The Extended TOPSIS Method for Multi-criteria Decision Making Based on Hesitant Heterogeneous Information

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Abstract—In the real world there often exist some decision situations with high degree of uncertainty where the decision makers hesitate among several values to provide their assessments. In such cases, the decision makers usually employ the hesitant fuzzy sets (HFSs) to express their assessments in the quantitative settings and the hesitant fuzzy linguistic term sets (HFLTSs) in the qualitative ones. This paper analyzes a hesitant heterogeneous multi-criteria decision making problem involved both HFSs and HFLTSs. To handle this sort of decision problems, an extended TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is proposed. In the proposed method the separations to the ideal solution and negative ideal solution under each criterion are calculated by using different distance measures, respectively. Then the separations under each criterion are normalized in order to ensure the compatibility of all criteria. Afterwards, the weight separations are derived, and the optimal alternative which is closest to the ideal solution and remotest from negative ideal solution is also identified. At length, an example is used to illustrate the proposed approach.

Keywords—Hesitant fuzzy sets; hesitant fuzzy linguistic term sets; multi-criteria decision making; TOPSIS

I. INTRODUCTION

Multiple criteria decision making (MCDM) is to make an optimal choice that has the highest degree of satisfaction from a set of all feasible alternatives characterized with multiple competing criteria. In the real decision process, these competing criteria in MCDM problems are usually of different nature, which may be qualitative and quantitative. According to the different nature of criteria and the decision maker (DM)’s knowledge area, the assessments provided by the DM may be taken different formats such as real numbers, intervals and linguistic variables, etc. In general, MCDM problems with multiple formats of information are called the heterogeneous MCDM problems. This sort of MCDM problems is very complex and interesting in applications of decision making theory. Many useful and valuable methods have been proposed to solve such MCDM problems [1] [4] [5] [6]. For example, Herrera et al. [4] proposed a method that converts all heterogeneous information into the 2-tuple linguistic information for solving the heterogeneous MCDM problems. Li et al. [6] developed a systematical approach that computes the distances to the positive ideal solution as well as negative ideal solution for each criterion and obtains the multi-attribute ranking index.

However, in the real world there often exist some decision situations with high degree of uncertainty where the DMs hesitate among several values to provide their assessments. When the decision criteria of MCDM problems are quite quantitative because of their nature, the hesitant fuzzy sets (HFSs) proposed by Torra [9] are usually used to manage this situation; while the decision criteria are quite qualitative, the hesitant fuzzy linguistic term sets (HFLTSs) introduced by Rodriguez et al. [7] are employed to capture the corresponding cases. Similar to the heterogeneous MCDM problems, the MCDM problems with multiple hesitant formats of information, such as HFSs and HFLTSs, are called the hesitant heterogeneous MCDM problems. For such situations, the previous methods cannot be used to manage them. To this end, Rodriguez et al. [8] proposed an approach that unifies the heterogeneous information in a linguistic domain by means of the 2-tuple linguistic representation [3] to manage this sort of MCDM problems. In their approach all the heterogeneous information is converted into the 2-tuple linguistic representation by using different transformation functions. However, these transformation processes are very complex and may lose much original information.

To preserve more original information, this study employs the main structure of TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) to proposed a generalization TOPSIS method for solving the hesitant heterogeneous MCDM problems. The proposed method computes the distances to the ideal solution and negative ideal solution under each criterion by using different distance measures, and determines the optimal alternative which is closest to the positive ideal solution and remotest from negative ideal solution. The structure of this paper is organized as follows: Section 2 briefly reviews some concepts of HFSs, HFLTSs and hesitant trapezoidal fuzzy numbers (HTrFNs). Section 3 presents a hesitant heterogeneous MCDM problem and proposes an extended TOPSIS method for solving such a MCDM problem. Section 4 employs a supplier selection example to demonstrate the implementation process of the proposed method. Section 5 presents our conclusions.

II. PRELIMINARIES

This section presents some basic concepts of HFSs, HFLTSs and HTrFNs which will be useful in the subsequent sections.
In [9], Torra introduced a concept of HFS which permits the membership degree of an element to a set to be represented as several possible values between 0 and 1. The biggest advantage of the HFS is that it can express the hesitancy of human beings efficiently, especially when two or more sources of vagueness appear simultaneously.

**Definition 2.1** [9]. Let $X$ be a reference set, a HFS $A$ on $X$ is defined in terms of a function $h_A(x) = [\alpha \in X]$ when applied to $X$ returns a subset of $[0,1]$. To be easily understood, Xia and Xu [10] expressed the HFS by a mathematical symbol:

$$ A = \{ < x, h_A(x) > | x \in X \} \quad (2.1) $$

where $h_A(x)$ is a set of some different values in $[0,1]$, representing the possible membership degrees of the element $x \in X$ to $A$. For convenience, they called $h_A(x)$ a hesitant fuzzy element (HFE) denoted by $h_A(x) = \gamma^f$ if $\gamma^f = \gamma^f \gamma_{1,2,\cdots,\#h}$ ( $\#h$ is the number of all elements in $h$).

**Assumption 2.1.** Two assumptions are made (see [2], [11], [13], [16-18]): (1) All possible values of the HFE $h_A$ are arranged in a decreasing order, and thus $\gamma^i$ is the $i^{th}$ largest value in $h$. (2) In order to have a correct comparison, the two corresponding HFEs should have the same length. For two HFEs $h_1$ and $h_2$, if there are fewer values in $h_1$ than in $h_2$, an extension of $h_1$ should be considered pessimistically by repeating its minimum value until it has the same length with $h_2$.

Drawing on the well-known Hamming and Euclidean distance measures, Xu and Xia [12] defined two hesitant fuzzy distance measures for HFEs:

**Definition 2.2.** For any two HFEs $h_i = \{ \gamma_i^f = \gamma_i^1, \gamma_i^2, \cdots, \gamma_i^{\#h} \}$ ($i = 1, 2$) with $\#h = \#h_i = \#h_2$, the hesitant fuzzy Hamming and Euclidean distances between them can be defined, respectively, as follows:

$$ d_h(h_1, h_2) = \frac{1}{\#h} \sum_{j=1}^{\#h} | \gamma_1^j - \gamma_2^j | \quad (2.2) $$

and

$$ d_e(h_1, h_2) = \left( \frac{1}{\#h} \sum_{j=1}^{\#h} (\gamma_1^j - \gamma_2^j)^2 \right)^{\frac{1}{2}} \quad (2.3) $$

Similar to the situations that are described and managed by HFSs [9] where the DMs may consider several possible values to define a membership function, Rodriguez et al. [7] introduced the concept of HFLTS to handle the situations in linguistic contexts where the DMs hesitant among several possible linguistic values to express their assessments.

**Definition 2.3** [7]. Let $S = \{ s_1, \cdots, s_k \}$ be a linguistic term set, a HFLTS $H_S$ is defined as an ordered finite subset of consecutive linguistic terms of $S$:

$$ H_S = \{ s^f | f = 1, \cdots, \#H_S \} \quad (s \in S) \quad (2.4) $$

where $\#H_S$ is the number of all linguistic values that compound the HFLTS $H_S$.

**Example 2.1.** Let $S = \{ s_0, \cdots, s_k \}$ be a linguistic term set with seven-point rating scales, so two different HFLTSs $H_S^1$ and $H_S^2$ might be as: $H_S^1 = \{ s_1, s_3 \}$, $H_S^2 = \{ s_1, s_4, s_5 \}$.

In the real decision process, the DMs usually employ the comparative linguistic expressions which are very close to human being’s cognitive model and also provide the DMs with greater flexibility to elicit linguistic expressions, to express their assessments. Rodriguez et al. [7] proposed a transformation function $E_{\omega}$ that transforms the comparative linguistic expressions into HFLTS as follows:

$$ E_{\omega}(s_i) = \{ s_j | s_j \in S \} $$
$$ E_{\omega}(\text{at most } s_i) = \{ s_j | s_j \leq s_i \text{ and } s_i, s_j \in S \} $$
$$ E_{\omega}(\text{at least } s_i) = \{ s_j | s_j \geq s_i \text{ and } s_i, s_j \in S \} $$
$$ E_{\omega}(\text{between } s_i \text{ and } s_j) = \{ s_k | s_k \leq s_i \text{ and } s_i, s_j, s_k \in S \} $$

**Example 2.2.** Let $S = \{ s_0, \cdots, s_k \}$ be a linguistic term set with seven-point rating scales, $l_l = \{ \text{at most } s_i \}$ and $l_r = \{ \text{between } s_i \text{ and } s_j \}$ be two comparative linguistic expressions. According to the above transformation function $E_{\omega}$, they can be converted into two HFLTSs as follows:

$$ E_{\omega}(l_l) = \{ s_0, s_1, s_2 \} $$
$$ E_{\omega}(l_r) = \{ s_1, s_2, s_3 \} $$

To effectively capture the semantics of the HFLTSs, Zhang and Xu [15] proposed a concept of HTrFN. The HTrFN benefited from both the superiority of the trapezoidal fuzzy number (TrFN) and the HFE, has strong ability to tackle the imprecise and ambiguous information in real-world applications, which is defined as below:

**Definition 2.4** [15]. Let $X$ be a fixed set, a hesitant trapezoidal fuzzy set (HTrFS) $\tilde{A}$ on $X$ is defined as follows:

$$ \tilde{A} = \{ < x, \tilde{h}_A(x) > | x \in X \} \quad (2.5) $$

where $\tilde{h}_A(x)$ is a set of several TrFNs, representing some possible membership degrees of the element $x \in X$ to $\tilde{A}$. 

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For convenience, \( \tilde{h}_i(x) \) is called a HTrFN, denoted by
\( \tilde{h} = \left[ \tilde{a}_i \right]_{f = 1,2,\ldots,# \tilde{h}} \), where \( \tilde{a}_i = T \left( \tilde{z}_i, \tilde{y}_i^l, \tilde{y}_i^u \right) \) is a TrFN and \( # \tilde{h} \) is the number of the TrFNs in \( \tilde{h} \).

**Assumption 2.2** [15]. Two basic assumptions are made:
1. All possible TrFNs of the HTrFNs \( \tilde{h} \) are arranged in a decreasing order, and thus \( \tilde{a}_i \) is the \( i^{th} \) largest TrFN in \( \tilde{h} \).
2. In order to have a correct comparison, the two corresponding HTrFNs should have the same length. For any two HTrFNs \( \tilde{h}_1 \) and \( \tilde{h}_2 \), if there are fewer TrFNs in \( \tilde{h}_1 \) than in \( \tilde{h}_2 \), an extension of \( \tilde{h}_1 \) should be considered pessimistically by repeating its minimum TrFN until it has the same length with \( \tilde{h}_2 \).

Two distance measures for HTrFNs were proposed by Zhang and Xu [15] as follows:

**Definition 2.5** [15]. Given two HTrFNs
\( \tilde{h}_1 = \left[ \tilde{a}_i' \right]_{f = 1,2,\ldots,# \tilde{h}_1} \)
and assume \( # \tilde{h}_1 = # \tilde{h}_2 \), the hesitant trapezoidal Hamming and Euclidean distances between them can be defined, respectively, as follows:

\[
d(\tilde{h}_1, \tilde{h}_2) = \frac{1}{6 # \tilde{h}} \left( \sum_{i=1}^{# \tilde{h}} \left( \tilde{a}_i' - \tilde{a}_i'' \right)^2 + 2 \left( \tilde{a}_i'' - \tilde{a}_i'' \right)^2 \right)^{1/2}
\]

(2.6)

and

\[
d(\tilde{h}_1, \tilde{h}_2) = \frac{1}{6 # \tilde{h}} \left( \sum_{i=1}^{# \tilde{h}} \left( \tilde{a}_i' - \tilde{a}_i'' \right)^2 + 2 \left( \tilde{a}_i'' - \tilde{a}_i'' \right)^2 \right)^{1/2}
\]

(2.7)

### III. HESITANT HETEROGENEOUS TOPSIS MULTIPLE CRITERIA ANALYSIS APPROACH

#### A. Hesitant heterogeneous MCDM problem

Consider a MCDM problem under a hesitant heterogeneous environment, let \( A = \{ A_1, A_2, \ldots, A_n \} \) \((m \geq 2)\) be a discrete set of \( m \) feasible alternatives, \( C = \{ C_1, C_2, \ldots, C_n \} \) be a finite set of criteria. The set of criteria \( C \) can be divided into two subsets, \( C_E \) and \( C_F \), representing the criteria whose values are in formats of HFEs and HFLTSS, respectively. Let \( C_E = \{ C_1, C_2, \ldots, C_e \} \), \( C_F = \{ C_{e+1}, C_{e+2}, \ldots, C_n \} \) where \( 1 \leq e \leq n \). Thus \( C_E \cup C_F = C \) and \( C_E \cap C_F = \emptyset \) (\( \emptyset \) is the empty set). For convenience, we denote the subscripts of these two subsets \( C_E \) and \( C_F \) as \( N_E = \{ 1,2,\ldots,e \} \) and \( N_F = \{ e+1, e+2, \ldots, n \} \) respectively, and let \( M = \{ 1,2,\ldots,m \} \), \( N = \{ 1,2,\ldots,n \} \), the ratings of the alternative \( A_i \) \(( i \in M) \) on the criteria \( C_j \) \(( j \in N) \) be denoted by \( x_{ij} \). Therefore, we obtain

\[
x_{ij} = \left\{ \begin{array}{l}
\tilde{h}_i = \left[ \tilde{y}_i' \right]_{f = 1,2,\ldots,# \tilde{h}_i}, i \in M, j \in N_e \\
H^e = \left[ \left( s_i \right) \right]_{f = 1,2,\ldots,# H^e}, i \in M, j \in N_e
\end{array} \right.
\]

(3.1)

In this paper, we employ the HTrFNs to represent the semantics of the HFLTSS. Thus the \( x_{ij} \) \(( i \in M, j \in N_F \) can further denoted by \( \tilde{h}_j = \left[ \tilde{a}_j' \right]_{f = 1,2,\ldots,# \tilde{h}_j} \), where \( \tilde{a}_j' \) is the TrFNs denoted by \( \tilde{a}_j = T \left( \tilde{z}_j, \tilde{y}_j^l, \tilde{y}_j^u \right) \) and \( # \tilde{h}_j = # H^e \) is the number of TrFNs in \( \tilde{h}_j \). Thus the Eq. (3.1) can be rewritten as:

\[
x_{ij} = \left\{ \begin{array}{l}
\tilde{h}_j = \left[ \tilde{y}_j' \right]_{f = 1,2,\ldots,# \tilde{h}_j}, i \in M, j \in N_e \\
H^e = \left[ \left( s_i \right) \right]_{f = 1,2,\ldots,# H^e}, i \in M, j \in N_e
\end{array} \right.
\]

(3.2)

Therefore, the hesitant heterogeneous MCDM problem can be concisely expressed in the matrix format as below:

\[
X = (x_{ij})_{m \times n} = A_1 x_{i1} x_{i2} \cdots x_{in} \\
\vdots \quad \vdots \quad \vdots \quad \vdots \\
A_m x_{m1} x_{m2} \cdots x_{mn}
\]

(3.3)

In practical decision process, the weights of criteria should be taken into account. Here we denote the criteria weighting vector by \( w = (w_1, w_2, \cdots, w_n)^T \), where \( w_j \) is the relative weight of the criterion \( C_j \) \(( j \in N) \), satisfying the normalization condition: \( \sum_{j=1}^{n} w_j = 1 \) and \( w_j \geq 0 \) \(( j \in N) \). Moreover, the criteria set \( C_E \) \(( C_F) \) can be further divided into two sets, \( C_E^b \) \(( C_F^b) \) \(( C_E^f \) \(( C_F^f) \), where \( C_E^b \) \(( C_F^b) \), \( C_E^f \) \(( C_F^f) \) represents a collection of benefit criteria (the larger the criteria values the better) and \( C_E^f \) \(( C_F^f) \) denotes a collection of cost criteria (the smaller the criteria values the better). To eliminate the effect of different physical dimensions and measurements on the final decision results, in the decision process we transform the criteria values of the cost type into the criteria values of the benefit type by using the following equation:
Thus the separation measures, \(d^+_i\) and \(d^-_i\), of each alternative \(A_i\), with respect to each criterion \(C_j\), from the HH-PIS \(A^+\) and the HH-NIS \(A^-\), respectively, are derived from the following formulas:

\[
d^+_i = \frac{1}{\sqrt{\sum_{j=1}^{m} (x_{ij} - y_{ij})^2}} \quad (i \in M) \\
d^-_i = \frac{1}{\sqrt{\sum_{j=1}^{m} (x_{ij} - y_{ij})^2}} \quad (i \in M)
\]

and

\[
d^*_i = \frac{1}{\sum_{j=1}^{m} \sum_{i=1}^{n} (x_{ij} - y_{ij})^2} \quad (i \in N) \\
d^-_i = \frac{1}{\sum_{j=1}^{m} \sum_{i=1}^{n} (x_{ij} - y_{ij})^2} \quad (i \in N)
\]

To ensure the compatibility of all criteria, we normalize the separations of each alternative \(A_i\) with respect to each criterion \(C_j\) from the HH-PIS \(A^+\) and the HH-NIS \(A^-\) using the following equations, respectively:

\[
(d^*_i)^* = \frac{d^*_i}{\sum_{i=1}^{n} d^*_i} \quad (i \in M) \\
(d^-_i)^* = \frac{d^-_i}{\sum_{i=1}^{n} d^-_i} \quad (i \in M)
\]

Thus the weight separation measures \(d^*_i\) and \(d^-_i\) of each alternative \(A_i\) from the HH-PIS \(A^+\) and the HH-NIS \(A^-\) can be determined respectively using the following formulas:

\[
d^*_i = \sum_{j=1}^{m} w_j (d^*_i)^* \quad (i \in M) \\
d^-_i = \sum_{j=1}^{m} w_j (d^-_i)^* \quad (i \in M)
\]

The relative closeness coefficient of each alternative \(A_i\), with respect to the HH-PIS \(A^+\) is defined as the following formula:

\[
CC_i = \frac{d^-_i}{d^*_i + d^-_i}
\]
where $0 \leq CC_i \leq 1 \ (i \in M)$.

Obviously, the alternative $A_i$ is closer to the HH-PIS $A^+$ and farther from the HH-NIS $A^-$ as the $CC_i$ approaches to 1. Therefore, by comparing the closeness coefficient $CC_i$, we can determine the ranking order of all alternatives and select the best one from a set of feasible alternatives.

C. The proposed algorithm

Based on the above analysis, the algorithm of the extended TOPSIS method for solving the hesitant heterogeneous MCDM problem can be summarized as:

Step 1. Form the hesitant heterogeneous MCDM problem and identify the corresponding decision matrix.

Step 2. Determine the HH-PIS $A^+$ and the HH-NIS $A^-$ by using Eqs. (3.5) and (3.6), respectively.

Step 3. Utilize Eqs. (3.7) and (3.8) to calculate the separation measures $d_i^+$ and $d_i^-$ of each alternative $A_i$ under each criterion $C_j$ from the HH-PIS $A^+$ and the HH-NIS $A^-$, respectively.

Step 4. Normalize the separation measures $d_i^+$ and $d_i^-$ by using the Eqs. (3.9) and (3.10), to obtain the normalized separation measures $(d_i^+)^*$ and $(d_i^-)^*$, respectively.

Step 5. Calculate the weight separation measures $d_i^{*j}$ and $d_i^{-j}$ of each alternative $A_i$ by using Eqs. (3.11) and (3.12), respectively.

Step 6. Compute the relative closeness coefficient $CC_i$ of each alternative $A_i$ to the HH-PIS $A^+$ by using the Eq. (3.13).

Step 7. Rank all the alternatives $A_i \ (i \in M)$ according to the relative closeness coefficients $CC_i \ (i \in M)$ to the HH-PIS $A^+$ and select the most desirable one(s).

IV. Case Illustration

To show the usefulness of the proposed method, this section presents a MCDM problem concerned with an automotive company which wants to select the most suitable green supplier by taking environmental performances into account.

Suppose that there are three potential green suppliers $A = \{A_1, A_2, A_3\}$ are evaluated by the following four important criteria $C = \{C_1, C_2, C_3, C_4\}$: $C_1$ Product cost, $C_2$ Product quality, $C_3$ Service performance and $C_4$ environmental performance. The criterion $C_1$ is the cost criterion, and the others are the benefit criteria. The subjective important of the criteria given by the DM is $w = \left(\frac{w_1}{w_2}, \frac{w_2}{w_3}, \frac{w_3}{w_4}\right)^T = (0.2, 0.3, 0.2, 0.3)^T$. Because the increasing complexity of the socio-economic context and the vagueness of inherent subjective nature of human think, it is difficult for the decision maker of the company to provide their assessments of all alternatives by means of crisp values in qualitative criteria and single linguistic terms for the qualitative criteria. Thus the decision maker may employ the HFEs and the comparative linguistic expressions based on HFLTs to express the assessments for the three potential suppliers. The evaluation results are listed in Table I. Linguistic terms and their corresponding TrFNs are described in Table II.

TABLE I. THE HESITANT HETEROGENEOUS DECISION MATRIX

<table>
<thead>
<tr>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>{0.7,0.5}</td>
<td>Between MG and G</td>
<td>Between MP and MG</td>
</tr>
<tr>
<td>$A_2$</td>
<td>{0.6,0.5,0.2}</td>
<td>At least G</td>
<td>F</td>
</tr>
<tr>
<td>$A_3$</td>
<td>{0.5,0.3}</td>
<td>G</td>
<td>Between F and MG</td>
</tr>
</tbody>
</table>

TABLE II. LINGUISTIC TERMS AND THE CORRESPONDING TRFNs

<table>
<thead>
<tr>
<th>Linguistic terms</th>
<th>Abbreviation</th>
<th>TrFNs</th>
</tr>
</thead>
<tbody>
<tr>
<td>sc: Very poor</td>
<td>VP</td>
<td>T(0.0,0.0,0.1,0.2)</td>
</tr>
<tr>
<td>s: Poor</td>
<td>P</td>
<td>T(0.1,0.2,0.2,0.3)</td>
</tr>
<tr>
<td>s: Medium poor</td>
<td>MP</td>
<td>T(0.2,0.3,0.4,0.5)</td>
</tr>
<tr>
<td>s: Fair</td>
<td>F</td>
<td>T(0.4,0.5,0.5,0.6)</td>
</tr>
<tr>
<td>s: Medium good</td>
<td>MG</td>
<td>T(0.5,0.6,0.7,0.8)</td>
</tr>
<tr>
<td>s: Good</td>
<td>G</td>
<td>T(0.7,0.8,0.8,0.9)</td>
</tr>
<tr>
<td>s: Very good</td>
<td>VG</td>
<td>T(0.8,0.9,1.0,1.0)</td>
</tr>
</tbody>
</table>

TABLE III. THE NORMALIZED HESITANT HETEROGENEOUS DECISION MATRIX

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>{0.5,0.3,0.3}</td>
<td>{T(0.7,0.8,0.8,0.9), T(0.5,0.6,0.7,0.8)}</td>
<td>{T(0.5,0.6,0.7,0.8), T(0.4,0.5,0.5,0.5)}</td>
<td>{T(0.7,0.8,0.8,0.9), T(0.7,0.8,0.8,0.9)}</td>
</tr>
<tr>
<td>$A_2$</td>
<td>{0.8,0.5,0.4}</td>
<td>{T(0.8,0.9,1.0,1.0), T(0.7,0.8,0.8,0.9)}</td>
<td>{T(0.4,0.3,0.5,0.6), T(0.4,0.5,0.5,0.6), T(0.4,0.5,0.5,0.6), T(0.4,0.5,0.5,0.6)}</td>
<td>{T(0.7,0.8,0.8,0.9), T(0.7,0.8,0.8,0.9)}</td>
</tr>
<tr>
<td>$A_3$</td>
<td>{0.7,0.5,0.5}</td>
<td>{T(0.7,0.8,0.8,0.9), T(0.7,0.8,0.8,0.9)}</td>
<td>{T(0.5,0.6,0.7,0.8), T(0.4,0.5,0.5,0.6), T(0.4,0.5,0.5,0.6), T(0.4,0.5,0.5,0.6)}</td>
<td>{T(0.7,0.8,0.8,0.9), T(0.7,0.8,0.8,0.9)}</td>
</tr>
<tr>
<td>$A^+$</td>
<td>{0.8,0.5,0.5}</td>
<td>{T(0.8,0.9,1.0,1.0), T(0.7,0.8,0.8,0.9)}</td>
<td>{T(0.1,0.2,0.2,0.3), T(0.5,0.6,0.7,0.8), T(0.4,0.5,0.5,0.6), T(0.4,0.5,0.5,0.6)}</td>
<td>{T(0.7,0.8,0.8,0.9), T(0.7,0.8,0.8,0.9)}</td>
</tr>
<tr>
<td>$A^-$</td>
<td>{0.5,0.3,0.3}</td>
<td>{T(0.7,0.8,0.8,0.9), T(0.5,0.6,0.7,0.8)}</td>
<td>{T(0.4,0.5,0.5,0.6), T(0.4,0.5,0.5,0.6), T(0.2,0.3,0.4,0.5), T(0.4,0.5,0.5,0.6)}</td>
<td>{T(0.7,0.8,0.8,0.9), T(0.7,0.8,0.8,0.9)}</td>
</tr>
</tbody>
</table>
In the following, we employ the proposed method to aid the automotive company select the most suitable green supplier. The solution process and the computation results are summarized as follows:

Firstly, we convert the HFLTSs into the HTrFNs and employ the Eq. (3.4) to transform criteria values of the cost type into the criteria values of the benefit type, the results are listed in Table III. Secondly, we utilize the Eqs.(3.5) and (3.6) to determine the corresponding HH-PIS \( A^+ \) and the HH-NIS \( A^- \) as in Table III. At length, we employ the Eqs. (3.7)-(3.13) to calculate the corresponding separation measures \( d_i^+ \) and \( d_i^- \), and the relative closeness coefficient \( CC_i \) \((i = 1, 2, 3)\), respectively. The results are presented in Table IV, together with the corresponding rankings on the basis of \( CC_i \).

It is easy to see that the optimal order for these three potential suppliers is \( A_1 \succ A_2 \succ A_3 \), and thus the supplier \( A_1 \) is the most desirable supplier. Obviously, the proposed method is not only simple and easy to understand but also reduces the loss of the original data information by using the unified methods proposed by Rodriguez et al. [8].

\[ \text{TABLE IV.} \quad \text{THE CLOSNESS COEFFICIENTS OF ALTERNATIVES ALONG WITH THEIR FINAL RANKING} \]

<table>
<thead>
<tr>
<th>Alternative</th>
<th>( d_i^+ )</th>
<th>( d_i^- )</th>
<th>( CC_i )</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>0.4783</td>
<td>0.2271</td>
<td>0.3219</td>
<td>3</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>0.2979</td>
<td>0.3343</td>
<td>0.5288</td>
<td>2</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>0.2238</td>
<td>0.4407</td>
<td>0.6632</td>
<td>1</td>
</tr>
</tbody>
</table>

V. CONCLUSIONS

TOPSIS method is one of the well-known methods for solving the MCDM problem. This paper proposes an extend TOPSIS method to handle the hesitant heterogeneous MCDM problem in which the ratings of alternatives take the different formats such as HFEs and HFLTSs. In the proposed method, the semantics of the HFLTSs are represented by the HTrFNs. Based on different distance measures, the separations under each criterion are normalized in order to ensure the compatibility of all criteria. Finally, the optimal alternative which is closest to the ideal solution and remotest from negative ideal solution is identified. The proposed method is not only very simple and easy to understand but also reduces the loss of the original data information by using the unified methods proposed by Rodriguez et al. [8]. Thus, the proposed approach provides us an effective and practical way to handle the hesitant heterogeneous MCDM problem in case of considering the DM’s hesitation.

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