Agent-Based Analysis of Annual Energy Usages for Domestic Heating based on a Heat Pump

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Abstract— **This paper describes an agent-based analysis approach to determine in which way a net zero house can be obtained. In particular, it addresses agent-based simulation to estimate annual energy usage for heating based on an air to water heat pump. Based on the introduced approach house owners will be able to decide on the specifications for further renewable energy production systems to be installed, for example, solar or wind energy production systems in order to obtain a net zero house in the present and in future years.**

Index Terms— **net zero house, annual energy usage, heat pump, SPF**

I. Introduction

One of the grand challenges of this time is to reduce the overall energy usage based on non-renewable sources of energy. A particular area to address this is that of domestic energy usage. In designing new houses nowadays, often the aim is to come as close as possible to an *energy neutral* or *net zero* house; e.g., [1–4]. Also for renovation of existing houses often the aim is to get close to a net zero house. A net zero house is a house that on an annual basis doesn't use any energy. During the year there may be times that such a house still uses energy, but in the same or other periods of the year it produces energy in such a way that the net total over the year is zero. For more details and overviews of net zero houses, see [5–7]. Given the dynamics of both environmental and household factors, the emerging system is a complex system and modelling both the energy consumption and energy production over time is a research challenge.

For households in many countries much of the annual energy usage is spent on domestic heating; overall it is estimated at 43% from the totel energy usage of heat related needs in the Europian Union in 2006 (cf. [8]). To get closer to the ideal of a net zero house, in addition to good isolation of the house, also more and more domestic heating systems are considered that allow the use of renewable energy, in contrast to most of the tradional heating systems that fully depend on non-renewable energy such as coal, gas and oil. Often heat pumps are suggested as an alternative; e.g., [9]. They take most of their energy (up to 80%) from the heat available in the ambient air, water or soil. The remaining energy usage concerns electrical energy to run the heat pump, which can be less than 40% of the energy usage based on traditional heating systems. Moreover, if the amount of electrical energy that is still needed is also produced based on renewable energy sources such as solar or wind energy in the same or different periods in a year, the total energy usage for heating can become net zero over a year. To obtain this, it is important to have an adequate estimation of the annual energy usage of a heat pump over years. Such an estimation can be taken into account when deciding, for example, for the dimensions of a solar energy production system to be installed. However, domestic heat pump users are facing a challenge to estimate their own energy usage on heating mainly because of the performance indicators given by heat pump manifacturers are far away from the dynamic conditions of using it (e.g. indoor and outdoor temperatures) and therefore, those measures are not directly helpful to plan an economical net zero house. This paper focuses on how agent-based methods can be used to get this estimation in case of an air to water heat pump (using the air as a source). Here the main agent considered is the (heat pump based) heating agent, and the main focus is on its ongoing interaction with the (strongly dynamical) environment. Other agents that play their role are energy production agents (e.g., based on a photovoltaic solar energy production system; e.g., [10]) and a thermostat agent.

Both the energy demand of a house and the efficiency of an air to water heat pump strongly depend on ambient temperature. This temperature varies much over the days of a year, and due to climate change it may change over the years as well. As the effect of variation of this temperature on the efficiency of a heat pump is nonlinear, simply taking average temperatures does not provide adequate estimations. In this paper, this variation of outdoor temperature is analysed and it is determined by agent-based simulation over days in different years how this variation affects the energy needed for heating. A main advantage of this approach is that by having some limited data for a short period of time the system itself is able to start to do the prediction, and over time of the system use the quality of the estimation will be naturally refined and improved. Furthermore, due to the continuouss analysis of energy consumption and energy generation, this methodology includes features like monitoring the performance of the heatpump and solar panels (this is useful for prompt repairs and even for replacements), form a small scale smart grid with neighbours and further reducing the risk of higher energy demands (in special situations) with low or zero cost.

In the paper, first in Section II some background theory on heating based on a heat pump is presented. Next in Section III it is shown how parameters representing characteristics of a given house and heat pump can estimated based on empirical data on energy usage and outdoor temperature. This provides a well-tuned model of the heat pump in the given house. In Section IV based on this model the (hypothetical) energy usages for the past 10 years are analysed using empirical data on the temperature over days in these years. Section V does the same for the future 10 years, thereby using prediction models for temperature variation over days and over years. Section VI includes a discussion and future directions.

II. Domestic Heating by a Heat Pump

In this section, some background knowledge on domestic heating is discussed, needed to model the heating agent. The modeling approach used here is single agent-based. This agent has the responsibility to take care of the heating via the action of controlling the water temperature of the central heating system. The agent is goal-directed and reactive to information about the indoor and outdoor circumstances. This information is acquired by the agent by sensing and monitoring the outdoor temperature and the indoor and heating system water temperature. Moreover, the goal concerns the desired indoor temperature. This is obtained by communication with the human(s) in the house (via the thermostat as a communication mean). Three important elements in domestic heating are:

- The characteristics of the environment
  E.g., how is the temperature over the year, how much wind is there
- The characteristics of the house
  E.g., how well isolated is the house, in how far it uses passive solar energy
- The characteristics of the heating system used
  E.g., efficency, performance depending on circumstances

Heat pumps have often been proposed as efficient heating systems, as they use renewable heat sources such as ambient air, water or soil, and to run use only a fraction of this as electrical energy [9]. The heating capacity and efficiency of a heat pump strongly depends on the temperature of the ambient heat source used. During a frost period in a winter season this ambient temperature may become quite low, compared to milder periods [11], thus implying a lower performance in times when most heating is needed. More specifically, the efficiency of a heat pump is closely related to the difference between the ambient temperature (heat source) and the output temperature of the heat pump [11], in addition to some other factors (cf. [12, 13]). A commonly used measure on the performance of a heat pump by those manufacturers is referred as Coefficient of Performance (COP): which is the ratio of the heat delivered by the heat pump (energy output) and the electrical energy supplied to it (energy input), both measured in kWh [14]:

$$COP = \frac{energy\ output}{energy\ input} \tag{1}$$

The main concern over the COP is that it is calculated under a set of controlled conditions with defined input and output temperatures: for the European standards (EN 14511) it is to be tested at 7ºC external temperature and 20ºC indoor temperature, with 35ºC output (hot water) from the pump [14, 15] (for American standards see [14]). These operating conditions are very different from real life situations: for example, outdoor temperature may vary widely and indoor temperature is always a subjective parameter that may includes a series of values over a day. This indication is far away from a heat pump's actual efficiency where the both indoor and outdoor temperatures are dynamically changing over the time and therefore, consumers may continuously observe that a given heat pump is consuming more electricity than they had been expected or informed. Seasonal Performance Factor *SPF* is a different measure for the efficiency of a heat pump which utilizes both the outdoor temperature and output temperature of heat pump into the calculation (usually this is considered for a particuler period of time: for a season (e.g. winter) or a year) [14, 15]. It is the main indicator of the efficiency of a heat pump relatively with more accuracy. For air to water heat pumps in the marketplace, the Seasonal Performance Factor usually varies between 2 and 5 (e.g., for outdoor temperatures between -5°C and 15°C) [16]. Often it is around 3 (e.g., for ambient temperatures between 0°C and 10°C). Given its strong dependence on the outdoor temperature, SPF can be approximated by a mathematical function of the outdoor temperature T_{od}. For this paper a linear approximation is used (adopted from [10, 17]):

$$SPF(T_{od}) = 7.5 - 0.1(T_{water} - T_{od}) \tag{2}$$

Here T_{water} is the heating system water temperature. The values of the parameters (i.e., the 7.5 and 0.1) in this approximation are in accordance with empirical data from www.liveheatpump.com (see also [10, 18]). For example, based on this function for T_{water} = 50°C it holds:

SPF(10)=3.5 SPF(0)=2.5 SPF(-10)=1.5

The energy usage also depends on the energy demand of the house. To determine the energy demand, equation (3) can be used:

$$tmed(t_1, t_2) = \int_{t_1}^{t_2} \frac{\varepsilon}{24}\big(T_{id}(\mathrm{t}) - T_{od}(\mathrm{t})\big)\mathrm{dt}$$

$$for24hrs\!: tmed_{avg} = \varepsilon\big(T_{id\_avg} - T_{od\_avg}\big) \tag{3}$$

Here $T_{od}(t)$ is the outdoor temperature at time t, $T_{id}(t)$ is the indoor temperature at time t, and T_{id_avg} is the average indoor temperature over a 24 hour, and ε is the energy loss per degree day (summation of individual deviations between the outdoor temperature T_{od} and a given indoor temperature T_{id_avg} in each time step over a 24 hours). The energy usage can be calculated from the energy demand *tmed* and *SPF* by equation (4); see also [10, 18]. By averaging over the day, with T_{od_avg} the average day temperature, this povides the energy usage for that day:

$$day\ energy\ usage = \frac{\varepsilon\left(T_{id\_avg} - T_{od\_avg}\right)}{7.5 - 0.1\left(T_{water} - T_{od\_avg}\right)} \quad (4)$$

In summary, overall the following characteristics are used in this model:

- The characteristics of the environment:
  T_{od_avg} over days
- The characteristics of the house:
  T_{id_avg} over days, ε
- The characteristics of the heating system:
  T_{water}, 0.1, 7.5

All these characteristics are represented in the model (4) for the heating agent. By tuning these model parameters to a specific situation, by simulation for the 365 days of a given year (given by the 365 day temperatures), the agent's year usage can be determined. This is what will be discussed in the next sections. Note that the first two types of characteristics depend on the house itself and on its location on the globe (which determines its environment). The last type of characteristics is more general and independent of these, and therefore can be taken over from other situations using the same technical equipment, for example those described at www.liveheatpump.com.

III. Parameter Estimation for the Usage of a Heat Pump

To determine the annual energy demand of a given house over a certain time period, first it is necessary to determine the constant values of the parameters in the heating agent model expressed in equation (4) (i.e., ε and T_{id_avg}) with dynamic changes of T_{od_avg}. Besides, T_{water} is set at 50°C. For the parameter estimation, three months (mid October 2013 to mid January 2014) of collected empirical data on daily energy usage of the heat pump in the given house (Amsterdam area Netherlands) was used together with the collected average outdoor day temperatures (T_{od_avg}) of each day for that period. In the process of parameter estimation, the sum of squares of residuals was used as the error function to be minimized. A residual is the deviation of the calculated energy usage through the model (4) on particular selected values for the above mentioned parameters, from the actual energy usage of that given day. The goal of this approach is to minimize the above mentioned error (sum of squares of residuals) with appropriate values for the parameters (least square method [19]). To implement the least square method the 'lsqcurvefit' function in MATLAB was used which is specifically recommended for nonlinear curve fitting [19]; a summery of the implementation is in Prog. Code 1.

It was found that the best values for the parameters are:

- $\varepsilon = 3.0160$
- $T_{id\_avg} = 17.6016$°C

Fig. 1 presents two graphs of the heat pump energy usage over average outdoor temperature (part a) and over days (part b) for both the real and predicted (with the mentioned parameter values) energy usage of the heat pump. These graphs clearly show that with the parameter values as determined, the predictions mostly align with the empirical data.

```
% Collected data on energy usage of heat pump for
domestic heating
actualEnergyUsage = [ ... ];
% Collected data of average outdoor temperature
tod_avg = [ ... ];
% To store time in days
day = 1 : 1 : length(actualEnergyUsage);
% Function F is: tmeu = (epsilon*(Tin_avg -
Tod_avg)) / (7.5 - 0.1*(50 - Tod)).
F = @(x,time)(x(1).*(x(2) - tod_avg(day))) ./
    (7.5 - (0.1.*(50.0 - tod_avg(day))));
% Initial parameter values (can be any random
values)
x0 = [3.5, 17];
% To find the best values for parameters which
makes the sum of squares of residuals the
smallest
[x, resnorm, ~, exitflag, output] =
lsqcurvefit(F, x0, day, actualEnergyUsage);
```

Prog. Code 1: MATLAB implementation for parameter estimation

The cumulative absolute error (from the residuals) is 0.2051. For all the different initial parameter value combinations that were tried, Prog. Code 1 always generated the same parameter value estimations, which gave some confidence that the obtained parameter values represent the global minimum. Thus, with these values it will be possible to predict the energy usage for the different days in any annual heating scenario for the given house with a higher degree of assurance. Though the current parameter estimation is based on three months of empirical data, this will be naturally extended in a real application mode. New data can be added at each day and the parameter estimation performed on the more extended data (for this the measurement of T_{water} value also can be periodically checked and used to eliminate the error from variable values of the model). Furthermore, it is clear that depending on the weather seasons (autumn, winter, spring, and summer), energy usage will be significantly different and when considering the annual energy usage it will be benificial to find this as a combination of different seasonal energy usage values, which is confirmed through this model.

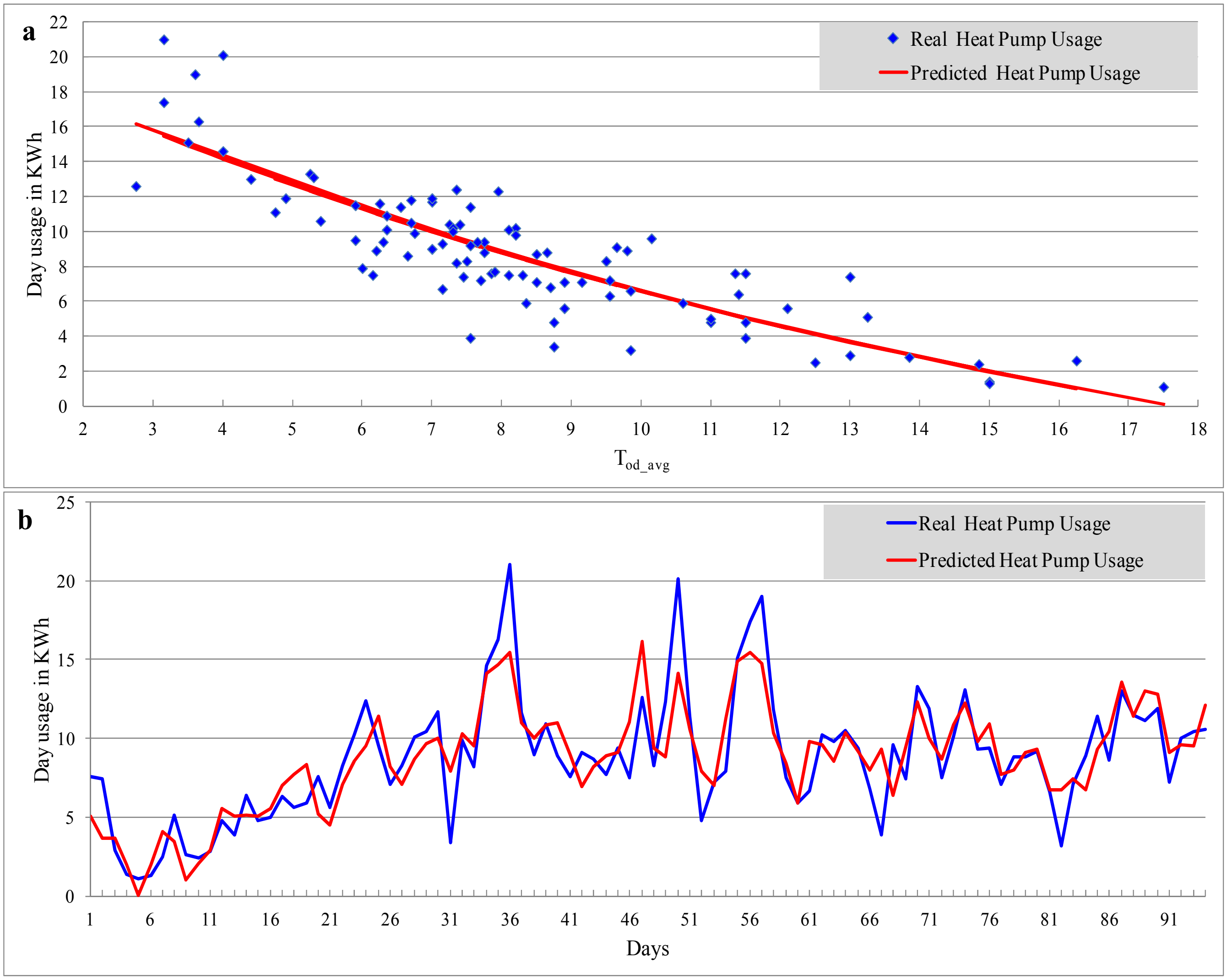

Fig. 1. Energy usage of the heat pump for the given empirical data set: (a) over the average outdoor day temperature, and (b) over the days

IV. Prediction Based on Simulation of Past Years

The heating agent model described by equation (4) in Section 2 with the parameter values found in Section 3 enables to predict the energy usage for heating of any year for which the relevant average daily outdoor temperatures are given. As a first step, the model was used to analyse what the usages would have been for the last ten years. Moreover, it can be analysed whether in this time period a certain trend can be found in the temperature variation that may relate to climate change. If such a trend can be observed from the empirical outdoor temperature data, this will be useful for future prediction as well.

The average outdoor temperature of each day of the years 2004 to 2013 were obtained from the Royal Netherlands Meteorological Institute (KNMI) archives, in particular for the area near Amsterdam (Schiphol). Based on the daily temperature data the yearly energy usage for heating was determined by using the model described by (4). The results are shown in Fig. 2. The average year usage over these 10 years is 2674 kWh. Moreover, it was found that there is an upward trend in these annual usages, so that roughly spoken there is an increase from 2500 kWh in 2004 to 3000 kWh in 2013 (see the straight line in Fig. 2), which gives a nonneglectable difference of 20%.

To analyse the background of the observed trend over the years in usage numbers, also the daily temperatures have been inspected further. A trend in usages most probably reveals a trend in daily temperatures over the years. Indeed this turns out to be the case as is shown in Fig. 3. It was found that there is indeed a decreasing trend of the yearly average day outdoor temperature from about 11.0°C in 2004 to about 10.3°C in 2013. Such a trend may indicate a (local) effect of climate change.

The additional trend information found highlights the necessity of studying the trend of temperature variation more statistically; in the next section this will be worked out for future predictions.

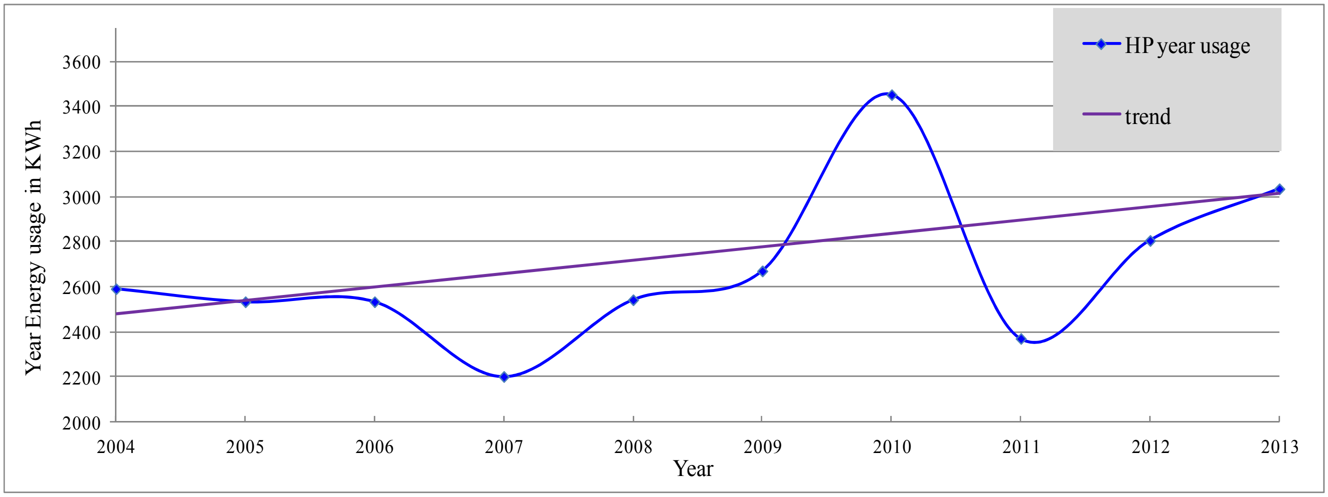

Fig. 2. Average outdoor temperature trend over the years 2004 to 2013

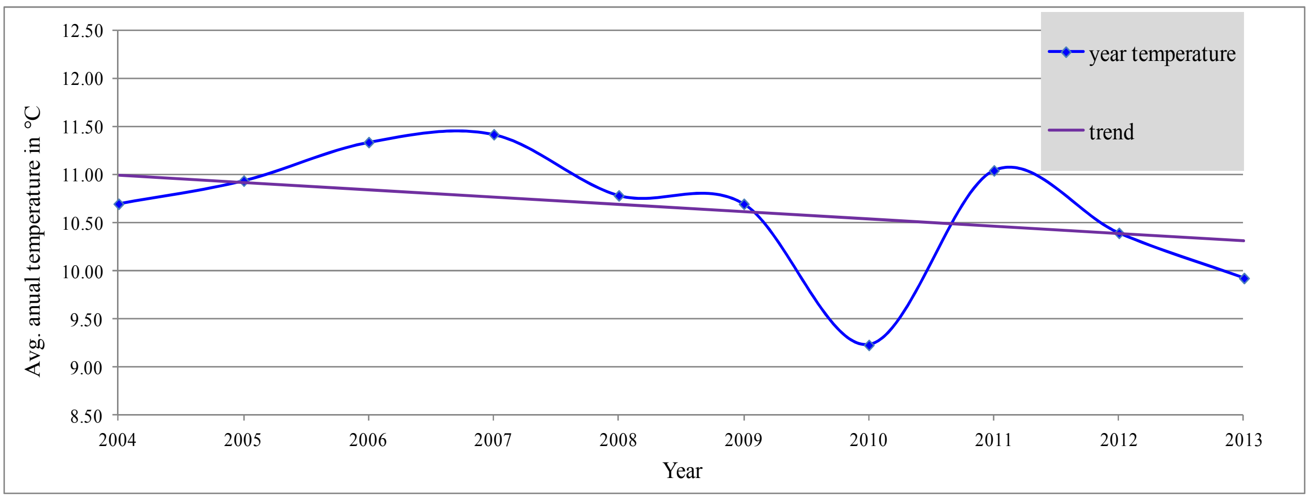

Fig. 3. Average year energy usage for heating and trend over the years 2004 to 2013

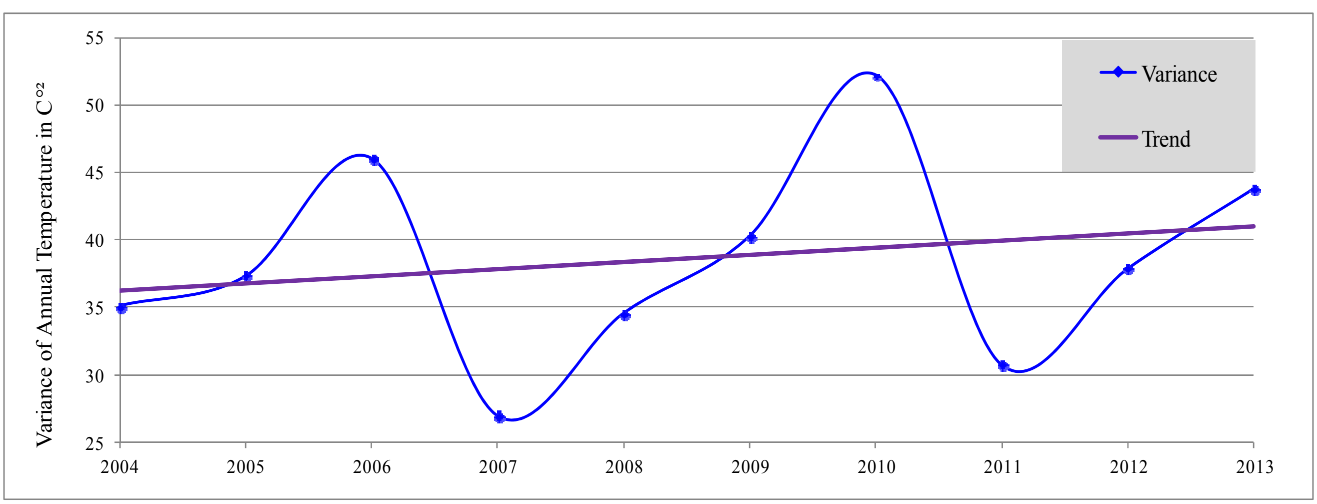

Fig. 4. Variance of daily temperatures over the years 2004 to 2013

V. Prediction Based on Simulation of Future Years

In this section, it will be discussed how the domestic energy usage in the future years (2014 to 2025) can be and actually was predicted by stochastic simulation of the heating agent model and its environment. To do this, first the distribution of daily temperature in previous years is analyzed, and the trend of changes was identified. In the next step, this information is used to predict daily temperatures over the future years and based on that the domestic energy usage in these years is predicted.

A. Analysing Variation in Daily Temperatures in the Last 10 years

As mentioned in Section 4, the average of the daily temperature in past 10 years has an overall decreasing trend. Upon further inspection it was found out that in the same period the variance is following an upward trend. Both effects might be local effects of climate change. Fig. 4 shows the variance of daily temperatures in the past 10 years and its upward trend.

By further analysis of the frequencies of the occurrencies of the daily temperatures during last 10 years (2004 to 2013) it was found out that they can be approximated by a mixture model (cf. [20]) obtained as a weighted average of two Normal distributions $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$:

$$Mixture\ Model = 0.29\ N(16.15, 8.84) + 0.71\ \mathrm{N}(7.72, 30.55) \quad (5)$$

The parameters of the above formula (weight, μ and σ^2 for each normal distribution) were calculated by the Expectation Maximization (EM) algorithm described in [20]. Fig. 5 represents as squares the frequencies of all occurrencies of daily temperatures in the past 10 years, obtained from empirical data. The proposed mixture model (5) is also depicted in this figure as a solid line. It should be noticed that this figure shows the diagram of 365 times the value obtained from (5), as the values of (5) are normalised at total sum 1, and here the sum is 365 days in a year.

B. Simulating Domestic Energy Usage for Future Years

After analyzing the trends for average and variance of daily temperatures, and proposing a combined model for the frequency distribution of the daily temperatures in the previous 10 years, these were used to predict daily temperatures and the domestic energy usages in the future years. To this end, for each year 365 random values were generated from the model providing the distribution of daily temperatures in that year. The used temperature distribution model for each year is like formula (5) (with the same weighs), but the mean and variance of each normal distribution are changed over the years according to the mentioned trends. It is clear that by having the daily temperature for a particular day, the energy usage of that day can be calculated from the formula (4). As a result, the energy usage of a year is estimated by summing up the estimation of energy usages in al of its days. This stochastic simulation was done 1000 times for the future years, from 2014 to 2025. Fig. 6 shows the average and standard deviation of year energy usage according to these 1000 simulations for each year.

VI. Discussion

This paper described an agent-based approach to estimate annual energy usage for heating when using an air to water heat pump. The important feature was that the prediction will be highly sensitive to the characteristics of three basic elements: the environment, the house, and the heating agent. These all have to be taken into account rather than only the given generic Coefficient of Performance (COP) of the heat pump as provided by the manufacturer, and average outdoor temperatures. Given the motivation and trends for net zero houses, such an approach will uplift the predictions of energy demands with more confidence. Therefore, house owners will be able to decide the specifications for other renewable energy

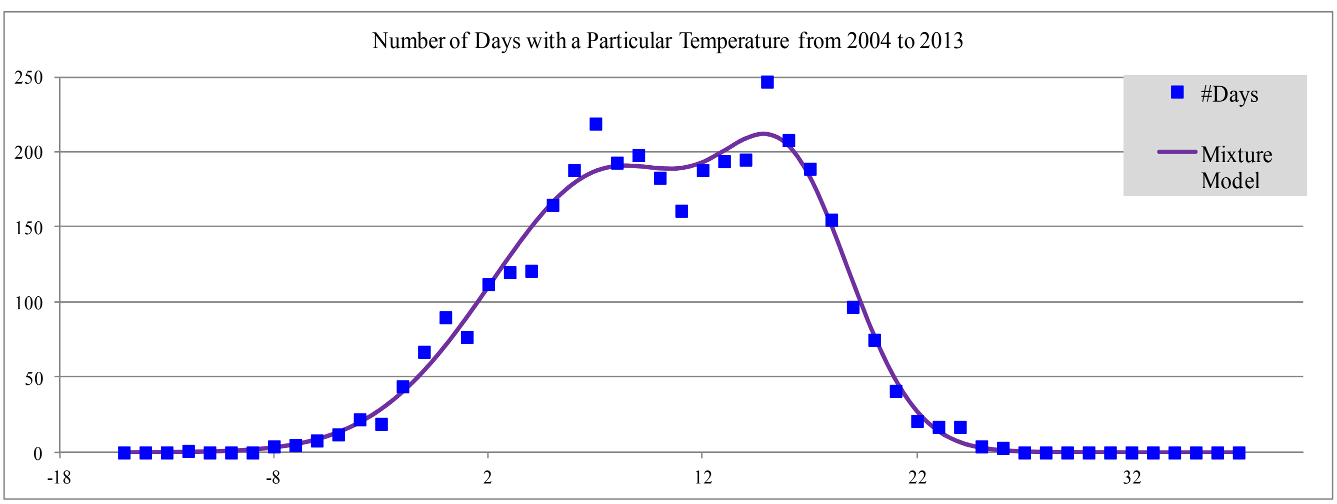

Fig. 5. Graph of the daily temperature distribution for the past 10 years

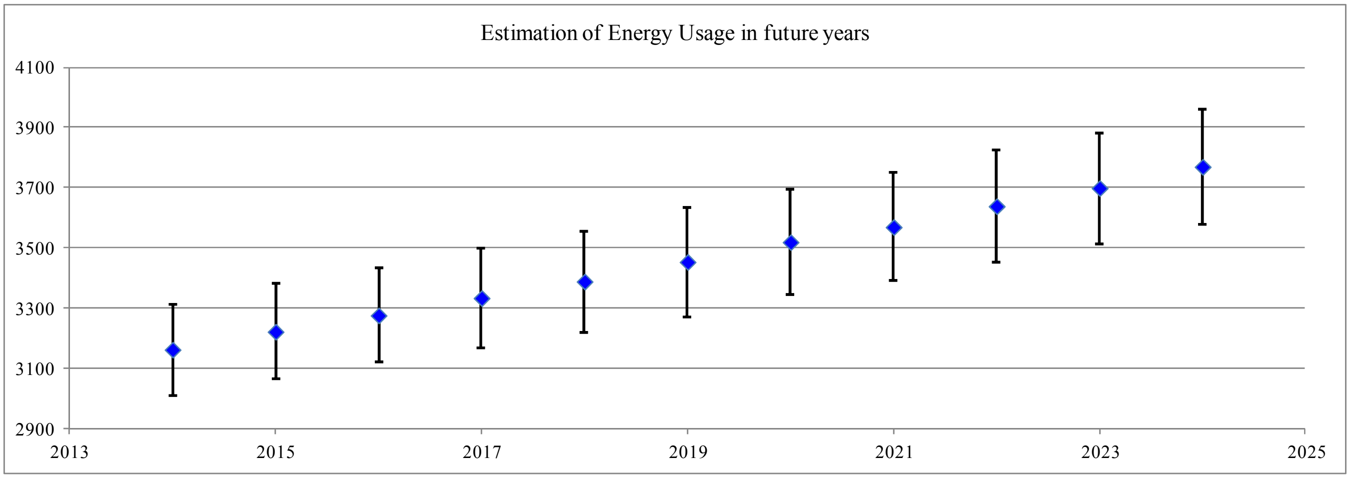

Fig. 6. Overview of simulation results for estimation of energy usage in future years

systems (e.g., solar or wind energy production systems) to obtain a net zero house. To this end, the outcomes of this approach can be combined with other energy consumption processes in the house.

More specifically, for the considered house, besides the heating per year around 1600 kWh is needed for other energy usages such as for the fridge, (dish) washing machines, light, computers, and cooking. From the outcomes in Fig. 6, it can be found that to become net zero with high probability for 2014 the total amount of compensating produced energy in 2014 should be at least 3400 kWh + 1600 kWh = 5000 kWh. For the location of the house in the Netherlands, this can be translated (based on a wellknown rule of thumb) into a photovoltaic solar energy production system of 5000/0.85 = 5900 Wp (Watt peak). When, for example, solar panels are used of 250 Wp, this implies that 24 of such panels are needed. However, when not only for 2014, but also for the years up to 2024 a net zero situation is aimed for, a bit more is needed (assuming that the 1600 kWh will not be reduced over time). In that case, the expected maximal overall energy usage per year is 4000 kWh + 1600 kWh = 5600 kWh. This can be translated into 5600/0.85 = 6600 Wp needed, which can be obtained by 27 solar panels (of 100 cm x 165 cm) of 250 Wp. A more refined calculation might also take into acount on the one hand that solar panels may become slightly less efficient over the years (e.g., by 5 or 10%), but on the other hand, also more advanced types of panels may be considered, for example of 270 Wp.

In a further practical setup based on this approach, three agents and their dynamical environment can be considered: a heat pump based heating agent, a thermostat agent, an energy production agent, and their dynamical environment with its variation in outdoor temperatures and other elements. The heating agent should generate energy for heating each day, whereas the thermostat agent monitors indoor and outdoor temperature of these days in parallel thereby using necessary sensory equipments and access to data sources related to the location of the house. By enabling a communication among these agents and interaction with the environment, the approach described in this paper will enable them to determine parameter values representing characteristics that are particular to that domestic heating energy usage behavior. Furthermore, the thermostat agent will be able to analyse the trends of the outdoor temperatures by access to the relevant data sources specific to that location of the house. As shown in this paper, these agents together will be able to predict the annual energy usage for heating specific to that context, and based on this provide advice to the house owner.

The approach discussed in the previous sections is agent based approach based on an agent specification by mathematical equations. Alternatively, the agent specification could be expressed in a rule-based format as is more common. However, semantically it would be the same agent. One of the reasons for the agent paradigm is the future extention of this work that will include energy usage optimization by allowing a communication between these agents and also with other agents which are necessary with entities essential in energy management (including the behavior aspects of the human agents involved). Energy related systems are naturally complex systems where the logical stability of the system is always far away from equlibirium and it is necessary to engage from situation to situation with enough details of data through a continuouss monitoring, communication and tuning process for a rational optimisation. For example, when considering the temperature settings of a house from evening to morning (for the winter season) for a comfortable stay and sleep, it is a question at what time the heat pump should start and how to control the temperature values over time so that the most economical energy usage with a satisfactory level of comfort is obtained (see [21]). This types of questions can be answered through agent based approaches more easily and proactively. Having a smart household agent based energy management system will not only provide more realistic predictions for annual energy usage but also:

- enable to construct detailed profiles of enery usage of a given household (which includes sufficient information to identify patterns of energy usage that may improved with various adaptation techniques such that energy usage will be minimized)

- promptly identifies problems of devices (e.g. heat pump, soler panel, etc.) and to do the necessary repairs or replacements to fullfil the needs for a net zero house
- extending this system with systems describing neigbouring houses in order to build a coherent system that includes mutual benefits (for example due to sudden energy demand needs it may possible to get the energy from others with low or zero cost, and further to move forward for a small scale wind plant that can be shared).

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