influence of Different Degree of Disposition Effect on Return Distribution

Due to the existence of the disposition effect, the investors adjust with higher frequencies in a bullish market than in a bearish market. *h* is used to express the degree of investor's disposition effect. It mainly exerts its influence on the left tail of the return distribution. With the increase of the value of *h,* the possibility of a continuous bearish market is getting less and less, and so is the investors' adjustment. Therefore, the thickness of the left tail should be decreasing with the increase of *h*. Moreover, with the increase of *h*, the kurtosis should be increasing if the possibility of a continuous down market *(n/N+h)* does not increase much; and with the increase of *h,* the kurtosis should be decreasing, if the possibility of continuous bearish market increases greatly. Let us see the differences between the daily return distributions with $h = 1$, $h = 2$ and $h = 3$, while holding other things constant.

The simulation results show that, the change in *h* does not have much influence on the kurtosis. With the increase of *h*, the kurtosis of the return distribution increases slightly, but not very obviously; but the influence of *h* is striking; while the change of *h* has an obvious influence on the left tail of the return distribution, but not much on the right tail. As *h* increases, the left tail tends to decrease, as the investors' adjustment frequency decreases with *h*.

Table 5 Influence of Change in h on Daily Return Distribution

	h =1	*h=2*	*h=3*
S	-0.4900*	-0.4085*	-0.5623*
K	9.9201*	10.6717*	15.7982*

5. Conclusion

Overconfidence and regret aversion bring investors' behavioral biases in their investment decisions, such as over-reaction, under-reaction and the disposition effect. From the numerical simulation, we find that these behavioral biases result in the following four characteristics of the return distributions: 1) sharper peaks and heavier tails than a normal distribution; 2) decreasing thickness of the tails as the time horizon extends; 3) decreasing kurtosis as the time horizon extends; and 4) left skewness with a heavier left tail, compared with a normal distribution.

We also find that the kurtosis of the return distribution increases remarkably as reaction delay time increases; but the skewness and the thickness of the right tail do not change much, while the left tail tends to decrease. As the investors' degree of under-reaction grows, the kurtosis of the return distribution will increase, the tail gets heavier, and the skewness and the kurtosis also increase as well. With the degree of over-reaction increasing, the right tail of the return distribution gets heavier, and the skewness and the kurtosis also increase at the same time. With the increase of the disposition effect, the kurtosis of the return distribution increases slightly; the skewness does not change much, as the right tail is not sensitive to the disposition effect, but the thickness of the left tail decreases.

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Appendix: Figures of simulated distributions when $n = 3, h = 2$, $k = 0.5, j = 0.2$. From left to right, overall, peak part, left tails, right tail.